

# Aggregate Supply and The Productivity of Money — A Transaction Cost Approach

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## I. Introduction

In the conventional discussion of the effects of monetary policy, based upon the IS-LM apparatus, an increase in nominal money balances will lower the market rate of interest to the extent that money holders adjust their portfolio in favor of bonds. This in turn will give rise to an increase in investment. It is this argument which has been perhaps most persuasive in linking money to aggregate supply in "Keynesian" theory. This mechanism seems, however, unnecessarily limitative, because while it regards money as an alternative asset affecting the interest rate and thereby investment, it ignores the fact that "money switches real resources from the exchange activity to the production activity." [Classen (1975)].

In this paper we present a general equilibrium model which, by incorporating the concept of transactions costs, includes this direct money-labor nexus. Utilizing this model, we attempt to explain the *ceteris paribus* effect of changes in real money balances on the labor market and aggregate supply. Although the effect of a change in money balances on the labor market through the wealth effect has been an integral part of macroeconomic theory

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[see Pesek and Saving (1967)], our model extends and modifies the standard result by explicitly considering the "direct" impact of changes in money balances on transactions costs and productive activity. Comparative static results are then derived which explicate the relationship among money balances, wage rates, employment, and aggregate supply.

It is a well known result that money as a medium of exchange increases transactions efficiency avoiding the costly search for acceptable terms of trade between buyers and sellers. The effect of this is a reduction of transaction time [Friedman (1959), Bailey (1971), Saving (1971, 1972), Dutton and Gramm (1973)]. It has been further argued that even in a fully monetized economy, money can be labor saving [Sinai and Stokes (1972)], and should be considered as a factor in the firm's production function [Mundell (1971), Bailey (1972), Yoo (1978)]. Dutton and Gramm (1973), in analyzing the behavior of households, have shown that they consider the timesaving role of money so important that the decision on the size of their money balances depends upon the price of time, the wage rate. That is, as the (real) wage rate rises, households which now face a higher opportunity cost of time, will tend to increase their money balances to increase transactions efficiency and reduce transactions time, *ceteris paribus*.

In this paper we extend the Claassen approach to explore the following questions *viz.*: How would economic units allocate the transactions time that is saved by holding larger money balances? Would they use it for production activities or leisure? How would the impact of changes in the money supply, which change real balances, affect the supply of productive labor and the wage rate; and how would this affect aggregate supply? To analyze these questions we utilize the insights of the above mentioned authors to develop a model where the wage rate is an endogenous variable affected by changes in the money supply and *a fortiori* real balances.

To begin, let us modify the traditional aggregate demand and supply model by a dichotomization of commodities into: (1) conventional goods and services and, (2) transactional services. Similarly, labor is partitioned into two categories, *viz.* productive labor and transactional labor. This will facilitate the development of a simple general equilibrium model of the commodity and labor markets. In order to illuminate the impact of the money

supply on the real wage rate, we assume that a change in the money supply is unanticipated by market participants so that the primary effect of the money supply increase (or decreases) is to change the level of real money balances, i.e., any price adjustment occurs only with a lag. In effect we assume that (unanticipated) changes in the nominal money supply translates initially into changes in the real money supply, i.e., prices do not instantaneously adjust.<sup>1</sup>

## II. The Model

To elucidate the interaction among productive labor, leisure, and transactions efforts due to an increase in the money supply, we introduce two categories of time uses: (1) productive labor ( $N_1$ ) and, (2) transactions labor ( $N_2$ ). Both are measured in work-time units, and are supplied by individuals, and demanded by firms. In our aggregate model, we assume for the sake of simplicity that the average economic unit is concomitantly a producer maximizing profit as well as a consumer maximizing utility. Firms will demand  $N_1$  as a production factor, and  $N_2$  (which is financed by whatever is produced by  $N_2$ ) as a transaction factor.<sup>2</sup> Firms also produce two types of commodities: conventional goods and services ( $X_1$ ), and transactional services ( $X_2$ ).  $X_1$  is produced by using capital ( $K$ ) and "productive" labor ( $N_1$ ), while  $X_2$  is produc-

<sup>1</sup> Our position here does not include the classical premise that money is "neutral" or "a veil". In the classical theory all real variables are completely independent of changes in nominal money balances and all nominal variables are "proportional" to the changes in the nominal supply of money. This is unrealistic as an empirical premise. The monetary transmission mechanism cannot be perfect (in the neutrality sense) or automatic, so that we do see "money illusion" in many sectors. In particular, if we consider the government sector whose activity in the bond market is unresponsive to the changed interest rate induced by a change in the supply of money, then there can be a net change in the equilibrium values of real variables, such as real income, real money balances, and real wealth. In particular, real money balances can increase or decrease depending upon the relative speed of adjustment in money and prices. Based upon recent research [e.g. Cagan-Gandolfi (1969), Laidler (1976), and Barro (1977)] and assuming that a change in the supply of money is unanticipated, we take the position that an increase in nominal money balances would increase real balances at least in the short run.

