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IS PUBLIC-PRIVATE PARTNERSHIP AN OPTIMAL MODE OF PROVISION OF INFRASTRUCTURE?*

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The present paper considers a closed economy model with infrastructure service which is an excludable, impure public good, e.g. metro railway service, electricity service, and telephone service etc. The physical capital required by infrastructure sector is provided by public-private partnership. Public and private investment may be complementary or substitute to each other in infrastructure production. We assume government runs a balanced budget. We find there exists unique, saddle path stable growth rate in both the cases. We find that PPP model is optimal in the provision of infrastructure no matter public capital and private capital are a substitute or complementary to each other. But, PPP solution is not growth maximizing in case of substitute relationship between the public capital and private capital. It also makes a comparative study of decentralized economy and command economy. We find that in case of substitute relationship between private capital and public capital, command economy growth rate is higher than the competitive economy growth rate but in the case of complementary relationship between two, command economy growth rate may not be higher than the competitive economy growth rate.

Keywords: Infrastructure, Public-Private Partnership, Endogenous Growth
JEL Classification: E62, H44, O40

1.INTRODUCTION

Infrastructure is one of the most important determinants of economic growth. The mode of financing infrastructure is considered to be an important issue in economic theories. Traditionally, infrastructure has been provided by the government in most of the countries. However, infrastructure bottleneck is an important concern for the government. One solution to this problem is the market provision of infrastructure. Privately provided infrastructure which includes road, power, water, transportation, irrigation and communications are quite common in the developed world (Chatterjee and

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Morshed, 2011). Governments of both developed and developing nations are considering the Public-Private Partnership (PPP) model as a solution to the problem of soft budget constraints faced by them in the provision of infrastructure. PPP is the collaboration of public and private investments which take place together. Recent trend shows that BRIC countries have been benefitted a lot from the implementation of the PPP model (Kateja, 2012). It is suggested that partnership with private entities offers significant advantages regarding enhancing efficiency through competition in the provision of services to users.

In this paper, we develop an endogenous growth model, with private and public capital to study the respective role of private and government investment in the infrastructure provision and consequently to growth. We extend Barro (1990)'s model by including private capital in the infrastructure sector. Bucci and Bo (2012) have extended Barro (1990)'s model but they consider infrastructure as a stock variable. They study the impact of the change in the degree of complementarity and substitutability between private capital and public capital investments on growth rate. It is well known that private investments and public investments are not independent of each other. Sometimes there is a crowding-out effect of public investment. Hence, public investment displaces private investment when public investments and private investments are substitutes to each other. But, when public capital and private capital are complementary to each other, there may be crowding in effect. In this case, public investment improves the productivity of private capital in production (Rashid and Ahmad, 2005). In the present paper, we consider both the cases of substitute and complementary relationship of public investment and private investment. In our paper, following Barro (1990), infrastructure is a flow variable. Here, the government partially finances public investment by imposing output tax and the private sector also finances a part of it. If the optimal tax rate to be imposed by the government or, the optimal private capital to be employed in infrastructure is found to be zero, public-private partnership (PPP) is not desirable, otherwise, it is.

We study the steady state growth paths for competitive and command economies for both complementary and substitute cases under the balanced budget fiscal rule assumption. We also analyse the transitional dynamics for both substitute and complementary case. Bom and Ligthart (2014), Chen and Guo (2016) also studied the dynamic macroeconomic effects of public infrastructure investment under a balanced budget fiscal rule. However, these works do not consider the possibility of PPP investment for infrastructure provision. The present paper starts with the complementary relationship between public investment and private investment and then proceeds to compare the results with a case where public investment and private investments are perfect substitutes in providing infrastructure. In public policy analysis, it is important to find growth-maximizing tax rate and welfare-maximizing tax rate. The private agent takes tax rate and public investment as given and accordingly makes its investment decision. The government takes into account its budgetary restriction and maximizes the welfare with respect to the tax rate along with other choice variables. There are several papers that focus on public investment and private investment in infrastructure but public-private partnership investment in infrastructure has been ignored in endogenous growth literature. Chatterjee and Morshed (2011) compare the impact of the private provision of infrastructure and government provision of infrastructure, both separately on the economy's aggregate performance. Barro and Sala-i-Martin (1992) study the effects of alternative fiscal policies in case of a publicly provided private good (rival and excludable), publicly provided public good (non-rival and non-excludable), publicly provided good which are subject to congestion (rival and non-excludable). There is a number of papers that consider a model where the output is produced using private and public capital e.g., Barro (1990), Futagami et al. (1993), Fisher and Turnovsky (1998), Devarajan et al. (1998). Bucci and Bo (2012) show how a change in the degree of complementarity/substitutability between public and private capital stock affect the optimal growth rate of the economy. However, they do not find out growth maximizing and optimal tax rate in a command economy and do not compare their results obtained in the competitive and command economy. In the micro-economic framework, Besley and Ghatak (2017) discuss the responsibility of public and private entities especially NGOs in infrastructure provision. Also, their paper is not the evaluation of PPP from the macroeconomic perspective.

In real life, there are number of instances where PPP is being successfully implemented, for example, metro rail system- New Delhi of India, roads in Chile-Argentina, United States of America, Hong Kong, Hungary and Italy, water system of Singapore, Airports of New Delhi and Mumbai of India, rural electrification of Guatemala, port expansion in Colombo, Sri Lanka, etc; are some examples of successful PPP projects. Though most of the infrastructure service may be non-rival in nature, most of these are excludable at least to some extent; for example, metro railway service, electricity, telephone service, etc. So the desirability of the PPP model is relevant for all these infrastructure services. This paper considers infrastructure as an impure public good and examines whether a public-private partnership in financing infrastructure service is optimal. Comparison between the growth-maximizing and welfare maximizing fiscal policy has been a central issue in the models of public finance and growth and is also important from policy-making. This paper attempts to find out growth-maximizing and welfare-maximizing policies in the context of PPP in infrastructure provision.

In our result, we find that in case of a complementary relationship between private and public investment, like Dasgupta (1999) command economy growth rate may be less than that of a competitive economy. But, in case of a perfect substitute relationship between public and private investment, the command economy growth rate is always higher than the competitive economy growth rate. However, when private capital and public capital are a perfect substitute for each other, the growth-maximizing tax rate is zero. Empirically, it is found that in most of the developing countries private investment and public investments are complementary to each other (Erden and Holcombe, 2006), which was also found to be true in case of Pakistan (Rashid and Ahmad, 2005). When we consider private investment and public investment to be complementary to each other in our paper, we find that there exists a unique, interior growth-maximizing tax rate and an optimal tax rate.

The structure of the paper is organized in the following manner. Section 2 describes the general model where public capital and private capital are complementary to each other. The special case when public capital and private capital are considered as a substitute for each other has been discussed in Section 3. Lastly concluding remarks are made in Section 4.

