The Time Series Properties of the Spot Exchange Rate:

A Multiple Input Transfer Function

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The goal of this paper is to investigate the time series properties of exchange rates in light of the asset market theory of exchange rate determination.[1] This study departs from previous Box-Jenkins time series studies of exchange rates in that a formal multipe input transfer function (MITF) is developed and estimated. The model itself is based on a simple asset market view of exchange rate determination.

Investigations of the time series properties of exchange rates have been reported by Levich (1977), Giddy and Dufey (1975), and Cornell and Dietrich (1978).[2] Levich studied the time series properties of weekly percentage changes in spot rates over the period from 1962 until 1975. Giddy and Dufey examined daily exchange rate data for three countries during two floating periods: post World War I and the early 1970's up to 1974. Using daily data from 1973 until 1975, Cornell and Dietrich examined the effici-

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1 For a useful introduction to the asset market theory of exchange rate determination, see the collection of papers in the *Scandinavian Journal of Economic*, vol. 78, no. 2 (1976), pp. 133-142, the papers collected in Frenkel and Johnson (1978), Dornbush (1980), and Healy (1980) pp. 7-50.

2 For a succinct survey of recent time series studies of exchange rates see Levich (1977).

ency of foreign exchange markets in six currencies relative to the U.S. dollar. However, their estimation procedure entertained lags up to only eight days, although, as in the other two studies cited, univariate time series models were developed and estimated. Yet another study using daily rates from 1973 to 1975 for nine countries was reported by Dooley and Shafer (1976). Formal time series models, however, were not estimated.

The analysis contained in this paper contributes to the previous work of these authors in using univariate time series models for daily exchange rates during the years 1975 and 1976. More importantly, this study advances from univariate analysis to develop and estimate a multiple input transfer function to analyze exchange rate movements.[3]

The thrust of the paper is to study the interrelationship through time among the spot exchange rate, the forward rate, and the domestic and foreign interest rates. This is an essentially empirical investigation on the asset market determination of exchange rates in the *very,* very short run. Daily exchange rate data and interest rate data are examined to test whether previous fluctuations in interest rates and the forward rate have a significant effect on the current spot rate.

Section I develops the model of exchange rate determination in the asset market on a day-to-day basis. A simple extension of the interest arbitrage equation is presented which emphasizes the role of a fixed supply of arbitrage funds available on any given day. The role that time series techniques will play in the empirical work is indicated. Section II presents the empirical representation of the individual variables as stochastic processes of autoregressive, moving average (ARIMA) form. Examination of the ARIMA structure for each variable is done in detail and the random walk hypothesis for daily exchange rates is tested. Likelihood ratio tests are employed to select the "best" ARIMA models. Section III estimates the multiple input transfer function (MITF) which dynamically relates the spot exchange rate to the forward exchange rate, the domestic interest rate, and the foreign interest rate. The impulse response patterns are derived. A discussion of causality and an examination of predictive performance of the model is included. Section IV contains concluding remarks.

3 This is the first study we are aware of that employs a MITF in exchange rate analysis.

SECTION I

Assume that there is perfect capital mobility between countries, that the financial assets of the home country are perfect substitutes with those of the rest of the world, and that coupon rates are fixed over the duration of the bonds. Given perfect capital markets and the absence of transaction costs, the familiar arbitrage equation follows:

$$\frac{(1 + r_t^d)}{(1 + r_t^*)} = \frac{e_t^f}{e_t^s} \tag{1}$$

where r_t^d is the domestic interest rate, r_t^* is the foreign interest rate, and e_t^s and e_t^f are the spot and forward exchange rates respectively, expressed as units of home currency per unit of foreign currency. All variables are evaluated at time t. When condition (1) is not met, the bond market is in disequilibrium as investors try to move into the asset with the higher rate of return. Arbitrage profits are earned until the rates of return are equalized.

To model exchange rate movements in a daily framework, the assumption will be made that the supply of investment funds is fixed over the course of a day or two.[4] This assumption leads to a modified version of the arbitrage equation:

$$\frac{1 + r_t^d}{1 + r_t^*} = \frac{e_t^f}{e_t^s} - \alpha \tag{2}$$

The usual approximation to (3) is given by:

$$r_t^d = r_t^* + \frac{e_t^f - e_t^s}{e_t^s} - \alpha \tag{3}$$

4 Branson (1969) and Einzig (1961) cite instances where departures from the strict interest arbitrage equation can be explained by supply elasticities. The basic concept is that there exists a point after which additional arbitrage funds are available only at increasing marginal cost. The marginal return from arbitrage transactions might be less than the marginal cost of acquiring the necessary funds. In this paper we emphasize another implication of a less than perfectly elastic supply of arbitrage funds. Given that funds are in limited supply, arbitrage opportunities might be deliberately passed up on a given day due to portfolio considerations concerning the expectations of future arbitrage opportunities and the variance of these expectations.

In (2) and (3) α is the "arbitrage opportunity factor." This concept needs amplification.

Consider the investor with a fixed amount of funds available to him in the short run (a few days, say). He allocates these funds between holding money and holding foreign and domestic securities. Assume that these securities are identical in all respects (risk class, maturity, etc.) except for the currency of their denomination. In choosing his mix of domestic and foreign bonds, the investor considers not only the rates of return of the foreign and domestic securities, but also the position he will be in to take advantage of future arbitrage opportunities.

Suppose in day t that there is no arbitrage opportunity, i.e., $r_t^d = r_t^* + (e_t^f - e_t^s)/e_t^s$, and the investor sinks all his funds in domestic securities. If the foreign interest rate rises at t + h, the investor is in a position to capture extra gains from an arbitrage operation. This will be the case if $r_t^d < r_{t+h}^* + \frac{(e_{t+h}^f - e_{t+h}^s)}{e_{t+h}^s}$ where $\frac{e_t^f - e_t^s}{e_t^s} = \frac{e_{t+h}^f - e_{t+h}^s}{e_{t+h}^s}$. He will simultaneously sell his domestic bonds, buy foreign exchange at the spot rate, buy foreign securities at the new interest rate, and buy the domestic currency forward, thereby covering his transactions. In this porcess, he will earn a total rate of return of $r_{t+h}^* + (e_t^f - e_t^s)/e_t^s$, and will increase his returns by $r_{t+h}^* + (e_t^f - e_t^s)/e_t^s - r_t^d$ compared to his original position in the absence of the arbitrage transactions.