<sup>2</sup> There are two possible ways to use transactions services. First, firms may hire clerical workers who take care of transaction activities; second, firms can concentrate on production and utilize sales and purchasing agents, i.e., middlemen, for transactional activities. In a world of perfect information these services would be of little value. However the need for these middleman services and the specialized information they provide takes on added importance where knowledge is imperfect, as Hayek (1945) has so persuasively argued.

ed by  $N_2$  and indirectly by real balances. It is further assumed that transactions services are needed for trading  $X_1$ , but no services are required for trading  $X_2$ .

### A. The Supply of Commodities

Conventional goods and services, ( $X_1$ ), are produced by utilizing capital ( $K$ ) and labor ( $N_1$ ) according to a Cobb-Douglas production function:

$$(1) \quad X_1^s = a(N_1^d)^\alpha (\bar{K})^{\alpha^*}$$

or simply:

$$X_1^s = a' (N_1^d)^\alpha$$

where  $a' = a\bar{K}^{\alpha^*}$  and superscripts "s" and "d" denote the quantity supplied and demanded, respectively. We assume capital ( $K$ ) is fixed in the short run, i.e.,  $K = \bar{K}$ .  $X_2$ , on the other hand, is produced by  $N_2$  in such a way that there exists a fixed coefficient  $\beta$ ,  $0 < \beta < 1$ , which shows that the production of transactions services is subject to the law of diminishing returns:<sup>2</sup>

$$(2) \quad X_2^s = b(N_2^d)^\beta, \quad 0 < \beta < 1$$

where  $b$  is a fixed constant. Since producers of  $X_1$  are profit maximizers, they will purchase labor input  $N_1$  up to the point where its marginal product equals the real wage rate ( $w$ ). Thus,

$$(3) \quad w = a\alpha(N_1^d)^{\alpha-1} \bar{K}^{\alpha^*} \\ = a' \alpha(N_1^d)^{\alpha-1}, \quad a' = a\bar{K}^{\alpha^*}$$

and the demand for productive labor  $N_1$  is

$$(4) \quad N_1^d = (Aw)^{\frac{1}{\alpha-1}}, \quad \partial N_1^d / \partial w < 0$$

where  $A = (1/a'\alpha)$ . Before proceeding, the demand for transactions efforts,  $N_2$ , needs further explication. Since it was assumed that labor is unnecessary for trading transactions services, the amount of goods and services requiring transactions services will consist of total goods and services ( $X_1^d$ ) plus total investment (I)

in any period. Now let " $\tau$ " denote the amount of transaction services needed for one unit of goods or services. The " $\tau$ " is the *nexus* between the labor market and monetary policy. That is, as real money balances, ( $m$ ), increase, the amount of transactions effort needed for one unit of output will decrease due to the fact that real balances and transactions labor are substitutes. As Claassen (1975) has argued, changes in real money balances will release "further amounts of labor,... for the use in the production process" (p. 432). Thus  $\tau$  can be written as a function of  $m$ :

$$(5) \quad \tau = \tau(m), \quad \tau_m < 0 \quad \tau_{mm} < 0$$

and

$$m = M/P$$

where  $M$  denotes nominal money balances and  $P$  the price level. Since both  $N_1$  and  $N_2$  are evaluated in work-time units, the demand for transactions time will be the total value of transaction services divided by the price of time, or the prevailing wage rate:

$$(6) \quad N_2^d = \frac{\tau(m) (X_1^d + I)}{w}$$

where  $I$  denotes the quantity of producers' goods purchased. Thus,  $N_2^d$  is a derived demand from the demand for  $X_1$  and  $I$ .

### B. The Demand for Commodities and Leisure

In the output market we posit a simple, linear aggregate consumption function similar in form to the absolute income consumption function.<sup>3</sup>

$$(7) \quad X_1^d = cX_1^s$$

<sup>3</sup> Two points need to be made explicit. First, a minor point: the constant term in the consumption function was removed for simplicity. This does not, however, affect our final results. Second, and more importantly, it is apparent that we have not included real money balances in the consumption function. The reason for the omission is that we are concerned with the *ceteris paribus* influence of real money balances on transaction time, the wage rate and aggregate supply. Furthermore, as can be observed in the appendix, inclusion of real money balances in the consumption function will generate ambiguous comparative static results since we are unable (without reliable empirical estimates) to measure the relative effect of a change in money balances on aggregate supply and demand. See the appendix for the comparative static results with real money balances in the consumption function.