2. THE MODEL

We consider a closed economy. The output is produced using private capital and infrastructure service. Following Barro (1990), in this model, infrastructure service enters into the production function as a flow variable. Production of infrastructure service requires physical capital. The labour is not considered as a factor of production in our model, because we focus only on the physical capital investment in the infrastructure sector and precisely would like to find out whether the PPP investment is an optimal solution in the long run or not. Usually, construction of infrastructure requires a lot more of physical capital as an input compared to labour. In this model, we consider only the physical infrastructure and not the social infrastructure (like education and health). Therefore, the inclusion of labour will not contribute much to the findings of the study. There are several other works on infrastructure investment in an endogenous growth framework where labour is not considered as an input. To mention a few of them are Greiner and Hanusch (1998), Dasgupta (1999), Mourmouras and Lee (1999), Devarajan et al. (1998), Dasgupta (2001), Chatterjee and Ghosh (2011) etc. Also, the omission of labour makes our model algebraically simple to deal with. The infrastructure services may be provided privately (by the representative agent), or publicly (by the government) or by a public-private partnership (by a collaboration of both). Initially, it is assumed that private capital and public capital are complements in producing infrastructure service. Government accumulates public capital by imposing a tax on output. It is also assumed that the government runs a balanced budget. The economy is populated by a large number of the infinitely lived household having perfect foresight.

An infinitely lived representative agent maximizes the present discounted value of utility from consumption. The utility function of the representative agent is given by

$$U = \int_0^\infty \frac{C^\gamma}{\gamma} e^{-\beta t} dt, \ -\infty < \gamma \leq 1, \ \beta > 0, \tag{1}$$

Infrastructure service is produced using capital provided partly by the government denoted by k_g and also by a fraction of private capital denoted by k. The production function of infrastructure service is assumed to take the Cobb-Douglas form,[1]

$$k_I^S = B(\theta k)^\alpha k_g^{1-\alpha}, \tag{2}$$

where k_g and k complement each other and θ is the fraction of privately owned physical capital allocated to infrastructure service production. The product exhaustion theorem states that in a competitive factor market since factors of production are paid a price equal to their value of marginal product, the payments to the factors will exhaust the value of total product. Here, we assume perfect competition in the product and the factor markets. So, in the absence of any external effect, the assumption of the constant returns to scale is necessary for product exhaustion and zero economic profits in the long-run. Increasing returns to scale is not compatible with the perfect competition, because the value of the marginal product cannot be distributed among the factors because that will over exhaust the total product. If there are diminishing returns to scale, then even after payments to the factors at competitive rate, there will be excess total product indicating super-normal profit in the economy which is compatible only with the imperfect competition in the product market. Though imperfect competition may be widely prevalent in the factor market and in the infrastructure provision, for simplicity, we have assumed perfect competition and consequently constant returns to scale. In the existing literature, Dasgupta (1999), Tsoukis and Miller (2003), Bucci and Bo (2012) also consider constant returns to scale in the infrastructure production function.

The final output (Y) is produced using the remaining fraction of private capital (k) and infrastructure service (k_I^S). The production function of the final output is given by

$$Y = A\{(1-\theta)k\}^\eta (k_I^S)^{1-\eta}, \; 0 < \eta < 1, \; A > 0. \tag{3}$$

In equation (3), $(1-\theta)$ is the fraction of privately owned physical capital allocated to the production of final goods. Post-tax disposable income over consumption and depreciation is invested as privately owned physical capital. Tax revenue over depreciation is accumulated as publicly owned physical capital. The rate of accumulation of private capital and public capital is governed by the following equations:

$$\dot{k} = (1-\tau)Y - C - \delta_k k, \tag{4}$$
$$\dot{k}_g = \tau Y - \delta_g k_g, \tag{5}$$

[1] According to Dasgupta (1999), under the Cobb-Douglas framework assumed, there is restriction on the model; this arises from the fact that the share of each factor in the Y-sector output must be a constant under competitive conditions. (page. 367)

where a dot over its variable indicates its time derivative. $\dot{k}$ is the change in the private physical capital stock per unit of time and $\dot{k}_g$ is the change in the public capital stock per unit of time, τ is the constant marginal tax rate on output used to finance the provision of infrastructure when private and public capital are complement to each other, δ_k and δ_g denote their corresponding depreciation rates, $(1-\tau)Y - C$ and τY measure the flow of new investments into the two capital goods, k and k_g respectively. We assume that depreciation rates of both private capital and public capital are positive.

2.1. Decentralized Economy

The representative agent maximizes the present discounted value of inter-temporal utility over an infinite time horizon subject to the resource constraints given by equation (3) and (4) and with respect to control variables C and θ_d. The subscripts d represent decentralized economy in the model. Private agents (households and firms) take fiscal policies as given when making private optimal decisions. The current-valued Hamiltonian of the representative agent is given by,

$$H_c = \frac{C^\gamma}{\gamma} + \lambda[(1-\tau_d)Y - C - \delta_k k]. \tag{6}$$

While maximizing their instantaneous inter-temporal utility function, the representative agent considers k_g to be given.

The first order conditions necessary for this optimization problem with respect to control variables C, θ_d are:

$$C^{\gamma-1} = \lambda, \tag{7}$$

$$\frac{\eta}{(1-\theta_d)} = \frac{(1-\eta)\alpha}{\theta_d}. \tag{8}$$

From equation (8) we get the value of θ_d as,

$$\theta_d = \frac{\alpha(1-\eta)}{\eta + \alpha(1-\eta)}. \tag{9}$$

In a decentralized economy, θ_d is the fraction of privately owned physical capital allocated to the infrastructure service production. Please note that θ_d is the output elasticity of private physical capital in the infrastructure production. θ_d has a negative relationship with η and positive relationship with α.

Time derivatives of the co-state variables satisfying the optimum growth path are:

$$\begin{aligned}\frac{\dot{\lambda}}{\lambda} = {} & \beta - (1-\tau_d)A\eta(k-\theta_d k)^{\eta-1}(1-\theta_d)\left\{B(\theta_d k)^\alpha k_g^{1-\alpha}\right\}^{1-\eta} \\ & -(1-\tau_d)A(k-\theta_d k)^\eta(1-\eta)\left\{B(\theta_d k)^\alpha k_g^{1-\alpha}\right\}^{-\eta} B\alpha(\theta_d k)^{\alpha-1}k_g^{1-\alpha}(\theta_d) + \delta_k,\end{aligned} \tag{10}$$

Dividing equation (10) by k, we have

$$\frac{\dot{\lambda}}{\lambda} = \beta - (1-\tau_d)A(1-\theta_d)^{\eta}(B\theta_d^{\alpha}u_d^{1-\alpha})^{1-\eta}[\eta + (1-\eta)\alpha] + \delta_k, \tag{11}$$

where u_d denotes (k_g/k), the ratio of public capital to private capital. The interior value of u_d is also necessary for the public-private partnership in the infrastructure provision.