If, however, the investor had put all his funds in foreign securities at time t and the foreign interest rate rose at time t + h the investor would be in no position to transact an arbitrage operation. It is true that $r_t^d < r_t^* + (e_t^f - e_t^s)/e_t^s$, but the investor is already in foreign currency and has no uninvested funds in the portfolio. Since all his funds are already in foreign exchange, he must watch this profit opportunity go by. He can not sell his foreign bonds and buy new foreign bonds with the higher interest rate since the capital loss he would incur by selling his bonds would just offset the increased interest rate gains on the new bonds he would buy. Since his access to investible funds is limited and these funds are already in foreign securities, he has no feasible arbitrage opportunity. He will keep the foreign securities which he originally purchased and his total return will be $r_t^* + \frac{e_t^f - e_t^s}{e_t^s}$ at maturity. This return is less than that which he would have earned if he had been in domestic securities originally and thus able to arbitrage into foreign ones when r rose. The return in this case is $r_{t+h}^* + \frac{e_t^f - e_t^s}{e_t^s}$

The foregoing argument suggests that with a finite supply of funds and expectations of future arbitrage opportunities, the relative demand for domestic securities vis-à-vis foreign securities will be a function of $r^d - r^* - (e^f - e^s)/e^s$, and also the expectations of the size and direction of future arbitrage opportunities. This follows from the recognition that in addition to offering current rates of return, bond holdings offer a *position* from which it is possible to capture additional arbitrage profits. The relative prices (and rates of return) of bonds should reflect this dimension. One should not expect the domestic security (say) to offer the same current rate of return as the foreign security if, in addition, the domestic security also offers a superior position from which expected arbitrage profits can be captured. The extra attractiveness of the domestic security should bring about an increased demand for it, and a higher price, and therefore, a lower current rate of return as compared with the foreign security. This discount factor is captured in the variable α in equation (3).[5]

It is clear from the foregoing that α is based on expected future arbitrage opportunities.[6] In order to make equation (3) operational

5 It might be argued that equation (1) still holds if securities are considered identical in all respects, including the position they offer for future arbitrage opportunities. This definition of perfect substitutes does not appear to be very useful. We continue to use equation (3) in the analysis.

6 It is well-known from price theory that perfect substitutes will have the same price in integrated markets due to commodity arbitrage. Imperfect substitutes would not, however, be expected to have the same price. The α in (3) represents the wedge between the rates of return on assets that are not perfect substitutes because the combination of short-run constraints on investment funds and expectations of future arbitrage opportunities may make one asset relatively more attractive than another by virtue of the speculative position it offers.

for empirical investigation it is assumed that α efficiently incorporates all present and prior information. Thus the arbitrage opportunity factor has the form

$$\alpha = g(\tilde{r}^d, \tilde{r}^*, \tilde{e}^f, \tilde{e}^s) \quad (4)$$

where the tilde over a variable denotes its current and past values.

Taking the logarithm of (3) and incorporation (4) suggests an equation of the following form:

$$\ln e_t^s = V_1 (B) \ln e_t^f + V_2 (B) \ln (1 + r_t^d) + V_3 (B) \ln (1 + r_t^*) + u_t \quad (5)$$

where $V_i(B)$ is a polynomial in the lag operator B, which acts on t, i.e., $V_i(B) = (V_i + V_{i1} B + V_{i2} B^2 + V_{i3} B^3 + \ldots)$ and $B^m X_t = X_{t-m}$. Note that this is the same general form of the arbitrage equation (1) except that there are lag structures and coefficient wieghts for each of the independent variables.[7]

Equation (5) is the equilibrium equation for the determination of the spot exchange rates in the asset markets in the very short run. In its general form it suggests that lagged values of interest rates and the forward exchange rates may exert some influence on the current spot rate. This need not be inferred. If all of the underlying probability mechanisms of the variables generate random walks then their lagged values will yield no information for future values. Equation (5) would then simplify to

$$\ln e_t^s = V_1 \ln e_t^f + V_2 \ln (1 + r_t^d) + V_3 (1 + r_t^*) + U_t .$$

Whether such a simplification is justified is an empirical question which we investigate.

A time series approach for investigating equation (5) seems appropriate for two reasons. First, much useful information is contained in an individual series which would otherwise be neglected if one considered only contemporaneous relationships with a few lags thrown in. Perhaps more importantly, time series

7. We are aware that there is risk of simultaneous equation bias in assuming that only e^s(and not e^f and e^s) will adjust to clear the bond market. This will be emphasized again in the text below.

analysis takes specific note of autocorrelation among the erros u_t. It makes no *a priori* assumption that u_t is a white noise process. By incorporating past information available in th series and by filtering the possibly autocorrelated error series, efficient estimates and optimal linear predictors are obtained.

SECTION II

The general ARIMA structure for a time series attempts to incorporate in parsimonious fashion as much information as is contained in the series itself. It is of the form

$$\phi (B) (1 - B)^d y_t = \theta (B) a_t \qquad (6)$$

where a_t is white noise and

$$\phi (B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$$

$$\theta (B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta^q B^q$$

The original series y_t is either covariance stationary or it is assumed that it can be suitably differenced $z_t = (1\text{-}B)^d y_t$ and/or transformed so that z_t is covariance stationary. An implication of covariance stationarity is that ϕ (B) has all zeroes outside the unit circle. As an identification restriction, the process must be invertible, i.e., that θ(B) has all zeroes outside the unit circle. The first stage in specifying an ARIMA structure is to deduce the order of the autoregressive parameters, p, the required degree of differencing for stationarity, d, and the order of the moving average parameter, q. This is done by analysis of the autocorrelation patterns, the partial autocorrelations, and the inverse autocorrelations.

Daily data extending for a two year period from January 1, 1975 to December 31, 1976 were employed in the study. Data for the U.S.-German exchange rates and the interest rates in each of the two countries were used. The interest rates were the ninety day U.S. treasury bill rate (r^d) and the ninety day German inter-bank rate (r^f); exchange rates were the spot($_t e_t^s$)and ninety day forward ($_t e^F_{t+90}$) U.S.-German exchange rates. The 1976-76 interval was chosen because it was relatively free of the structural changes and disturbances of earlier periods. The abandonment of the Bretton Woods system in 1971 and the world monetary disturbances following the formation of the OPEC cartel in the fall of 1973 made these earlier periods unsatisfactory for analysis. No such abrupt shocks occurred in 1975-76 and so one can better study the typical workings of the exchange market under fairly constant structural conditions.

The ARIMA identification[8] was done for each of these data series. Examples of the autocorrelation and partial autocorrelation patterns are shown in Figures 1 and 2. Lags are given along the horizontal axis while the estimated autocorrelations or partial autocorrelations are on the vertical axes. The dotted lines indicate $2\hat{\sigma}$ bands for the variation in the estimated (partial) autocorrelations. The models for the spot and ninety day forward rates exhibited long lags. The autocorrelations were significant at lags out through 30 and exhibited some signs of weekly seasonality after a delay of about a week. To test to see whether the series were autocovariance stationary, the sample was split in half and each half was separately estimated. The autocorrelation patterns were remarkably robust in each of the period subsets. Our findings indicate that disturbances in the foreign exchange market take up to seven weeks to work their way through the system.