Investment (I) is assumed to be a function of the interest rate (r):

$$I = I(r)$$

The demand for transactions services ( $X_2^d$ ) should be equal to the amount of services required per unit of output times total transactions:

$$(8) \quad X_2^d = \tau(m) \cdot (X_1^d + I) = N_2^d \cdot w$$

Since leisure can be assumed to be something one "purchases" in the labor market, the demand function for leisure will be:

$$(9) \quad L = L(w, m) \quad L_w < 0 \quad L_m > 0$$

### C. The Equilibrium Conditions

Given the above relations, the equilibrium conditions for the commodity and labor markets can now be stated. First, in the commodity market, consumption plus investment during the given period should equal  $X_1$  plus  $X_2$ , i.e., the goods and services produced plus the transactions services necessary for trading  $X_1$  during the same period:

$$(10a) \quad X_1^d + I = X_1^s$$

$$(10b) \quad X_2^d = X_2^s,$$

and adding both sides we obtain:

$$(10c) \quad X_1^d + X_2^d + I = X_1^s + X_2^s$$

Substituting (7) and (8) into the left hand side of (10c), and substituting (2), (3), (4) and (6) into the right hand side we get:

$$(11) \quad cX_1^s + \tau(X_1^d + I) + I = a'(N_1^d)^\alpha + b(N_1^d)^\beta$$

Utilizing the assumption that firms will employ transactions-services up to the point where the marginal product of the transactions-services is equal to the wage rate we have:

$$(12) \quad b\beta(N_2^d)^{\beta-1} = w$$

or

$$b(N_2^d)^\beta = \frac{w}{\beta}(N_2^d)$$

Substituting (3), (6), and (7) into (12) we obtain:

$$\begin{aligned} (13) \quad b(N_2^d)^\beta &= \frac{w}{\beta}(N_2^d) \\ &= \frac{w}{\beta} \left[ \frac{\tau(X_1^d + I)}{w} \right] \\ &= \frac{1}{\beta} \left[ \tau \left\{ ca' (Aw)^{\frac{\alpha}{\alpha-1}} + 1 \right\} \right] \end{aligned}$$

Now equation (4) states that:

$$N_1^d = (Aw)^{\frac{\alpha}{\alpha-1}}$$

So by substituting (4) and (13) into (11), we obtain the output market equilibrium:

$$\begin{aligned} (14) \quad a' (Aw)^{\frac{\alpha}{\alpha-1}} \left\{ (c-1) + c \tau \left(1 - \frac{1}{\beta}\right) \right\} \\ + \left\{ 1 + \tau \left(1 - \frac{1}{\beta}\right) \right\} = 0 \end{aligned}$$

Second, in the labor market, total time demanded should be equal to total time available for either productive labor or transactions labor:

$$(15) \quad N_1^d + N_2^d = N^s$$

Substituting (1a), (4), (6), and (7), the left side of (15) becomes:

$$(16) \quad N_1^d + N_2^d = (Aw)^{\frac{\alpha}{\alpha-1}} + \frac{1}{w} \tau(m) \{ X_1^d + I \}$$

While, the right hand side of (15) can be written as:

$$(17) \quad N^S = T - L(w, m) \quad L_w < 0 \quad L_m > 0$$

where  $T$  denotes the total amount of time available, and  $L$ , leisure. The equilibrium condition in the labor market therefore is:

$$(18) \quad T - L(w, m) - (Aw)^{\frac{1}{\alpha-1}} - \frac{\tau(m) \{X_1^d + I\}}{w} = 0$$

Thus equation (14) and (18) constitute a two market general equilibrium system with  $w$  and  $r$  as endogenous variables. Assuming that Walras' law holds we do not have to consider the money market. [Cagan (1958), Tucker (1971)].