Taking the log and derivative of equation (7), we have,

$$(\gamma - 1)\frac{\dot{c}}{c} = \frac{\dot{\lambda}}{\lambda}. \tag{12}$$

From equations (11) and (12), the growth rate of consumption is obtained as,

$$\frac{\dot{c}}{c} = \frac{(1-\tau_d)A(1-\theta_d)^{\eta}B^{1-\eta}\theta_d^{\alpha(1-\eta)}u_d^{(1-\alpha)(1-\eta)}[\eta+\alpha(1-\eta)]-\delta_k-\beta}{(1-\gamma)} = g_d. \tag{13}$$

We find that the growth rate of consumption depends on the ratio of public capital to private capital, u_d.

2.1.1. Steady-state Growth for the Decentralized Economy

Steady-state growth path is defined as a path along which consumption, public physical capital, and private physical capital grow at a constant rate and the fraction of private capital devoted for infrastructure production is constant. For the existence of steady-state balanced growth equilibrium, $\dot{c}/c$ must be constant. We also assume, τ_d, θ_d and g_d to be constant along the steady state. If g_d is constant, then u_d must also be constant. If u_d and $\dot{k}/k$ are constant in the equation (14) then c/k is also constant.

Therefore, in steady-state balanced growth, $\dot{c}/c = \dot{k}/k = \dot{k_g}/k_g = g_d$

$$\frac{\dot{k}}{k} = (1-\tau_d)A(1-\theta_d)^{\eta}B^{1-\eta}{\theta_d}^{\alpha(1-\eta)}u_d^{(1-\alpha)(1-\eta)} - \frac{c}{k} - \delta_k, \tag{14}$$

$$\frac{\dot{k_g}}{k_g} = \tau_d A\,(1-\theta_d)^{\eta}B^{1-\eta}\theta_d^{\alpha(1-\eta)}u_d^{-\alpha(1-\eta)-\eta} - \delta_g. \tag{15}$$

Now equating demand-side growth rate given by equation (13) and supply-side growth rate given by equation (15), we have

$$\begin{aligned}(1-\gamma)\tau_d A(1-\theta_d)^{\eta}B^{1-\eta}\theta_d^{\alpha(1-\eta)}u_d^{-\alpha(1-\eta)-\eta} \\ = (1-\tau_d)A(1-\theta_d)^{\eta}B^{1-\eta}\theta_d^{\alpha(1-\eta)}u_d^{(1-\alpha)(1-\eta)}[\eta+\alpha(1-\eta)] \\ -\delta_k - \beta + \delta_g(1-\gamma).\end{aligned} \tag{16}$$

In the above equation, there is only one unknown variable that is, u_d. We can solve for the equilibrium rate of u_d^* graphically. Let left-hand side of equation (16) be $f_1(u_d)$ and right-hand side of the equation be $f_2(u_d)$. For the existence and uniqueness of the equilibrium solution u_d^*, we differentiate f_1 and f_2 with respect to u_d, we find an interior solution as given in the figure 2.

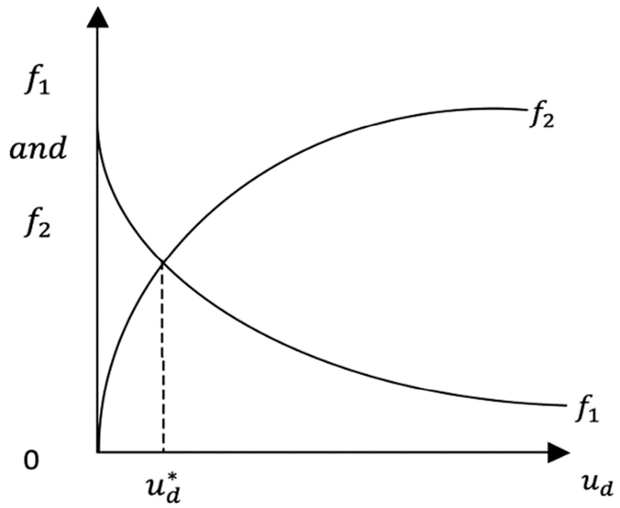

Figure 1. Existence of Unique u_d

Figure 1 shows that there exists a unique equilibrium u_d^* for the competitive economy under the complementary relation between the public capital and private capital.

Proposition 1. *There exists a unique steady state balanced growth rate in the decentralized economy when public and private capital complements each other. The growth maximizing tax rate is positive for financing of infrastructure services.*

2.1.2. Transitional Dynamics for the Complementary Case.

To study the dynamic behaviour of the model when there is complementary relationship between the public capital and private capital, we analyse the transitional dynamics in this section. Before we analyse the model around the steady state, we define a new variables, $x = c/k$ and $u = k_g/k$. From equation (9) we find that θ is always a constant. Differentiating these variables with respect to time leads to a two dimensional system of differential equations. Therefore, the first-order differential equation system in two variables in the general form is given as,

$$\frac{\dot{x}}{x} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = f(x,u), \tag{17}$$

$$\frac{\dot{u}}{u} = \frac{\dot{k}_g}{k_g} - \frac{\dot{k}}{k} = g(x,u). \tag{18}$$

From equation (17)–(18), we have,

$$\frac{\dot{x}}{x} = \frac{[(1-\tau)A(1-\theta)^{\eta}\{B\theta^{\alpha}u^{1-\alpha}\}^{1-\eta}\{[\eta+(1-\eta)\alpha]-(1-\gamma)\}-\delta_k\gamma-\beta+x(1-\gamma)]}{(1-\gamma)}, \tag{19}$$

$$\frac{\dot{u}}{u} = A(1-\theta)^{\eta}B^{1-\eta}\theta^{\alpha(1-\eta)}u^{-\alpha(1-\eta)-\eta}[\tau-(1-\tau)u]-\delta_g+\delta_k+x. \tag{20}$$

At steady state, $\dot{x}/x = 0$ and $\dot{u}/u = 0$. The relationship between x and u are given by equations (19) and (20) respectively,

$$x^* = \frac{\delta_k\gamma+\beta}{(1-\gamma)} - \frac{(1-\tau)}{(1-\gamma)}A(1-\theta)^{\eta}\{B\theta^{\eta}u^{1-\alpha}\}^{1-\eta}\{\eta+\gamma+(1-\eta)\alpha-1\}, \tag{21}$$

$$x^* = \delta_g - \delta_k - A(1-\theta)^{\eta}B^{1-\eta}\theta^{\alpha(1-\eta)}u^{-\alpha(1-\eta)-\eta}[\tau-(1-\tau)u]. \tag{22}$$

We show the qualitative transitional dynamic analysis with the help of phase diagram using equations (21)–(22).