Once tentative models were identified for each series, parameters were estimated by iterative minimization of the sum of squared residuals, $\sum_{t=1}^{T} a_t^2$. Such a procedure yields maximum likelihood estimates under the assumption that a_t follows a Gaussian white noise process. Two diagnostic checks were applied to check the adequacy of the models. The Box-Pierce statistic[9] was used in a test to determine whether the estimated residuals, when taken as a group, are white noise. The second diagnostic check, the Kolmogorov-Smirnov goodness-of-fit test, checks for the presence of periodic non-randomness in the estimated noise process. Additional measures such as overfitting and testing the stability of estimates in different time periods were also carried out. Models which failed either the Box-Pierce chi-squared test or the Kolmogorov-Smirnov tests were rejected.

Table 1 gives a listing of the random walk model for each series plus all those models which passed both the Box-Pierce test and the

8 Computing was done by programs developed by or under the direction of Charles Nelson. We gratefully acknowledge his work. For a good discussion of ARIMA techniques see Nelson (1973).

9 For a more thorough treatment of the Box-Pierce statistic and the Kolmogorov-Smirnov test see Box and Jenkins (1976), 289-298.

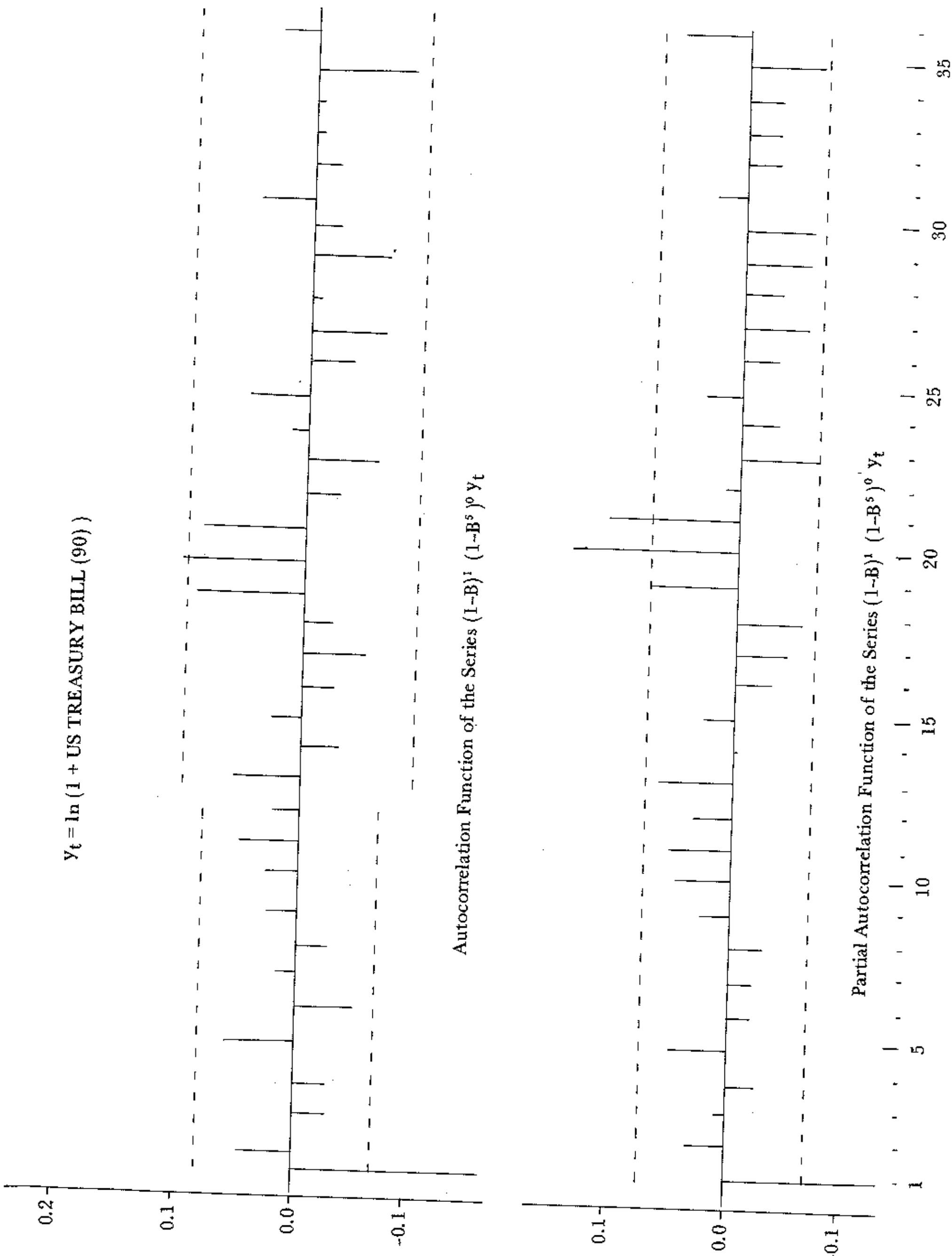
y_t = ln (1 + US TREASURY BILL (90))
Autocorrelation Function of the Series (1-B)^1 (1-B^5)^0 y_t
0.2
0.1
0.0
-0.1
Partial Autocorrelation Function of the Series (1-B)^1 (1-B^5)^0 y_t
0.1
0.0
-0.1
1
5
10
15
20
25
30
35