#### *D. The Comparative Statics of the Model*

To demonstrate the effect of a change in the money supply on the real wage rate through the transactions-time-saving mechanism we totally differentiate (14) and (18) with respect to  $w$ ,  $r$ , and  $m$  where  $m = M/p$  and  $P$  is assumed exogenous.<sup>4</sup>

From (14):

$$(19) \quad a' \left( \frac{\alpha}{\alpha-1} \right) (Aw)^{\frac{1}{\alpha-1}} \left[ (c-1) + c \tau \left( 1 - \frac{1}{\beta} \right) \right] dw \\ + \left[ 1 + \tau \left( 1 - \frac{1}{\beta} \right) \right] I_r dr \\ + \left[ a' (Aw)^{\frac{\alpha}{\alpha-1}} c \tau_m \left( \frac{1}{p} \right) \left( 1 - \frac{1}{\beta} \right) \right]$$

<sup>4</sup> Given our assumption that changes in the nominal money supply ( $M$ ) are unanticipated, the price level,  $P$ , will be insensitive to money supply changes in the "short run" so that changes in the nominal money supply will equal changes in the real money supply. See footnote 1.



$$+ I \tau_m \left( \frac{1}{p} \right) \left( 1 - \frac{1}{\beta} \right) \Big] dM = 0$$

From (18):

$$(20) \quad \left[ \frac{1}{A^{\alpha-1}} \frac{1}{(\alpha-1)w^{\alpha-1}} + \frac{1}{w^2} \left\{ \tau_c a' A w^{\frac{\alpha}{\alpha-1}} \frac{1}{(\alpha-1)} - I \right\} \right. \\ \left. + L_w \right] dw + [\tau I_r] dr \\ + \left[ \frac{1}{w} (X_1^d + I) \tau_m \left( \frac{1}{p} \right) + L_m \left( \frac{1}{p} \right) \right] dM = 0$$

In matrix notation, let  $a_{11}$ ,  $a_{12}$ ,  $a_{13}$ , and  $a_{21}$ ,  $a_{22}$ ,  $a_{23}$  be the coefficients of  $dw$ ,  $dr$ , and  $dM$  in (19) and (20), respectively.

By inspection, it is obvious that:

$$a_{11} > 0 \quad a_{12} < 0 \quad a_{13} > 0$$

$$a_{21} > 0 \quad a_{22} > 0 \quad a_{23} < 0^5$$

Summarizing (19) and (20):

$$(21) \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dw/dM \\ dr/dM \end{bmatrix} = \begin{bmatrix} -a_{13} \\ -a_{23} \end{bmatrix}$$

5 We know that the third term of  $a_{23}$  is positive since it represents an increase in leisure from transactions time saved due to an increase in  $M$ . The first and second terms are negative and their sum is the total amount of transaction time saved. Since the latter should be greater than the former it follows that  $a_{23}$  should be negative.

By Cramer's Rule:

$$(22) \quad \frac{dw}{dM} = \frac{\begin{vmatrix} -a_{13} & a_{12} \\ -a_{23} & a_{22} \end{vmatrix}}{|A|} = \frac{(-a_{13} \cdot a_{22} + a_{12} \cdot a_{23})}{|A|} < 0$$

(23)

$$\frac{dr}{dM} = \frac{\begin{vmatrix} a_{11} & -a_{13} \\ a_{21} & -a_{23} \end{vmatrix}}{|A|} = \frac{(-a_{11} \cdot a_{23} + a_{13} \cdot a_{21})}{|A|} < 0$$

where  $|A|$  is the determinant of the coefficient matrix in (21), which is unequivocally negative.<sup>6</sup>

### III. Interpretation

The result obtained in equation (22), i.e.,  $\frac{dw}{dM} < 0$ , can be explained as follows. This negative sign implies that as (real) money balances increase, *ceteris paribus*, the equilibrium wage rate will fall. This effect can be interpreted as follows: first, productive and transaction labor *taken together* will decrease as real money balances increase:

<sup>6</sup> Although the sign of (23) is ambiguous we know that in a general equilibrium model with price rigidity the relationship between  $M$  and  $r$  will be negative. See Patinkin (1965), pp. 236-244.

$$(17) \quad N_1 + N_2 = N^s = T - L(w, m)$$

$$L_w < 0 \quad L_m > 0$$

Second, from equation (6) we know that the demand for transactions labor,  $N_2^d$ , will decrease because  $\partial \tau / \partial m < 0$ . However,  $N_1^d$  must remain unchanged since its demand is determined by its marginal product as shown in equation (4). It follows therefore from equation (22) that the shift in  $N_2^d$  must have been greater than the shift in  $N^s$ , causing a fall in the equilibrium real wage rate.