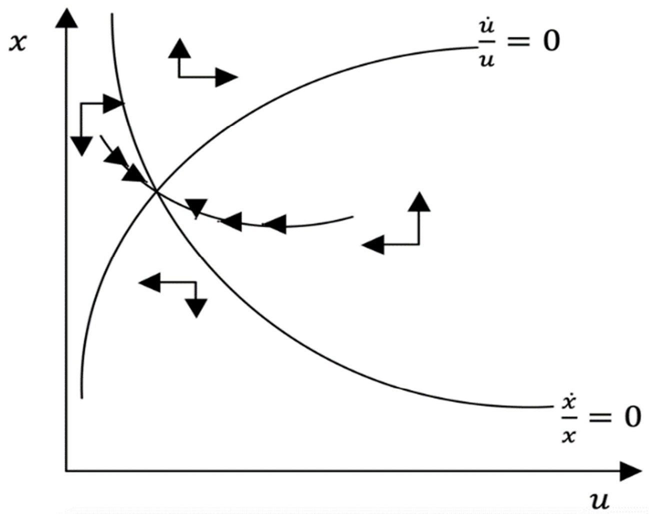

Figure 2. Saddle Path Stability when There is Complementary Relation between the Private Capital and Public Capital

To determine the local behaviour around the steady state, we linearize the dynamic system. The characteristic equation is given by,

$$J_E = \begin{bmatrix} 1-\lambda & M \\ 1 & N-\lambda \end{bmatrix} = 0, \tag{23}$$

where, $M = ((1-\tau)A(1-\theta)^\eta\{B\theta^\alpha\}^{1-\eta}(1-\eta)(1-\alpha)u^{(1-\alpha)(1-\eta)-1}\{[\eta+(1-\eta)\alpha] - (1-\gamma)\}/(1-\gamma))N = -A(1-\theta)^\eta B^{1-\eta}\theta^{\alpha(1-\eta)}u^{-\alpha(1-\eta)-\eta}(1-\tau)[\{\alpha(1-\eta)+\eta\}\tau/u(1-\tau)+2]$, λ is the characteristic root of the dynamic system. From the matrix given in equation (23), we obtain the characteristic equation,

$$\lambda^2 - \lambda(1+N) - (M-N) = 0. \tag{24}$$

The characteristic roots depend critically on the expression $(1+N)$ and $(N-M)$. Where, $(1+N)$ is the sum of the principal – diagonal elements of the Jacobian (or trace of Jacobian) and $(N-M)$ is the determinant of Jacobian. Now, the characteristic roots can be expressed as,

$$r_1, r_2 = \frac{trJ_E \pm\sqrt{(trJ_E)^2-4|J_E|}}{2}. \tag{25}$$

Thus, in order for this dynamic system to be stable, there must at least one stable eigen-value or characteristic root. The trace of the Jacobian is given by

$$trJ_E = 1 - A(1-\theta)^\eta B^{1-\eta}\theta^{\alpha(1-\eta)}u^{-\alpha(1-\eta)-\eta}(1-\tau)\left[\frac{\{\alpha(1-\eta)+\eta\}\tau}{u(1-\tau)} + 2\right]. \tag{26}$$

The trJ_E is positive, implying sum of the roots are positive. So, for the dynamic system to be stable $detJ_E$ must be negative.

$$detJ_E = -[(1-\tau)A(1-\theta)^\eta B^{1-\eta}\theta^{\alpha(1-\eta)}u^{-\alpha(1-\eta)-\eta} \{\frac{(1-\eta)(1-\tau)([\eta+(1-\eta)\alpha](1-\alpha\gamma)-(1-\gamma))}{(1-\gamma)} + \frac{[\eta+(1-\eta)\alpha]\tau}{u(1-\tau)} + 2\}]. \tag{27}$$

Since $[\eta+(1-\eta)\alpha] > 0$, $detJ_E$ is negative. Hence the dynamic system is saddle path stable when the private physical capital and public capital are complementary to each other in infrastructure provision.

Proposition 2. *The dynamic system is saddle path stable.*

2.1.3. Growth Maximizing Tax Rate

From equation (16), we have,

$$A(1-\theta_d)^\eta B^{1-\eta}\theta_d^{\alpha(1-\eta)}u_d^{-\alpha+\alpha\eta-\eta}\left[\frac{(1-\tau_d)}{(1-\gamma)}\{\eta+\alpha(1-\eta)\}u_d - \tau_d\right] = \frac{\delta_k+\beta}{(1-\gamma)} - \delta_g. \tag{28}$$

Differentiating equation (28) with respect to τ_d, wefind that,

$$\left(\frac{\partial u_d}{\partial \tau_d}\right) = \frac{(1-\gamma)+\{\eta+\alpha(1-\eta)\}u_d}{\{\eta+\alpha(1-\eta)\}\left[(1-\gamma)\frac{\tau_d}{u_d}+(1-\tau_d)(1-\eta)(1-\alpha)\right]} > 0, \tag{29}$$

$(\partial^2 u_d/\partial \tau_d^2)$ is given in equation (30) of the Appendix A1.

Now for the existence of growth maximizing tax rate, the first order condition and second order condition given by equation (29) and (30) must be positive. To find the growth- maximizing tax rate, equation (13) is differentiated with respect to τ_d. We set $\partial g_d / \partial \tau_d = 0$.

$$\frac{\partial g_d}{\partial \tau_d} = AB^{1-\eta}\{\eta + \alpha(1-\eta)\}(1-\theta_d)^{\eta}\theta_d^{\alpha(1-\eta)}u_d^{(1-\alpha)(1-\eta)-1}$$
$$[-u_d + (1-\tau_d)(1-\eta)(1-\alpha)\frac{\partial u_d}{\partial \tau_d}] = 0. \tag{31}$$

From equation (31), we obtain the value of $u_d = [(1-\tau_d)(1-\eta)(1-\alpha)\partial u_d / \partial \tau_d]$. The growth-maximizing tax rate, τ_d^* for the decentralized economy is obtained after substituting the value of $\partial u_d/\partial \tau_d$ in u_d. Hence, the growth maximizing tax rate for the decentralized economy is a function of the marginal productivity of public capital and the output elasticity of the public capital in the production of infrastructure services.

$$\tau_d^* = (1-\eta)(1-\alpha). \tag{32}$$

Now, the second order condition must be negative for the existence of growth maximizing tax rate,

$$\frac{\partial^2 g_d}{\partial \tau_d^2} = N u_d^{(1-\alpha)(1-\eta)-2}[(1-\alpha)(1-\eta)\{-2u_d\frac{\partial u_d}{\partial \tau_d} + (1-\tau_d)$$
$$\{(1-\alpha)(1-\eta)-1\}(\frac{\partial u_d}{\partial \tau_d})^2 + (1-\tau_d)\frac{\partial^2 u_d}{\partial \tau_d^2}\}] < 0, \tag{33}$$

where $N = (AB^{1-\gamma}/(1-\gamma))\{\eta+\alpha(1-\eta)\}(1-\theta_d)^{\eta}\theta_d^{\alpha(1-\eta)}$ which is positive.