Figure 1

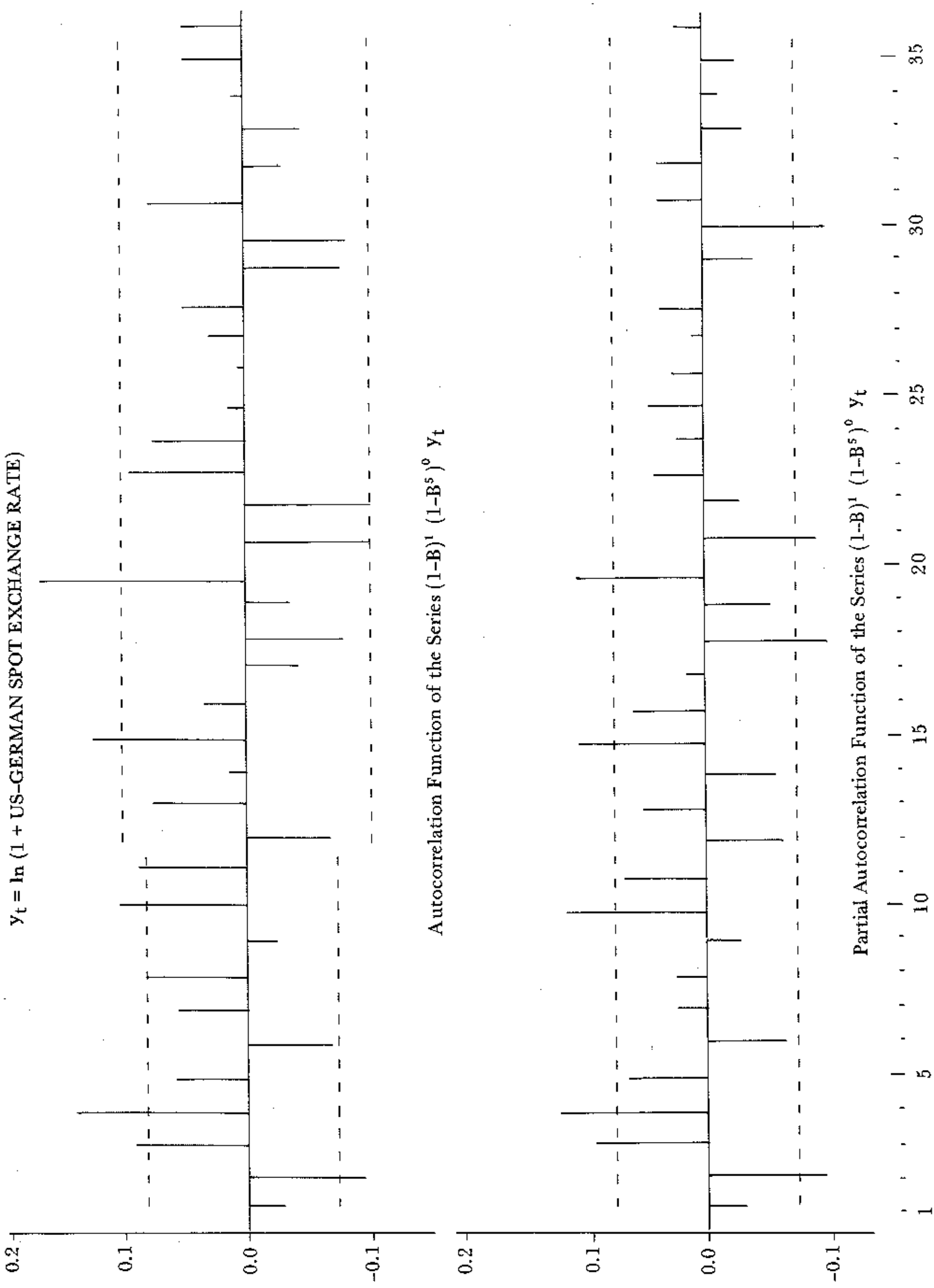
y_t = ln (1 + US-GERMAN SPOT EXCHANGE RATE)
Autocorrelation Function of the Series (1-B)^1 (1-B^5)^0 y_t
Partial Autocorrelation Function of the Series (1-B)^1 (1-B^5)^0 y_t
0.2
0.1
0.0
-0.1
1
5
10
15
20
25
30
35

Figure 2

Table 1

MODEL	RSS (residual sum of squares)	DF	CONSTANT	PARAMETERS				
			ln (1 + GERINTBK (90))					
(0, 1, 0)	$.1005014 \times 10^{-2}$	520	$-.54118 \times 10^{-4}$ $(.6090 \times 10^{-4})$					
(0, 1, 10)*	$.95617 \times 10^{-3}$	511	S (suppressed)	MA1 .0943 (.0437)	MA2 .0134 (.0438)	MA3 .0791 (.0438)	MA4 –.0319 (.0438)	MA5 –.1219 (.0437)
				MA6 –.0883 (.0437)	MA7 .0459 (.0438)	MA8 .0543 (.0437)	MA9 .0725 (.0436)	MA10 –.0654 (.0436)
(0, 1, 12)	$.95306 \times 10^{-3}$	509	$-.5346 \times 10^{-4}$ $(.5276 \times 10^{-4})$	MA1 .0978 (.0438)	MA2 .0152 (.0440)	MA3 .0753 (.0440)	MA4 –.0368 (.0440)	MA5 –.1165 (.0443)
				MA6 –.0726 (.0443)	MA7 .0530 (.0442)	MA8 .0443 (.0440)	MA9 .0583 (.0439)	MA10 –.0741 (.0438)
				MA11 .0333 (.0439)	MA12 .0467 (.0439)			

MODEL	RSS	DF	CONSTANT	PARAMETERS				
			1n (1 + USTB (90))					
(0, 1, 0)	$.429978 \times 10^{-3}$	520	$-.4770 \times 10^{-4}$ ($.3984 \times 10^{-4}$)					
				MA1				
(0, 1, 1)*	$.42408 \times 10^{-3}$	520	S	.12394				
				(.04340)				
				AR1	MA1			
(1, 1, 1)	$.423177 \times 10^{-3}$	519	S	-.32552	-.19533			
				(.3056)	(.3160)			
			1n (spot US-German exchange rate)					
(0, 1, 0)	$.83406 \times 10^{-2}$	520	$.48034 \times 10^{-4}$ ($.1755 \times 10^{-3}$)					
				MA1	MA2	MA3	MA4	MA5
(0, 1, 15) x (0, 1, 4)*	$.73533 \times 10^{-2}$	506	S	.0156	.0669	-.0975	-.1493	S
				(.0424)	(.0430)	(.0417)	(.0424)	
				MA6	MA7	MA8	MA9	MA10
				.1105	S	-.1356	S	-.1335
				(.0425)		(.0424)		(.0686)
				MA11	MA12	MA13	MA14	MA15
				-.1068	.0779	-.0615	S	.2329
				(.0420)	(.0428)	(.0416)		(.0651)
				SEMA1	SEMA2	SEMA3	SEMA4	
				-.0035	.0243	-.3179	-.1290	

MODEL	RSS	DF	CONSTANT	PARAMETERS				
(0, 1, 15) x (0, 0, 5)	$.73537 \times 10^{-2}$	506	S	MA1 S	MA2 .0672 (.0430)	MA3 -.0979 (.0416)	MA4 -.1484 (.0422)	MA5 S
				MA6 .10876 (.0424)	MA7 S	MA8 -.1350 (.0421)	MA9 S	MA10 -.1555 (.1042)
				MA11 -.1132 (.0408)	MA12 .0746 (.0423)	MA13 -.0617 (.0415)	MA14 S	MA15 .2377 (.0661)
				SEMA1 -.00432 (.0445)	SEMA2 -.04559 (.1094)	SEMA3 -.3216 (.0672)	SEMA4 -.1345 (.0475)	SEMA5 .0223 (.0710)
(0, 1, 15) x (0, 1, 6)	$.73867 \times 10^{-2}$	506	S	MA1 S	MA2 S	MA3 -.0970 (.0415)	MA4 -.1528 (.0421)	MA5 S
				MA6 .0980 (.0419)	MA7 S	MA8 -.1242 (.0421)	MA9 S	MA10 -.15144 (.1117)
				MA11 -.1121 (.0413)	MA12 .0630 (.0417)	MA13 -.0708 (.0416)	MA14 S	MA15 .2473 (.1020)
				SEMA1 -.0077 (.0443)	SEMA2 .0310 (.1184)	SEMA3 -.3386 (.1072)	SEMA4 -.1417 (.0476)	SEMA5 .0186 (.0714)
				SEMA6 -.0205 (.0755)				