In Figure I this is shown graphically. The initial equilibrium in the labor market occurs at point A, i.e., the intersection of  $N^s$  and  $N^d$ . At this point the equilibrium wage is  $w$ , and the equilibrium values for productive labor, transaction labor and leisure are  $ON_1^d$ ,  $N_1^d N^e$  and  $N^e T$ , respectively. Since  $T$  indicates the total amount of time available,  $N^s$  becomes vertical at  $TT$ . After an increase in real money balances, the labor supply curve will shift from  $N^s$  to  $N^{s'}$ . But the increase in real money balances decreases the demand for  $N_2$ , causing  $N^d$  to shift down to  $N^{d'}$  with a decrease in the wage rate. (Since we have assumed  $N_1^d$  is solely a function of its marginal product and therefore independent of  $m$ , it remains unchanged). The new equilibrium will occur at point B with a wage of  $w'$ . The new equilibrium values of productive labor, transactions labor and leisure are  $ON_1^{d'}$ ,  $N_1^{d'}$   $N^{e'}$  and  $N^{e'} T$ , respectively. At the new equilibrium the quantity of productive labor and leisure demanded has increased, while the demand for transactions labor has decreased., i.e., labor has been transferred from transactions services to production. In Figure II, this increase in  $N_1^d$  is depicted in the lowest panel with a larger real money balance (curve  $X_1^{s'}$ ). The conventional labor input is  $N$ , but the net change in productive labor is actually  $(N_1^{d'} - N_1^d)$ .

This result has interesting economic implications. At the now lower wage rate ( $w'$ ), firms will demand more productive labor [equation (1)], i.e., there has occurred, *ceteris paribus*, an increase in aggregate supply brought about by the change in real money balances. From equations (1a) and (7), the demand for com-

Figure I

THE EFFECT TO AN INCREASE IN REAL MONEY  
BALANCES ON AGGREGATE SUPPLY

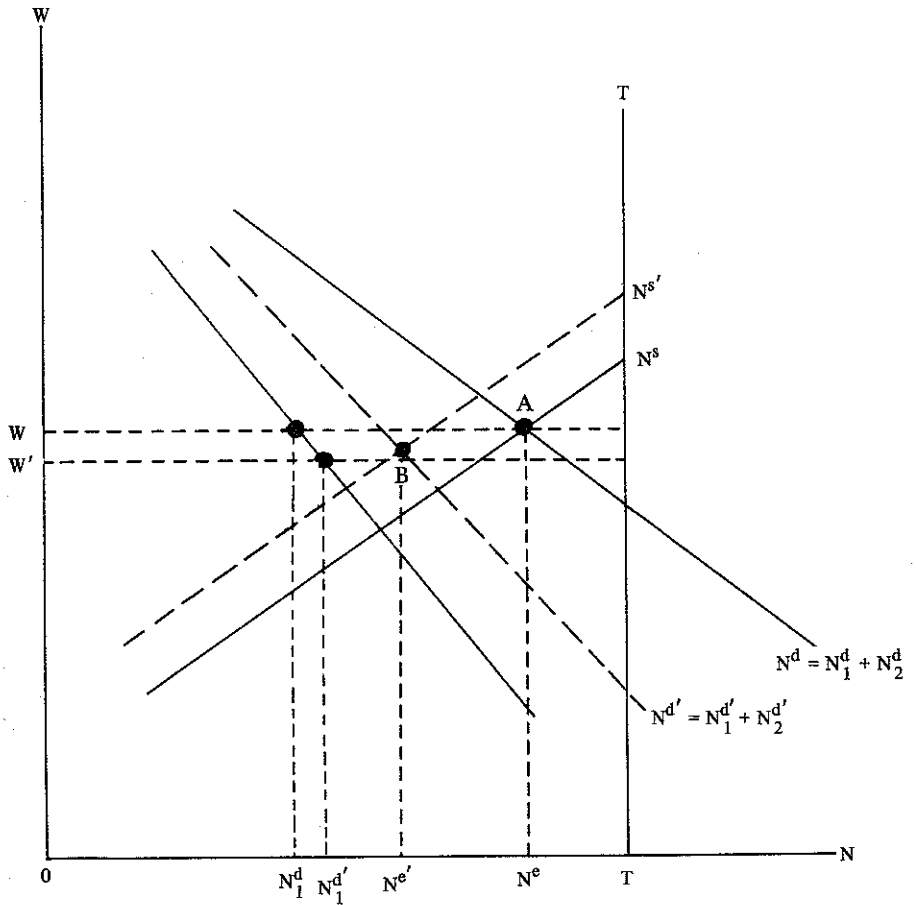
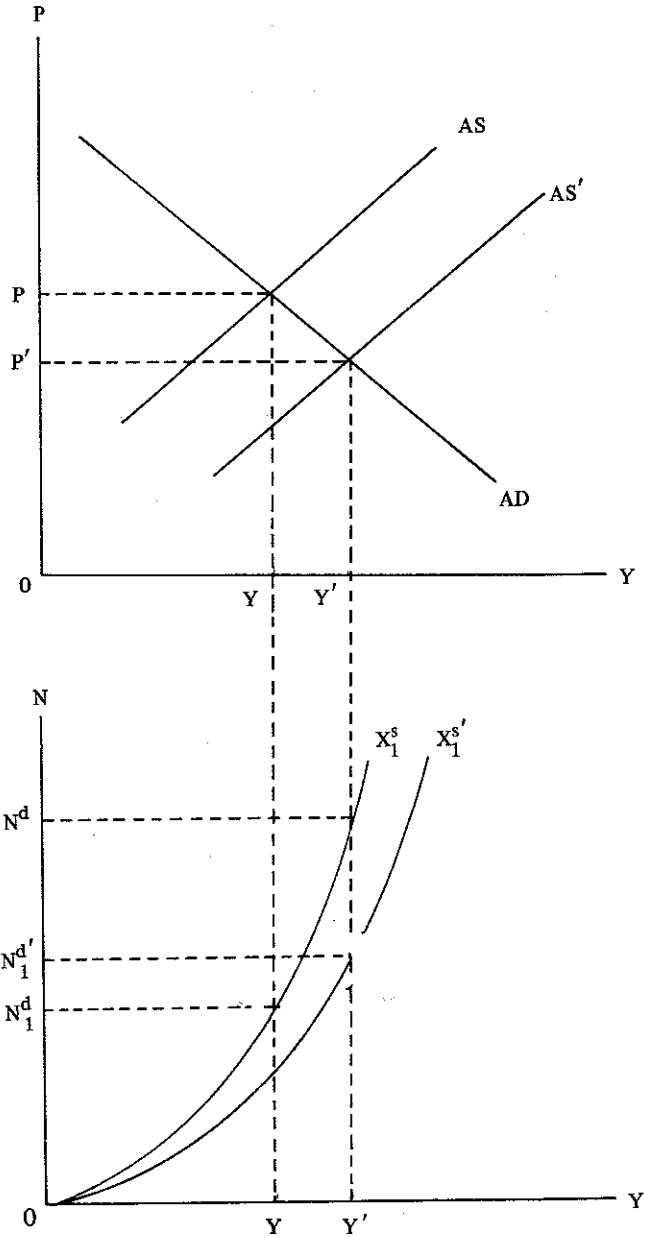