The interior values of θ_d and τ_d imply public-private partnership in infrastructure. While α is the parameter and the equilibrium value of θ is determined in competitive economy and τ are determined optimally in command economy problem.

Proposition 3. *When public and private capital complements each other, there exists a unique, positive growth-maximizing tax rate for the financing of infrastructure services.*

The main concern for the policy-makers today in the developing countries is to accelerate growth. The present research points out that maximization of the growth rate in the PPP model requires setting the tax rate equal to the marginal productivity of public capital that is the product of marginal productivity of infrastructure services in output and the output elasticity of the public capital in the production of infrastructure services.

2.2. Command Economy

The difference between competitive economy and command economy is that: in competitive economy τ and k_g are considered to be given; but, in command economy while optimizing the present discounted value of utility the dynamic constraint $\dot{k}_g$ is taken into consideration and tax rate (τ) is one of the choice variables of the social planner. Because of the difference in optimization procedure the results obtained in competitive and command economy are different.

The command economy maximizes the utility function over the infinite time horizon given by equation (1), subject to the resource constraints (4) and (5), and with respect to the control variables C, θ_c, τ_c, where the subscript c represents command economy in the model. The current value Hamiltonian is,

$$H_c = \frac{C^\gamma}{\gamma} + \lambda_1[(1-\tau_c)Y - C - \delta_k k] + \lambda_2[\tau_c Y - \delta_g k_g], \tag{34}$$

λ_1 and λ_2 are the co-state variables of k and k_g respectively, representing their shadow prices.

The first order conditions with respect to control variables, τ_c, θ_c are given by the following equations:

$$C^{\gamma-1} = \lambda_1, \tag{35}$$
$$\lambda_1 = \lambda_2, \tag{36}$$
$$\frac{\partial Y}{\partial \theta_c}[\lambda_1(1-\tau_c) + \lambda_2\tau_c] = 0. \tag{37}$$

From equation (37), we obtain the optimal value of θ_c,

$$\theta_c = \frac{\alpha(1-\eta)}{\eta+\alpha(1-\eta)}, \tag{38}$$

Note that the above-mentioned θ_c is same with θ_d given by equation (9) implying that, the share of private investment in the infrastructure provision in the PPP model for both decentralized economy and command economy is same in the case of a complementary relationship between private capital and public capital.

Time derivatives of the co-state variables satisfying the optimum growth path are given by following:

$$\frac{\dot{\lambda}_1}{\lambda_1} = \beta - \frac{\partial Y}{\partial k} + \delta_k, \tag{39}$$

$$\frac{\dot{\lambda_2}}{\lambda_2} = \beta - \frac{\partial Y}{\partial k} + \delta_k. \tag{40}$$

Using equation (36) and equating equations (39) and (40), we get,

$$\frac{\delta_k - \delta_g}{A(1-\theta_c)^{\eta} B^{1-\eta} \theta_c^{\alpha(1-\eta)}} = u_c^{(1-\alpha)(1-\eta)} [\{\eta + \alpha(1-\eta)\} - (1-\alpha)(1-\eta)\frac{1}{u_c}]. \tag{41}$$

Taking the log and derivative of equation (35), we get,

$$(\gamma - 1)\frac{\dot{C}}{C} = \frac{\dot{\lambda}_1}{\lambda_1}. \tag{42}$$

Therefore, the growth rate of consumption for command economy is given by,

$$\frac{\dot{c}}{c} = \frac{A(1-\theta_c)^{\eta} B^{1-\eta} \theta_c^{\alpha(1-\eta)} u_c^{(1-\alpha)(1-\eta)} \{\eta + \alpha(1-\eta)\} - \beta - \delta_k}{(1-\gamma)} = g_c. \tag{43}$$

2.2.1. Steady-state Growth for the Command Economy

In steady-state balanced growth equilibrium, $\dot{c}/c$ must be constant. Since growth rate of consumption depends on u_c, therefore if $\dot{c}/c$ is constant then u_c is also constant. Therefore in steady-state balanced growth, $\dot{k}_g/k_g = \dot{k}/k = \dot{c}/c = g_c$.

In equation (41), there is only one unknown variableu$_c$. Hence, we can solve for the equilibriumu$_c$graphically. On the left-hand side of equation (41), we do not have u_c. Let the left-hand side of the equation be represented as J and the right-hand side of the equation be represented as $f(u_c)$. Differentiating $f(u_c)$ with respect to u_c we find the existence and uniqueness of the equilibrium u_c^*, which is illustrated in figure 3 below.

$$f'(u_c) = (1-\alpha)(1-\eta)u_c^{-\eta-\alpha(1-\eta)} \left[\frac{(1+u_c)(\alpha(1-\eta)+\eta)}{u_c}\right] > 0, \tag{44}$$

$$f''(u_c) = (1-\alpha)(1-\eta)u_c^{-\eta-\alpha(1-\eta)-2}[\alpha(1-\eta)+\eta]\left[\left(-\eta-\alpha(1-\eta)\right)(1+u_c)+u_c\right] < 0. \tag{45}$$

Figure 3, shows that there exists a unique equilibrium u_c^*, for the command economy under the complementary relation between the public capital and private capital.

Proposition 4. *There exists a unique growth rate in the command economy when public and private capital complements each other. Also, there exists an optimal tax rate that maximizes the welfare.*

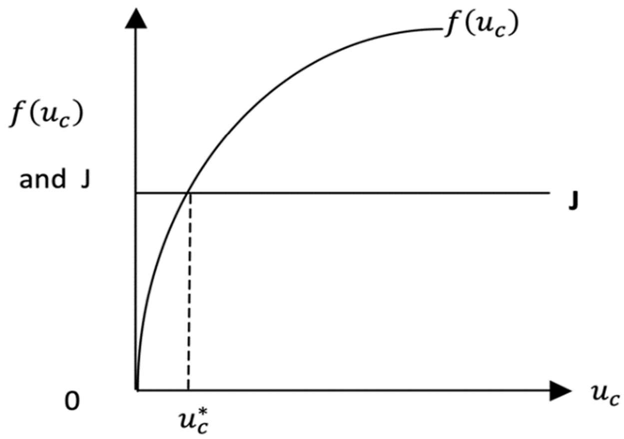

Figure 3. Existence of Unique u_c

2.2.2. Optimal Tax Rate

Equating (26) and (43) i.e., $\dot{k}_g/k_g = g_c$, we have,

$$\tau_c A(1-\theta_c)^\eta B^{1-\eta}\theta_c^{\alpha(1-\eta)}u_c^{-\alpha(1-\eta)-\eta} - \delta_g$$
$$= \frac{1}{(1-\gamma)}\left[A(1-\theta_c)^\eta B^{1-\eta}\theta_c^{\alpha(1-\eta)}u_c^{(1-\alpha)(1-\eta)}\{\eta+\alpha(1-\eta)\} - \beta - \delta_k\right]. \quad (46)$$

