Estimating the Parameters in the Box-Cox Type Equations

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Fairly often we encounter a "nonconvergent" situation when we estimate the parameters in a Box-Cox type equation. However, under appropriate circumstances we can find the estimate of the equation, using the scaling technique.

I

This technique is effective when the transformation parameters are insensitive with respect to the scaling in the data.** By using this, we can have a better guess for the intercept term in terms of the absolute magnitude over which the computer has to search for a higher value of likelihood of the equation under investigation. And since we are dealing with relatively smaller digits by this, we are less likely to hit the capacity limit of the available computer on the way in traveling to the highest value of likelihood attainable in the equation.

II

Write the unconstrained Box-Cox equation as

$$\frac{q^{\theta 1}-1}{\theta 1} = \alpha + \beta \frac{S^{\theta 2}-1}{\theta 2} + \gamma \frac{t^{\theta 3}-1}{\theta 3} + \epsilon \quad \ldots \ (1)$$

where ε displays all the properties upon which we can obtain max-

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** The sensitiveness of the capital intensity with respect to the scaling in the CES production function (2) with Box-Cox transformation (3) gives difficulties in using this technique.

imum likelihood estimates for α, β, γ given $\theta 1$, $\theta 2$, $\theta 3$, which are to be estimated within the same framework of the maximum likelihood method, and where q, S, t are the data, and q takes higher digits than S and t.

Let

$$B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}, \quad Y = \begin{bmatrix} \dfrac{q^{\theta 1} - 1}{\underline{\theta 1}} \end{bmatrix}, \text{ and}$$

$$X = \begin{bmatrix} \underline{1} & \dfrac{S^{\theta 2} - 1}{\underline{\theta 2}} & \dfrac{t^{\theta 3} - 1}{\underline{\theta 3}} \end{bmatrix}.$$

hen $\hat{B} = (X'X)^{-1} X'Y$. .... (2)

Assuming $\theta 1$, $\theta 2$, $\theta 3$ display stable patterns with respect to changes in the scale of q in (1), the change of $\hat{B}$ comes from the different scale in Y while $(X'X)^{-1} X'$ remain unchanged.

In case that we want to scale down the unit of q by 1,000 for example, we can have the following steps.

STEP 1. Using all the available informations and knowledges

about θ, B, where $\theta = \begin{bmatrix} \theta 1 \\ \theta 2 \\ \theta 3 \end{bmatrix}$, $B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$,

make the initial guesses for the estimates in the equation (1). Call them $\tilde{\theta}$, $\tilde{B}$, where

$$\tilde{\theta} = \begin{bmatrix} \widetilde{\theta 1} \\ \widetilde{\theta 2} \\ \widetilde{\theta 3} \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\gamma} \end{bmatrix}$$

STEP 2. With the set of prices scaled down by 1,000, obtain the estimates for the parameters with the guesses $\tilde{\theta}$ and $\tilde{B}$, where

$$\tilde{\tilde{B}} = \tilde{B} \cdot \frac{\left\{(q/1{,}000)^{\widetilde{\theta 1}} - 1\right\} / \widetilde{\theta 1}}{(q^{\widetilde{\theta 1}} - 1)/\widetilde{\theta 1}} = \tilde{B} \cdot \frac{(q/1{,}000)^{\widetilde{\theta 1}} - 1}{q^{\widetilde{\theta 1}} - 1} \qquad (3)$$

where for practical purposes, we use the mean value of the vector, q.
Call the resulting estimates $\hat{\hat{\theta}}$ and $\hat{\hat{B}}$.

STEP 3. Returning to the original q prior to scaling down, obtain the estimates of the equation (1), with the guesses, $\hat{\hat{\theta}}$ and $B^{\star}$
where

$$B^{\star} = \hat{\hat{B}} \cdot \frac{q^{\hat{\hat{\theta}}_1} - 1}{(q/1{,}000)^{\hat{\hat{\theta}}_1} - 1}, \text{ where } B^{\star} = \begin{pmatrix} \alpha^{\star} \\ \beta^{\star} \\ \gamma^{\star} \end{pmatrix} \quad \ldots\ldots (4)$$

moving from STEP 2 to STEP 3, intermediate steps may be used as many times as needed, if necessary.

References

Box, G. E. P., and D. R. Cox, "An Analysis of Transformations", J. Roy. Statist. Soc., Series B, 26, 1964, pp 211-43

Brown, Murray, *On the Theory and Measurement of Technological Change,* Cambridge, 1968

Zellner, Arnold, *An Introduction to Bayesian Inference in Econometrics,* John Wiley & Sons, Inc., 1971