MODEL	RSS	DF	CONSTANT	PARAMETERS				
				1n (US-German exchange rate (90))				
(0, 1, 0)	$.92200 \times 10^{-2}$	520	$.3804 \times 10^{-4}$					
			($.1843 \times 10^{-3}$)					
				MA1	MA2	MA3	MA4	MA5
(0, 1, 15) x	$.83518 \times 10^{-2}$	506	S	.0101	.1169	-.0788	-.1425	S
(0, 0, 4)*				(.0441)	(.0439)	(.0436)	(.0436)	
				MA6	MA7	MA8	MA9	MA10
				.0638	S	-.0918	S	-.1440
				(.0438)		(.0436)		(.1164)
				MA11	MA12	MA13	MA14	MA15
				-.0989	.0836	-.0700	S	.0774
				(.0434)	(.0445)	(.0437)		(.0952)
				SEMA1	SEMA2	SEMA3	SEMA4	
				.00026	.0754	-.1627	-.1262	
				(.0446)	(.1174)	(.09822)	(.0511)	
				MA1	MA2	MA3	MA4	MA5
(0, 1, 15) x	$.83525 \times 10^{-2}$	506	S	S	.1196	-.0800	-.1408	S
(0, 0, 5)					(.0437)	(.0430)	(.0431)	
				MA6	MA7	MA8	MA9	MA10
				.0675	S	-.0892	S	-.1825
				(.0433)		(.0431)		(.1267)
				MA11	MA12	MA13	MA14	MA15
				-.1035	.0848	-.0680	S	.1300
				(.0420)	(.0441)	(.0431)		(.0917)
				SEMA1	SEMA2	SEMA3	SEMA4	SEMA5
				-.00196	.1106	-.2120	-.1300	.0360
				(.0444)	(.1299)	(.0933)	(.0551)	(.0673)

MODEL	RSS	DF	CONSTANT	PARAMETER				
(0, 1, 15) x (0, 0, 6)	$.845717 \times 10^{-2}$	506	S	MA1	MA2	MA3	MA4	MA5
				S	S	-.0767	-.1430	S
						(.0438)	(.0438)	
				MA6	MA7	MA8	MA9	MA10
				.0373	S	-.0812	S	-.1547
				(.0437)		(.0445)		(.1844)
				MA11	MA12	MA13	MA14	MA15
				-.0989	.0704	-.0802	S	0.0535
				(.0434)	(.0435)	(.0450)		(.1430)
				SEMA1	SEMA2	SEMA3	SEMA4	SEMA5
				-.0106	.0770	-.0418	-.1438	-.0259
				(.0443)	(.1890)	(.1450)	(.0620)	(.0540)
				SEMA6				
				.0248				
				(.0534)				

Explanation to Table 1

(p,d,q) × (P,D,Q) refers to an ARIMA model with d simple differences, an autoregressive polynomial of order p and a moving average polynomial of order q. The second set of parentheses indicates seasonal parameters: seasonal AR or order P, seasonal differencing of order D and seasonal MA of order Q. For our daily data, the relevant seasonal interval is 5 days, a business week. The numbers in parentheses under the parameter estimates are the standard error of the parameters under the linear hypothesis. The asterisk indicates the model which was finally selected for each series on the basis of the likelihood ratio tests.

GERINTBK (90) ≡ German inter-bank rate for 90 day loans
USTB (90) ≡ rate of interest on 90 day U.S. Treasury bills

Kolmogorov-Smirnov test. In some trial runs it was discovered that the constant term was never significant. Hence it was usually suppressed in subsequent estimations. A random walk is expressed as (0, 1, 0), i.e., zero order autoregressive and moving average process plus a first difference to induce stationarity. The random walk model is included for each of the series because it provides a benchmark, though it should be noted that the random walk never left white noise residuals. For the random walk model all four series failed the Kolmogorov-Smirnov goodness-of-fit test. The U.S. treasury bill rate did reasonably well on the Box-Pierce test; however, the other three failed decisively. The failure of the random walk model to adequately depict exchange rate movements is consistent with the results in the studies mentioned above by Levich, Dooley and Shafer, and Giddy and Dufey. In each of these studies, time series analysis suggested departures from the random walk model for the exchange rates of most of the countries under study.[10]

When there was a plurality of models which passed both tests and if the models were nested, the likelihood ratio (or the posterior odds ratio) tests[11] were carried out.

Table 2 presents the results of these tests. The asterisk in Table 1 indicates the model which was finally selected for each series. It will be noted that both the spot and the ninety-day forward exchange rates always involve models with fifteen parameters.[12] Many other models with autoregressive terms and lower order moving averages were attempted. None however (except those listed) passed both tests.

Aside from the long moving average processes characterizing the two exchange rates and to a certain extent the German interbank interest rate, it is noteworthy that only one of the series can

10 For example, Levich investigated the percentage change in exchange rates for nine industrial countries and found strong evidence for the random walk model for only two countries Italy and Switzerland.

11 These tests were developed by Zellner (1971), 291-318. In our calculation the simplified expression = $(S_0^2/S_1^{2-T/2}$ is used. This assumes diffuse prior information as to the relative suitability of the two models, a large number of observations, and a symmetric loss function. Zellner and Palm (1974) also contains a useful discussion of this technique as well as applications.

12 It would be desirable to overfit the model with more than fifteen parameters and then conduct a likelihood ratio test. However, the estimation program which we used allowed a maximum of fifteen parameters to be estimated. Selection in the case of nonnested hypotheses was made on the basis of the lowest sum of squared residuals.

Table 2

	$\lambda = \frac{L(X\|H_1)}{L(X\|H_0)}$	$2 \ln \lambda$	r	Critical points for χ^2		
ln (1 + GERINTBK (90))					α	
				.05	.10	.25
H_0 (0, 1, 0) vs H_1 (0, 1, 10)	414900.25	25.87	10	18.307	15.987	12.549
H_0 (0, 1, 0) vs H_1 (0, 1, 12)	1038245.	27.71	12	21.026	18.549	14.845
H_0 (0, 1, 10) vs H_1 (0, 1, 12)	2.5024	1.8345	2	5.991	4.605	2.773
ln (1 + USTB (90))					α	
				.05	.10	.25
H_0 (0, 1, 0) vs H_1 (0, 1, 1)	36.686	7.2048	1	7.879	6.635	5.024
H_0 (0, 1, 0) vs H_1 (1, 1, 1)	64.1796	8.3234	2	10.597	9.210	7.378
H_0 (0, 1, 1) vs H_1 (1, 1, 1)	1.7494	1.1186	1	7.879	6.635	5.024

r ≡ degrees of freedom

be satisfactorily modelled by using an autoregressive process. Even then, the (1, 1, 1) process for the U.S treasury bill rate is rejected in favor of a pure moving average process.