From equation (46), we find the optimal tax rate, which is welfare maximizing,

$$\tau_c = \frac{\frac{A(1-\theta_c)^\eta}{(1-\gamma)}B^{1-\eta}\theta_c^{\alpha(1-\eta)}u_c^{(1-\alpha)(1-\eta)}\{\eta+\alpha(1-\eta)\} - \frac{(\beta+\delta_k)}{(1-\gamma)} + \delta_g}{A(1-\theta_c)^\eta B^{1-\eta}\theta_c^{\alpha(1-\eta)}u_c^{-\alpha(1-\eta)-\eta}}. \quad (47)$$

This is the first best solution of the optimal tax rate in the command economy. We can also achieve command economy solution through the decentralized economy by equating steady- state growth rates obtained in the command economy as expressed in equation (43) and steady state growth rate obtained in the market economy as expressed by equation (13) and imposing the tax rate that equals both the growth rates. Hence, the second best tax rate is $\tau^* = 1-(u_c/u_d)^{(1-\alpha)(1-\eta)}$. Note that, u_d must be greater than u_c for the tax rate to be positive.

2.3. Zero-depreciation Rate of Physical Capital

In this section, we assume that the depreciation rate of both the public capital and the private physical capital are zero.

In the decentralized economy, when $\delta_k = \delta_g = 0$, there is not a specific value of u_d but graphical solution shows that there will be a reduction in the value of u_d. However, in the case of a command economy, we get the value of u_c as,

$$u_c = \frac{(1-\alpha)(1-\eta)}{\{\eta+\alpha(1-\eta)\}}. \tag{48}$$

When $\delta_k = \delta_g = 0$, the decentralized economy growth rate and the command economy growth rate are given respectively as,

$$g_d = \frac{(1-\tau_d)A(1-\theta_d)^{\eta}B^{1-\eta}\theta_d^{\alpha(1-\eta)}u_d^{(1-\alpha)(1-\eta)}[\eta+\alpha(1-\eta)]-\beta}{(1-\gamma)}, \tag{49}$$

$$g_c = \frac{A(1-\theta_c)^{\eta}B^{1-\eta}\theta_c^{\alpha(1-\eta)}u_c^{(1-\alpha)(1-\eta)}[\eta+\alpha(1-\eta)]-\beta}{(1-\gamma)}. \tag{50}$$

Therefore, comparing equation (49) and (50), we see that the growth rate of the command economy is greater than the growth rate of the decentralized economy if $u_d < \{(1-\alpha)(1-\eta)\}\{\eta+\alpha(1-\eta)\}^{-1-(1-\alpha)(1-\eta)/(1-\alpha)(1-\eta)}$.

In general case, when depreciation rates of physical capital are not zero, the command economy growth rate is greater than the growth rate of the decentralized economy, if$[\eta+\alpha(1-\eta)]u_d^{(1-\alpha)(1-\eta)} < u_c^{(1-\alpha)(1-\eta)}$.

Proposition 5. *Command economy growth rate may not be higher than the competitive economy growth rate.*

In a model of non-rival infrastructure, Dasgupta (1999) finds a similar result, where the market economy grows faster than the command economy, though the latter dominates in welfare. In our paper, if the marginal productivity of private capital in output andthe output elasticity of private capital in the production of infrastructure services are high then the market economy would allocate more resources (even more than what is optimal) to private capital investment and thus result into faster growth rate than the command economy.

3. PRIVATE CAPITAL AND PUBLIC CAPITAL ARE PERFECT SUBSTITUTES

In this section, we consider the case when private capital and public capital are perfect substitutes in producing infrastructure service.

Therefore the production function of infrastructure service is given by

$$k_I^S = \theta k + k_g, \tag{51}$$

θ is the fraction of privately owned physical capital allocated to infrastructure service production. k_I^S, k and k_g denote flow of infrastructure services, private capital and public capital respectively.

The final output (Y) is produced using a fraction of private capital (k) and infrastructure service (k_I^S). The public capital and private capital are perfect substitutes in the production of infrastructure services, as shown in equation (51).

The production function of the final output is given by,

$$Y = A\{(1-\theta)k\}^{\eta}\left(\theta k + k_g\right)^{1-\eta}, \quad 0 < \eta < 1,\ A > 0. \tag{52}$$

In equation (52), $(1-\theta)$ is the fraction of privately owned physical capital allocated to the production of final goods.

3.1. Decentralized Economy

The representative agent maximizes the inter-temporal utility over an infinite time horizon as given in equation (1) subject to the resource constraints given by equation (3) and (4) and with respect to control variables C and θ_d. Private agents (households and firms) take fiscal policies as given when making private optimal decisions. The current-valued Hamiltonian of the representative agent is given by,

$$H_c = \frac{C^{\gamma}}{\gamma} + \lambda\,[(1-\tau_d)Y - C - \delta_k\, k]. \tag{53}$$

While maximizing their instantaneous inter-temporal utility function, the representative agent considers k_g to be given.

The first order conditions necessary for this optimization problem with respect to control variables C, θ_d are:

$$C^{\gamma-1} = \lambda, \tag{54}$$

$$\eta\left(k_g + k\right) = (1+\theta_d)k. \tag{55}$$

From equation (55), we obtain constant value of the share of private investment in the infrastructure provision by PPP mode in this case too. Therefore, for the decentralized economy, θ_d is given as,

$$\theta_d = 1 - \eta - \eta u_d. \tag{56}$$

Time derivatives of the co-state variables satisfying the optimum growth path are:

$$\frac{\dot{\lambda}}{\lambda} = \beta - (1-\tau_d)A(k-\theta_d k)^{\eta-1}\left(\theta_d k + k_g\right)^{-\eta}$$

$$\left[\eta(1-\theta_d)\left(\theta_d k + k_g\right) - (1-\eta)\theta_d(k-\theta_d k)\right] + \delta_k. \tag{57}$$

Dividing equation (57) by k and denoting the ratio of public to private capital by u_d, we have

$$\frac{\dot{\lambda}}{\lambda} = \beta - (1-\tau_d)A(1-\theta_d)^{\eta-1}(\theta_d+u_d)^{-\eta}$$
$$[\eta(1-\theta_d)(\theta_d+u_d) - (1-\eta)\theta_d(1-\theta_d)] + \delta_k. \tag{58}$$

Taking the log and derivative of equation (54), we get,

$$(\gamma-1)\frac{\dot{c}}{c} = \frac{\dot{\lambda}}{\lambda}. \tag{59}$$

Using equation (58) and (59), the growth rate of consumption for the substitute cum decentralized case is given by,

$$\frac{\dot{c}}{c} = \frac{(1-\tau_d)A(1-\theta_d)^{\eta-1}(\theta_d+u_d)^{-\eta}[\eta(1-\theta_d)(\theta_d+u_d)+(1-\eta)\theta_d(1-\theta_d)]-\delta_k-\beta}{(1-\gamma)}. \tag{60}$$

We find that growth rate of consumption depend on the ratio of public capital to private capital (u_d).