Figure II

CHANGES IN THE LABOR MARKET AFTER REAL MONEY BALANCES INCREASE



modities will also increase and the commodity market will reach equilibrium.

The effect of unanticipated changes in the money supply on aggregate supply can now be readily seen. If, for example, the monetary authorities suddenly decrease the money supply or decrease the growth rate of the money supply, the demand for transactions services may increase. This will cause, *ceteris paribus*, a rise in the wage rate while the employment level will be falling. In other words, because of the increase in transactions time and services required, an overly contractionary monetary policy will cause the wage rate to rise in equilibrium instead of fall while actual employment is decreasing, causing aggregate supply to decrease. Of course for unanticipated increases in the money supply, the opposite result will obtain, *mutatis mutandis*.

#### IV. Conclusion

By dichotomizing the labor and commodity markets between conventional labor ( $N_1$ ), conventional goods ( $X_1$ ) and transactions labor ( $N_2$ ), transactions services ( $X_2$ ) and introducing explicitly the role of the money balances and transactions services into the firms production function [Bailey (1972), Claassen (1975), Mundell (1971), Yoo (1978)], we have obtained results about the relations between monetary growth, the wage rate and aggregate supply which differ from standard macroeconomic models [e.g. Pesek and Saving (1967)].

Assuming that both the wage of transactional labor and the opportunity cost of leisure equal the wage rate of productive labor, we have determined that money's *ceteris paribus* contribution (in the short run) to the equilibrium quantity of productive labor and to changes in aggregate supply is unambiguously positive.