SECTION III

While an individual time series y_t can be parsimoniously represented by an ARIMA process, it is often the case that y_t is correlated with other time series x_t which may be helpful in predicting more accurately the future values $y_{t+\ell}$, $\ell = 1,\ldots,N$. Of course, correlation does not imply causality nor does causality imply correlation. It is essential to have a clear criterion for establishing whether the real world, i.e., the data, reveals causal relationships. Under the perhaps not so obvious assumption that the future does not cause the past, Granger and Newbold [1976, pp. 224-226] propose the following criterion, which we adopt, to establish whether series x_t causes series y_t. Let $P(y_t|\Omega_{t-1})$ denote the conditional probability of y_t given Ω_{t-1}, where Ω_{t-1} denotes the universe of information available at time t-1. If

$$P\,(y_t|\Omega_{t-1}) = P\,(y_t\,|\,\Omega_{t-1} - x_{t-1}) \tag{8}$$

then x_t does not cause y_t, where $\Omega_{t-1}-x_{t-1}$ represents the universe of information at time t-1 but excluding the past realizations of the x series. Instantaneous causality exists if

$$P\,(y_t|\Omega_t - y_t) > P\,(y_t\,|\,\Omega_t - y_t - x_t) \tag{9}$$

Feedback is present if x causes y (non-instantaneously) and y causes x (non-instantaneously). Of course such definitions are non-operational since we rarely have enough information to estimate a conditional distribution function. In practice we restrict ourselves to looking at the mean of the forecast and its variance. The variance of the forecast will in turn be influenced by the cross-correlation functions $\Upsilon_{yx}(k)$ where k indicates the lag between the series.

As a first approximation to a determination of the spot rate, a more general version of equation (5) is considered. This is a single output, multiple input transfer function:

$$y_t = \sum_{i=-\infty}^{\infty} v_{1i}\,B^i x_{1t} + \sum_{i=-\infty}^{\infty} v_{2i}\,B^i x_{2t} + \sum_{i=-\infty}^{\infty} v_{3i}\,B^i x_{3t} + \epsilon_t \tag{10}$$

where

y_t = natural log of the spot exchange rate

x_{1t} = natural log of the ninety day forward exchange rate

x_{2t} = natural log of 1 plus the three month German interbank rate

x_{3t} = natural log of 1 plus the three month U.S. treasury bill rate

ε_t = a zero-mean stationary process

The following procedure was followed to examine the causality relationship characterizing the various series. First, the ARIMA filter for an input was used to prewhiten the input series. Second, the output series was prewhitened using each input filter and the cross-correlation patterns between the prewhitened input and output were studied. Such a procedure preserves the causality relationship between the two variables. This exercise showed only meagre evidence of any correlation at all. In particular, as evidenced by insignificant cross correlations when the output leads the input, there was no evidence to suggest a feedback relationship between the spot exchange rate and the other three variables. The cross correlation betwen the spot rate and ninety-day forward exchange rate evidences very strong instantaneous causality as well as a pronounced lag relationship (spot following the forward rate) at twelve days. The instantaneous causality reflects the simultaneous determination of exchange rates and interest rates in the asset markets. The transfer function represents a first approximation to the system. We are aware of the difficulty of simultaneous equations bias. However, neither the theoretical framework nor the required data are available for the joint estimation procedure which would eliminate the bias.

The possibility of causal relationships among these variables as well as between the spot and forward exchange rates was investigated further. Allowing for the possibility of significant lag relationships but excluding feedback relationships. A more parsimonious version of (10) is:

$$y_t = \sum_{i=0}^{K_1} v_{1i} B^i x_{1t} + \sum_{i=0}^{K_2} v_{2i} B^i x_{2t} + \sum_{i=0}^{K_3} v_{3i} B^i x_{3t} + \epsilon'_t$$

$$= v_1(B)x_{1t} + v_2(B)x_{2t} + v_3(B)x_{3t} + \epsilon'_t \qquad (11)$$

where $v_j(B)$ are polynomials in B which operate on t. The order of the $v_j(B)$ may be quite high since both the forward exchange rate and the German interbank rates had high order ARIMA representations. Although it is not assumed that the error process is white noise, nevertheless consistent estimates of the v_{ji} can be obtained by ordinary least squares regression. The spot exchange rates were regressed on the forward rates, the German interbank rates and the U.S. treasury bill rates, each of these lagged from zero to forty-six.

Given the estimates for the $v_j(b)$ functions, a more parsimonious representation was sought by means of the ratio of two polynomials:

$$v_j (B) \approx \frac{\omega (B)}{\delta \ (B)}$$

where ω and δ, are polynomials in B of order r and s respectively. Orders of ω and δ, as well as those individual polynomial coefficients to be suppressed, are chosen so that they reproduce in a general fashion the v_{ji}.

To actually estimate the values of the ω and δ coefficients, one must model the error process. Inspection of the autocorrelation and partial autocorrelation functions for the residuals from the ordinary least squares regression indicates that the residuals follows a MA(2) process:

$$\epsilon'_t = (1 - \theta_1 \ B - \theta_2 \ B^2)a_t$$

where a_t is a white noise process. If we further assume that a_t is Gaussian, then minimizing the sum of these squared residuals, conditional on starting values y_0, $x_{1,0}$, $x_{2,0}$, $x_{3,0}$, and a_0, gives a good approximation to the maximum likelihood estimates.

Several different models were fitted and the generic results (excluding parameter values) are given in Table 3. ω^j refers to the numerator polynomial in B of input j whereas δ^j refers to the denominator polynomial of j. The numbers in parentheses indicate the polynomial coefficients which are actually estimated; all other coefficients of that polynomial are constrained to be zero. The exception is that $\delta^j(0) = 1$ for all j. Thus $\omega^3(0, 6, 18)$ indicates the numerator polynomial for the U.S. treasury bill rate (input 3). It has the form

$$\omega^3 \ (0, 6, 18) = \omega^3_0 - \omega^3_6 \ B^6 - \omega^3_{18} \ B^{18}$$

Table 3

MODEL	DESCRIPTION	SSR	F	R^2
1	$\omega^1(0, 2)$ $\omega^2(0, 2, 8, 18)\delta^2(1, 2)$ $\omega^3(0, 3, 15)\delta^2(1, 2)b = 3$	$.96673 \times 10^{-3}$	$F(15,488) = 245$	.879
2	$\omega^1(0, 2)$ $\omega^2(0, 2, 8, 18)\delta^2(1, 2)$ $\omega^3(0, 6, 18)\delta^3(1, 2)b = 0$	$.96534 \times 10^{-3}$	$F(15,488) = 245$	.879
3	$\tilde{c}, \omega^1(0, 2)$ $\omega^2(0, 2, 18)\delta^2(1, 2)$ $\omega^3(0, 3)\delta^3(1, 2)b = 3$	$.9995 \times 10^{-3}$	$F(14,488) = 252$	.875
4	$\omega^1(0, 2)$ $\omega^2(0, 2, 8, 18)\delta^2(1)$ $\omega^3(0, 3)\delta^3(1, 2)b = 3$	$.10005 \times 10^{-2}$	$F(13,490) = 273$	.875
5	$\omega^1(0, 2)$ $\omega^2(0, 2, 8, 18)\delta^2(1)$ $\omega^3(0, 6)\delta^3(1, 2)b = 0$	$.96961 \times 10^{-3}$	$F(13,490) = 282$	.879