3.1.1. Steady-State Growth for the Decentralized Economy

For steady-state balanced growth equilibrium to prevail (the growth rate of consumption to be constant), the ratio of public capital (k_g) to private physical capital (k)must also remain constant. Let the growth rate of consumption in the decentralized economy (when private and physical capital are substitutes) is g_d, i.e.; $\dot{c}/c = g_d$ and the growth rate of private physical capital be g_k,

If g_d is constant, $\dot{k}/k = \dot{k}_g/k_g = g_k$ is also constant. From equation (56), we have a constant θ_d, which is the share of private investment in the PPP model of infrastructure provision in the decentralized economy.

$\dot{k}/k$ and $\dot{k}_g/k_g$ are obtained in equation (61) and (62),

$$\frac{\dot{k}}{k} = (1-\tau_d)A(1-\theta_d)^{\eta}(\theta_d+u_d)^{1-\eta} - \frac{c}{k} - \delta_k\,, \tag{61}$$
$$\frac{\dot{k}_g}{k_g} = \tau_d\, A(1-\theta_d)^{\eta}(u_d)^{-\eta}\left(\frac{\theta_d}{u_d}+1\right)^{1-\eta} - \delta_g. \tag{62}$$

In steady-state, if $\dot{k}/k$ and u_d are constant in equation (61) then c/k must also be constant.

Equating the growth rates of public capital accumulation given by equation (62) with the growth rate of consumption given by equation (59), we get, the equilibrium values of

u_d and g_d respectively,

$$u_d = \frac{\tau_d A\eta^{\eta}(1-\eta)^{1-\eta}(1-\gamma)}{A\eta^{\eta}(1-\eta)^{1-\eta}(1-2\tau_d+\tau_d\gamma)-\delta_k-\beta+\delta_g(1-\gamma)}. \tag{63}$$

The conditions for u_d to be positive are $\gamma \leq 1$ and $\tau_d \geq (\delta_k - \delta_g + \gamma\delta_g + \beta - A\eta^{\eta}(1-\eta)^{1-\eta})/(A\eta^{\eta}(1-\eta)^{1-\eta}(2-\gamma))$. These conditions are both necessary and sufficient condition for $u_d \geq 0$.

The steady-state balanced growth rate of the competitive economy for the substitute case is given by,

$$g_d = \frac{1}{(1-\gamma)}[(1-\tau_d)A\eta^{\eta}(1-\eta)^{1-\eta} - \delta_k - \beta]. \tag{64}$$

3.1.2. Transitional Dynamics for the Substitute Case

To study the dynamic behaviour of the model when there is substitute relationship between the public capital and private capital, we analyse the transitional dynamics in this section. Before we analyse the model around the steady state, we define a new variables, We denote $x = c/k$ and $u = k_g/k$. From equation (56) we find that θ is always a constant. Differentiating x and u with respect to time leads to a two dimensional system of differential equations. Therefore, the first-order differential equation system in two variables in the general form is given as,

$$\frac{\dot{x}}{x} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = f(x,u), \tag{65}$$

$$\frac{\dot{u}}{u} = \frac{\dot{k}_g}{k_g} - \frac{\dot{k}}{k} = g(x,u). \tag{66}$$

From equation (65)-(66), we have,

$$\frac{\dot{x}}{x} = \frac{[(1-\tau)A(1-\theta)^{\eta}(\theta+u)^{-\eta}\{[\eta(\theta+u)-(1-\eta)\theta]-(\theta+u)(1-\gamma)\}-\beta-\delta_k\gamma+x(1-\gamma)]}{(1-\gamma)}, \tag{67}$$

$$\frac{\dot{u}}{u} = A(1-\theta)^{\eta}(\theta+u)^{1-\eta}\left[\frac{\tau}{u} - (1-\tau)\right] + \delta_k - \delta_g + x. \tag{68}$$

At steady state, $\dot{x}/x = 0$ and $\dot{u}/u = 0$. The relationship between x and u are given by equations (69) and (70) respectively,

$$x^* = \frac{\delta_k\gamma+\beta}{(1-\gamma)} - \frac{(1-\tau)}{(1-\gamma)}A(1-\theta)^{\eta}(\theta+u)^{1-\eta}\left[(\eta-1+\gamma) - \frac{(1-\eta)\theta}{(\theta+u)}\right], \tag{69}$$

$$x^* = \delta_g - \delta_k - A(1-\theta)^{\eta}(\theta+u)^{1-\eta}\left[\frac{\tau}{u} - (1-\tau)\right]. \tag{70}$$

We show the transitional dynamic analysis with the help of phase diagram using equations (69)-(70).

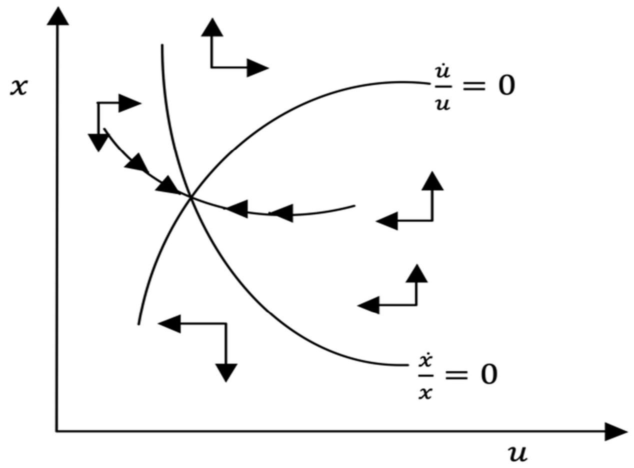

Figure 4. Saddle Path Stability when There is Substitute Relation between the Private Capital and Public Capital