## Appendix

In this appendix we present the result of including real money balances in the consumption function. For the sake of brevity, the modified equations are presented with the comparative static results. All terms have the same interpretation as given in the main body of the paper with only new terms being defined. Equations (1) – (6) will remain unchanged while all modified equations are signified by equation numbers with an asterisk.

$$(7^*) \quad X_1^d = cx_1^S + gm$$

where  $g$  is the marginal real balance effect.

$$(8) \quad X_2^d = \tau(m) (x_1^d + I) = N_2^d \cdot w$$

$$(9) \quad L = L(w, m), L_w < 0, L_m > 0$$

$$(10) \quad X_1^d + X_2^d + I = X_1^S + X_2^S$$

$$(11^*) \quad cx_1^S + gm + \tau(x_1^d + gm + I) = a'(N_1^d)^\alpha + b(N_2^d)^\beta$$

$$(12) \quad N_1^d = (Aw)^{\frac{1}{\alpha-1}}$$

$$(13^*) \quad b(N_2^d)^\beta = \frac{w}{\beta} (N_2^d) \\ = \frac{w}{\beta} \left[ \frac{\tau(x_1^d + gm + I)}{w} \right] \\ = \frac{1}{\beta} \left[ \tau \left\{ ca' (Aw)^{\frac{\alpha}{\alpha-1}} + gm + I \right\} \right]$$

substituting (12) and (13a) into (11a):

$$(14^*) \quad a' (Aw)^{\frac{\alpha}{\alpha-1}} \left\{ (c-1) + c\tau \left(1 - \frac{1}{\beta}\right) \right\} + \left\{ \left(1 + \tau - \frac{1}{\beta}\right) (gm + I) \right\} = 0$$

$$(15) \quad N_1^d + N_2^d = N^S$$

substituting (1), (5), (6), into (7\*), (15) becomes:

$$(16^*) \quad N_1^d + N_2^d = (Aw)^{\frac{1}{\alpha-1}} + \frac{\tau(m) \{ca' (N_1^d)^\alpha + gm + I\}}{w}$$

$$(17) \quad N^S = T - L(w, m)$$

Labor market equilibrium:

$$(18^*) \quad T - L(w, m) - (Aw)^{\frac{1}{\alpha-1}} - \tau(m) \left\{ ca' (N_1^d)^\alpha + gm + I \right\} = 0$$

$$a' \left(\frac{\alpha}{\alpha-1}\right) (Aw)^{\frac{1}{\alpha-1}} A \left\{ (c-1) + \tau c \left(1 - \frac{1}{\beta}\right) \right\} dw$$

$$+ \left[ -\frac{M}{P^2} g \left\{ 1 + \tau \left(1 - \frac{1}{\beta}\right) \right\} - \left(\frac{M}{P^2}\right) \tau_m \right. \\ \left. \left\{ ca' (Aw)^{\frac{\alpha}{\alpha-1}} + gm + I \right\} \left(1 - \frac{1}{\beta}\right) \right] dp$$



$$\begin{aligned}
 (19^*) \quad & + \left[ \frac{m}{p} \left(1 - \frac{1}{\beta}\right) \left\{ ca' (AW)^{\frac{\alpha}{\alpha-1}} + gm + I \right\} \right. \\
 & \left. + \left\{ 1 + \tau \left(1 - \frac{1}{\beta}\right) \right\} \frac{g}{p} \right] dm = 0 \\
 & \left[ -L_w - A \left(\frac{1}{\alpha-1}\right) (AW)^{\frac{2\alpha}{\alpha-1}} - \left(\frac{1}{w^2}\right) ca' \left(\frac{\alpha}{\alpha-1}\right) \right. \\
 & \left. (AW)^{\frac{1}{\alpha-1}} \right] dw \\
 (20^*) \quad & + \left[ -L_m \left(-\frac{M}{p^2}\right) - \left(\frac{1}{w}\right) \left(-\frac{M}{p^2}\right) \left\{ \tau_m (ca' (AW)^{\frac{\alpha}{\alpha-1}} \right. \right. \\
 & \left. \left. + gm + I) + g\tau \right\} \right] dp + \left[ -L_m \left(\frac{1}{p}\right) - \left(\frac{1}{w}\right) \left(\frac{1}{p}\right) \right. \\
 & \left. \left\{ \tau_m (ca' (AW)^{\frac{\alpha}{\alpha-1}} + gm + I) + g\tau \right\} \right] dm = 0
 \end{aligned}$$

Let us use the shorthand notations for the coefficients of (19\*) and (20\*). Let  $a_{11}$ ,  $a_{12}$ , and  $a_{23}$  be that of  $dw$ ,  $dr$ , and  $dm$  in (20\*), respectively. It is obvious that:

$$a_{11} > 0, \quad a_{12} < 0, \quad a_{13} > 0, \quad a_{21} > 0^1$$

However, the signs of  $a_{13}$ ,  $a_{22}$  and  $a_{23}$  are not intuitively obvious. Under some assumptions, however, the signs of  $a_{22}$  and  $a_{23}$  can be determined. Looking at  $a_{22}$  and  $a_{23}$  in equation (20\*), one can find two conflicting terms in the brackets of the second terms;