MODEL	DESCRIPTION	SSR	F	R^2
6	$\bar{c}, \omega^1 (0, 2)$ $\omega^2 (0, 2, 8, 18)\delta^2 (1)$ $\omega^3 (0, 3)\delta^3 (1, 2)b = 3$	$.10004 \times 10^{-2}$	F(13,489) = 272	.875
7	$\omega^1 (0, 2)$ $\omega^2 (0, 8, 18)\delta^2 (1, 2)$ $\omega^3 (0,6,18)\delta^3 (1, 2)b = 0$	$.96542 \times 10^{-3}$	F(14,489) = 263	.879
8	$\omega^1 (0, 2)$ $\omega^2 (0, 2, 8, 18)$ $\omega^3 (0, 6, 18)\delta^3 (1, 2)b = 0$	$.96723 \times 10^{-3}$	F(13,490) = 283	.879
9	$\omega^1 (0, 2)$	$.10970 \times 10^{-2}$	F(4,515) = 850	.867
10	$\omega^2 (0, 8, 18)\delta^2 (1, 2)$	$.76490 \times 10^{-2}$	F(7,496) = 5.42	.058

For each model the noise process was $\epsilon_t = (1-\theta_1 B-\theta_2 B^2)a_t$. $\bar{c}$ means a constant was estimated.

The order of the delay impulse operator is given by b. If, for example, b = 3, then the indicated input, input 3 in our case, is constrained to have no effect on the output until three periods have elapsed. All of the models, except 9 and 10, performed well on the diagnostic checks, which are referred to in detail below.[13] To help facilitate the choice of models, likelihood ratio tests were conducted on nested hypotheses. The results are given in Table 4.

Table 4 indicates that there are a number of models at our disposal among which we cannot choose a "best one." On balance, model 2 and model 7 seem attractive. Since a likelihood ratio test indicates that, under the assumption that model 7 is the correct model, it cannot be rejected in favor of model 2, model 7 was chosen for further analysis.

Before presenting the parameter estimates for model 7, we turn to some of the diagnotic chekcs, which offer further evidence that the model is approximately correct. The residuals of the nonlinear estimation procedure produce no autocorrelation at any lags. This is the case both when individual lags are considered and when one groups the autocorrelations over lags from 1 to 60. This indicates that there is practically no information in the residuals which could be useful in providing a better predictor for the output variable.

Other important signs that both the noise process and the transfer function are correctly modeled are to be found in the cross-correlation functions between the residuals and the input variables. If there is residual autocorrelation and cross correlation with the input, then this is evidence that the transfer function has been modeled incorrectly. If there is residual autocorrelation but not cross correlation, this points to a misspecification of the noise process.[14] Our results indicate no such problems.

When one inspects the cross correlations between the estimated residuals and the three input series, the results are very satisfactory. Both in the case of the residuals leading the input series and when the residuals lag the input series, the Box-Pierce statistics are good and the individual cross correlations are practically never significant up to sixty leads and lags. Only in the case of the U.S. treasury bill rate is there some indication that at short leads

13 All the models calculated were both stationary and noninvertible, i.e., all the roots of ω^j and δ^j lay outside the unit circle. As it turns out stationarity is a necessary assumption in a MITF though invertibility is not.

14 This is a trivial generalization of the analysis in Box and Jenkins (1976), 392-393.

Table 4

	λ	$21n\lambda$	r	Critical values of χ^2 for α = .05	.10	.25
H_0 4 vs H_1 6	1.02633	.051977	1	3.84146	2.70554	1.3233
H_0 5 vs H_1 2	3.15039	2.29505	2	5.99147	4.60517	2.77259
H_0 6 vs H_1 3	1.26366	.46802	1	3.84146	2.70554	1.3233
H_0 8 vs H_1 2	1.66287	1.01709	2	5.99147	4.60517	2.77259
H_0 7 vs H_1 2	1.0217797	.04309	1	3.84146	2.70554	1.3233
H_0 9 vs H_1 7	2.6760077×10^{14}	66.44103	10	18.3070	15.9871	12.5489
H_0 10 vs H_1 7	greater than 10^{100}		9	16.9190	14.6837	11.3887

(because the cross correlations between $\hat{a}_{t+k}$ and x_{3t} for $k = 4$ and $k = 6$ are barely significant) there may be some slight misspecification in the model. The other input series behave in exemplary fashion.

Model 7, about which we can be reasonable confident, has the following parameter values:

$$\ln \ell_t = \underset{(.01402)}{(.91433} + \underset{(.01403)}{.0776B^2)} \ln e_{t+90}$$

$$+ \frac{(\underset{(.04144)}{.14967} + \underset{(.04191)}{.04387B^8} + \underset{(.03963)}{.068976B^{18}})}{1 + \underset{(.24134)}{.23570B} + \underset{(.24351)}{.41304B^2}} \ln (1 + \text{GRINTBK}(90))$$

$$+ \frac{(\underset{(.02559)}{-.11593} + \underset{(.02470)}{.04998B^6} + \underset{(.01345)}{01701B^{18}})}{1 - \underset{(.1343)}{.07786B} - \underset{(.12603)}{.78939B^2}} \ln (1 + \text{USTB}(90))$$

$$+ (1 - \underset{(.03990)}{.39692B} - \underset{(.03947)}{.50351B^2})a_t \qquad (12)$$

Though the model is much more complicated than the simple interest arbitrage equation (3′), it nevertheless retains some of its characteristics. The coefficient of $\ln \ell_{t+90}$ is fairly close to 1, though the 95% confidence interval, based on the linear hypothesis, does not include 1. The signs on the coefficient of the unlagged German interbank rate and the unlagged U.S. treasury bill rate are respectively positive and negative, which is what one would expect from the simple story. Though the absolute values of the coefficients are not close to 1, they are about equal in magnitude and the absolute values of the 95% confidence intervals overlap.

Dividing ω^j by δ^j one obtains the $v_{ji}B^i$ and gets a feel for the way in which the inputs drive the output. The forward exchange rate expends all its influence after just two periods. The weights on the German interbank rate begin positive for the unlagged operator and then decline in magnitude and alternate in sign with two negative, two positive, two negative, etc. They are of significant size out to about lag 25. The weights on the U.S. treasury bill rate are all negative and roughly declining out to lag 18, at which point they take a jump and then continue to decline again, but now alternatively positive and negative. They remain significant out to approximately 35 legs.

The steady state gain $g_j = \sum_{i=0}^{\infty} v_{ji}$ for each of the input series is perhaps more properly considered the analog to the coefficient of the simple interest arbitrage equation. Since $g_1 = .992$, the total effect over time of the forward rate is that predicted by (3'). On the other hand, $g_2 = .076$ and $g_3 = -.369$ and these bear little resemblance to 1 and -1 in (3').[15]

All this indicates that values of the input series going back as far as seven weeks provide useful information about the present value of the spot exchange rate. It is not surprising that the impulse (v) weights are significant at long lags for the interest rates but only at short lags for the forward exchange rate. The ARIMA processes for the spot and forward exchange rates were very similar. Thus the forward rate would not be expected to contribute much information at long lags to the determination of the spot rate. On the other hand the spot ARIMA process was such that information on the a_t up to thirty periods in the past was useful in determining the present spot rate. Apparently both interest rates are sufficiently different from, yet related to, the spot rate so that interest rates at rather long lags offer useful information about the present spot exchange rate.

A good measure of the multiple input transfer function's predictive power has been suggested by Pierce (1975). He proposes that we look at a modified R^2. It is modified because we are not really interested in its explanatory power for current values of the output variable, but rather for future values. More importantly we know that the output y_t may already contain considerable information in its own history which can be captured in a suitable ARIMA process. The interesting question is: what can the inputs x_{it} explain of y_t which is not already explained by y_t's own ARIMA process? Put anther way; how much of the variance of the innovation process of y_t can the multiple input transfer function eliminate? The proper measure of this is

15. The computer program to estimate MITF does allow one to constrain the gain to a certail value. This would, however, adversely affect the predictive power of the estimated equation.

$$R_{+}^{2} = \frac{v_1(1) - v_2(1)}{v_1(1)} \tag{13}$$

where $v_1(1)$ is the one step ahead forecast variance of the single variable ARIMA process and $v_2(1)$ is the one step ahead forecast variance of the multiple input transfer function. Since the one step ahead forecast variance is simply the variance of the innovation process R_{+}^{2} is easily calculated in our case to be .864. Thus the multiple input transfer function has a residual variance substantially smaller than that of the univariate ARIMA process.

SECTION IV

A model which explicitly considered the effects of short-run constraints on the supply of arbitrage funds and of the uncertainty of future arbitrage opportunities suggested that lagged values of interest rates and exchange rates may have significant influence on the current spot exchange rate. The main purpose of this paper was to investigate this issue empirically. A multiple input transfer function was successfully estimated and each time series was modeled by a suitable ARIMA process as well.

All of the individual series were characterized as moving average processes. The spot and forward exchange rate series had long lags which indicated that disturbances in the foreign exchange market may take up to seven weeks to work their way through the system. The two interest rate series were rather different from each other. The German interbank rate was modeled as a tenth order moving average process, while the U.S. treasury bill rate required only a first order moving average process. It is interesting to note that the random walk hypothesis was rejected for all four series.

The major contribution of the paper was to advance beyond univariate analysis to develop and estimate a multiple input transfer function. This technique allowed the efficient estimation of the effects of lagged interest rates and the forward exchange rate on the current spot rate. The evidence suggests that lagged values of the German interbank rate and the U.S. treasury bill rate significantly effect the value of the current spot rate. The weights on the German interbank rate were of significant size out to a lag of about 25 days, while the U.S. treasury bill rate had weights that remained significant out to approximately 35 lags. The forward rate expeded its infuence on the spot rate with a lag up to only two days.

Pierce's predictive R^2_+ was calculated and indicated that the MITF is superior to the univariate process for the spot exchange rate. Thus, the evidence suggests that lagged values of the interest rates significantly effect the current spot exchange rate. Furthermore, incorporating this information into a MITF results in a model with a predictive power that is superior to the univariate ARIMA model for the spot exchange rate. Additional work needs to be carried out along this line.

Forecasts over multi-period horizons remain to be performed. For such work, the ARIMA structures would be useful since they provide forecasts for the input variables as a first step toward obtaining unconditional forecasts of the output series. Then it would be appropriate to forecast, via the MITF, future spot exchange rates and compare them with the present forward exchange rate in an effort to determine whether the forward rate is a "rational" forward rate, i.e., a forward rate based on all the information in the system. We leave such analysis to further work.

References

Box, G. E. P. and Gwilym M. Jenkins, *Time Series Analysis: Forecasting and Control,* Holden-Day, San Franciso, 1976.

Branson, William H., "The Minimum Covered Interest Differential Needed for International Arbitrage Activity," *Journal of Political Economy,* 77, 1028-1035, 1969.

Cornell, W. Bradford and J. Kimball Dietrich, "The Efficiency of the Market for Foreign Exchange Under Floating Exchange Rates," *The Review of Economics and Statistics,* KX, 111-120, 1978., Dooley, Michael P. and Jeffrey R. Shafer, "Analysis of Short Run Exchange Rate Behavior: March, 1973 to September, 1975," International Finance Discussion Papers, No. 76 (Federal Reserve System: Washington, D.C., February, 1976).

Dornbusch, Rudiger, "Expectations and Exchange Rate Dynamics," *Journal of Political Economy,* 84, 1161-1176, 1976.

Dornbusch, Rudiger, "Exchange Rate Economics: Where Do We Stand?" *Brookings Papers on Economic Activity* 1, pp. 143-185, 1980.

Einzig, P.A., *A Dynamic Theory of Foreign Exchange,* MacMillan, London 1961.

Frenkel, Jacob A. and Harry G. Johnson, eds., *The Economics of Exchange Rates,* Addison-Wesley, 1978.

Giddy, Ian H. and Gunter Dufey, "The Random Behavior of Flexible Exchange Rates," *Journal of International Business Studies,* 1, 1-32, 1975.

Granger C. W. J. and Paul Newbold, *Forecasting Economic Time Series,* Academic Press; New York, 1977.

Healy, James P., "The Asset Market Determination of Exchange Rates in A Multi-Country Setting," Unpublished Ph. D. dissertation, Princeton University, 1980.

Healy, James P., and John J. Piderit, "The Interest Arbitrage Equation as a Multiple Input Transfer Function," Econometric Research Program, Princeton University, Research Memorandum No. 237, 1978.

Hume, David, 1752, "Of the Balance of Trade," in R. N. Copper, ed., *International Finance: Selected Readings,* Penguin Books: Baltimore, 25-37, 1969.

Levich, Richard M., "The International Money Market: Tests of Forecasting Models and Market Efficiency," unpublished Ph. D. dissertation, University of Chicago, 1977.

Nelson, Charles R., *Applied Time Series Analysis for Managerial Forecasting,* Holden-Day, San Francisco, 1973.

Pierce, David A., "Forecasting in Dynamic Models with Stochastic Regressors," *Journal of Econometrics,* 3, 349-374, 1975.

Zellner, A., *An Introduction to Bayesian Inference in Econometrics,* John Wiley and Sons, New York, 1971.

Zellner, A. and F. Palm, "Time Series Analysis and Simultaneous Equation Econometric Models," *Journal of Econometrics,* 2, 17-54, 1974.