1 To determine the signs of  $a_{12}$  and  $a_{13}$ , we used the assumption that in the neighborhood of equilibrium, the transactions service demand ( $X_2^d$ ) is approximately a linear function of ( $N_2^d$ ) so that  $\beta$  is very close to unity.

$$\tau_m \left[ ca' (Aw)^{\frac{\alpha}{\alpha-1}} + gm + I \right]$$

which is negative and

$g\tau$  which is positive. The first term indicates a decrease (increase) in transactions time due to an increase (decrease) in real money balances, and the second part indicates an increase (decrease) in consumptive transactions time due to an increase (decrease) in real money balances. Let us call the first term a *transaction saving effect* and the second a *real balance effect*. It is not obvious which of these two effects dominates. However, since our purpose in this study is to test the conformity of our extended model with the conventional macro theory, where the transaction saving effects are not considered at all, we may assume that the transactions saving effects are smaller than or equal, at the most, to the real balance effects.<sup>2</sup> Thus,

$$(21^*) \left| \tau_m \left[ ca' (Aw)^{\frac{\alpha}{\alpha-1}} + gm + I \right] \right| \leq g\tau$$

Using condition (21\*), we may determine the signs of  $a_{22}$  and  $a_{23}$  such as

$$a_{22} > 0, \quad a_{23} < 0.$$

Summarizing (19\*) and (20\*), we get

$$(22^*) \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dw/dm \\ dp/dm \end{bmatrix} = \begin{bmatrix} -a_{13} \\ -a_{23} \end{bmatrix}$$

Using Cramer's Rule,  $dw$  and  $dm$  are obtained:

<sup>2</sup> If we adopt the assumption that the transactions saving effects dominate the real balance effects, the stability conditions in the system are not clearly satisfied, namely the sign of the 2x2 coefficient determinant is indeterminate.

$$(23^*) \quad \frac{dw}{dm} = \frac{\begin{vmatrix} -a_{13} & a_{12} \\ -a_{23} & a_{22} \end{vmatrix}}{|A|} = \frac{-a_{13} a_{22} + a_{12} a_{23}}{|A|} = ?$$

$$(24^*) \quad \frac{dp}{dm} = \frac{\begin{vmatrix} a_{11} & -a_{13} \\ a_{21} & -a_{23} \end{vmatrix}}{|A|} = \frac{-a_{11} a_{23} + a_{13} a_{21}}{|A|} > 0$$

where  $|A|$  is the determinant of the coefficient matrix in (22\*), which is unequivocally positive.

Considering equations (23\*) and (24\*), the sign of (23\*) is ambiguous, but that of (24\*) is unambiguously positive. The effects of an increase in nominal money balances on the equilibrium real wage rate is not readily apparent, but expansionary monetary policy will be inflationary according to (24\*).

### References

- Bailey, M., *National Income and Price Level*, New York, 1972.
- Barro, R., "Unanticipated Money Growth and Unemployment in the U.S.," *American Economic Review*, March 1977.
- Cagan, P. and Gandolfi, Arthur, "The Lag in Monetary Policy as Implied by the Time Pattern of Monetary Effects on Interest Rates," *American Economic Review*, May 1969.
- Claasen, E., "On the Indirect Productivity of Money," *Journal of Political Economy*, April 1975.

- Friedman, M., "The Demand for Money: Some Theoretical and Empirical Results," *Journal of Political Economy*, August 1959.
- Gramm, W.P. and Dutton, D., "Transactions Costs, the Wage Rate, and the Demand for Money," *American Economic Review*, September 1973.
- Hayek, F.A., "The Use of Knowledge in Society," *American Economic Review*, September 1945.
- Laidler, D., "Inflation in Britain: A Monetarist Perspective," *American Economic Review*, September 1976.
- Mundell, R., *Monetary Theory*, Pacific Palisades, California: Goodyear, 1971.
- Pesek, B.P. and Saving, T.R., *Money, Wealth, and Economic Theory*, Macmillan, New York, 1967.
- Saving, T., "Transactions Costs and the Firm's Demand for Money," *Journal of Money, Credit, and Banking*, May 1972.
- , "Transaction Cost and the Demand for Money," *American Economic Review*, June 1971.
- Sinai, A. and Stokes, H., "Real Money Balances: An Omitted Variable from the Production Function," *Review of Economics and Statistics*, 54, August 1972.
- Yoo, Jang H., "Real Money Balances as Factor Saving Media," *Atlantic Economic Journal*, July 1978.