To determine the local behaviour around the steady state, the characteristic equation of the reduced linearization is given by,

$$J_E = \begin{bmatrix} 1-\lambda & M \\ 1 & N-\lambda \end{bmatrix} = 0, \tag{71}$$

where $M = (1-\tau)A(1-\theta)^{\eta}(\theta+u)^{-\eta}\{\eta(\theta+u)[1-\gamma-\eta]+\eta(1-\eta)\theta+\eta+\gamma-1\}$, $N = A(1-\theta)^{\eta}(\theta+u)^{-\eta}[(1-\eta)\tau/u-(1-\tau)(1-\eta)-(\theta+u)\tau/u^2]$, λ is the characteristic root of the dynamic system. From the matrix given in equation (71), we obtain the characteristic equation,

$$\lambda^2 - \lambda(1+N) - (M-N) = 0. \tag{72}$$

The characteristic roots depend critically on the expression $(1+N)$ and $(N-M)$. Where, $(1+N)$ is the sum of the principal–diagonal elements of the Jacobian (or trace of Jacobian) and $(N-M)$ is the determinant of Jacobian. Now, the characteristic roots can be expressed as,

$$r_1, r_2 = \frac{trJ_E \pm \sqrt{(trJ_E)^2 - 4|J_E|}}{2}. \tag{73}$$

Thus in order for this dynamic system to be saddle path stable, there must be two stable and one unstable eigen-values.

$$trJ_E = 1 + A(1-\theta)^{\eta}(\theta+u)^{-\eta}\left[(1-\eta)\frac{\tau}{u} - (1-\tau)(1-\eta) - \frac{(\theta+u)\tau}{u^2}\right]. \tag{74}$$

The trJ_E is positive and now for the BGP (balanced growth path) to be stable $detJ_E$ must be negative.

$$detJ_E = -[A(1-\theta)^{\eta}(\theta+u)^{-\eta}(1-\tau) \{\frac{\tau}{u(1-\tau)}(\frac{\theta}{u}+\eta) + \gamma + \eta(\theta+u)(1-\gamma-\eta) + \eta(1-\eta)\theta\}], \tag{75}$$

$detJ_E$ is negative if $1-\gamma-\eta > 0$ holds, hence this condition is sufficient for the saddle path stability.

Proposition 6. *If $\gamma + \eta < 1$ implying high output elasticity of infrastructure in final goods production, the dynamic system in substitute case is saddle path stable.*

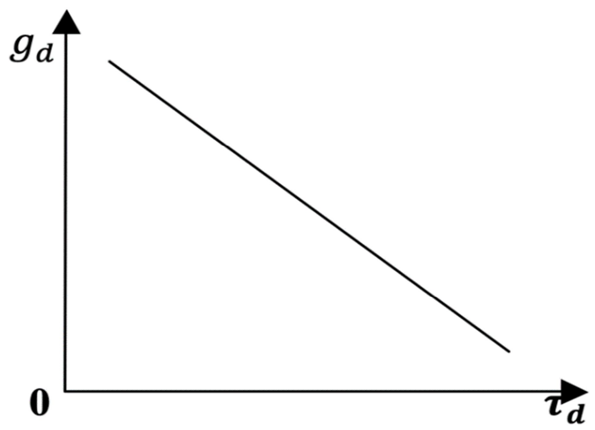

Figure 5. Growth Maximizing Tax Rate $(\tau_d)$is Zero

3.1.3. Growth Maximizing Tax Rate

Differentiating equation (64) with respect to τ_d, we find that the growth-maximizing tax rate is zero in the case of a decentralized economy, when public and private capitals are perfect substitutes. The following figure depicts the relationship between growth rate (g_d) and tax rate (τ_d) in the decentralized economy.

***Proposition** 7. When private capital and public capital are the perfect substitutes, there exists unique steady state balanced growth rate in the decentralized economy. There exists a feasible range of tax rate for which growth rate is a positive and public-private partnership in infrastructure investment happens. However, the unique steady state growth falls with a rise in tax rate. Hence in the decentralized equilibrium, the growth-maximizing income tax is zero, and this suggests complete privatization.*

The reason is quite obvious. As public capital and private capital are perfect substitute and usage of public capital requires taxation that creates a distortionary effect, we obtained the result that growth-maximizing tax rate is zero. Our result is similar to the results obtained by Dasgupta (1999), Fischer and Hof (2000) where they find growth-maximizing tax rate to be zero too.

3.2. Command Economy

The command economy maximizes the present discounted value of utility by taking into account the equation of motion of both private physical capital and public capital with respect to the choice variables including the tax rate.

The command economy maximizes the present discounted value of utility over the infinite time horizon given by equation (1), subject to the resource constraints (4) and (5), and with respect to the control variables C, θ_c, τ_c, where the subscript c stands for a command economy. The current value Hamiltonian is,

$$H_c = \frac{C^\gamma}{\gamma} + \lambda_1[(1-\tau_c)Y - C - \delta_k k] + \lambda_2[\tau_c Y - \delta_g k_g], \tag{76}$$

λ_1 and λ_2 are the co-state variables of k and k_g respectively, representing their shadow prices.

The first order conditions with respect to control variables, τ_c, θ_c are given by the following equations:

$$C^{\gamma-1} = \lambda_1, \tag{77}$$
$$\lambda_1 Y = \lambda_2 Y, \tag{78}$$
$$\frac{\partial Y}{\partial \theta_c}[\lambda_1(1-\tau_c) + \lambda_2 \tau_c] = 0. \tag{79}$$

From equation (79), we obtain the optimal value of θ_c in terms of u_c. Therefore,

$$\theta_c = 1 - \eta - \eta u_c. \tag{80}$$

Note that the above-mentioned θ_c is same with θ_d given by equation (56) implying that, the share of private investment in the infrastructure provision in the PPP model for both decentralized economy and command economy is same also in the case of a perfect substitute relationship between the private capital and the public capital.

Time derivatives of the co-state variables satisfying the optimum growth path are given by following,

$$\frac{\dot{\lambda}_1}{\lambda_1} = \beta - \frac{\partial Y}{\partial k} + \delta_k, \tag{81}$$

$$\frac{\dot{\lambda}_2}{\lambda_2} = \beta - \frac{\partial Y}{\partial k_g} + \delta_g. \tag{82}$$

From equations (81) and (82), we get the equilibrium value of u_c,

$$u_c = \frac{(\delta_k - \delta_g) - A\eta^\eta(1-\eta)^{2-\eta}}{A\eta^\eta(1-\eta)^{2-\eta}}. \tag{83}$$

Note that for u_c to be positive,the condition $(\delta_k - \delta_g) \geq A\eta^\eta(1-\eta)^{2-\eta}$ must hold true.

From the condition that $u_c \geq 0$ and $\theta_c \geq 0$, we find $1 \leq \delta_k - \delta_g / A\eta^\eta(1-\eta)^{2-\eta} \leq 1/\eta$.

Taking the log and derivative of equation (77), we get,

$$(\gamma - 1)\frac{\dot{C}}{C} = \frac{\dot{\lambda}_1}{\lambda_1}. \tag{84}$$

Using equation (81) and equation (82), the growth rate of consumption for command economy for the substitute case is given by

$$\frac{\dot{C}}{C} = \frac{A\eta^\eta(1+u_c)^\eta(\theta_c+u_c)^{-\eta}[\eta(\theta_c+u_c)-(1-\eta)(1-\eta-\eta u_c)]-\delta_k-\beta}{(1-\gamma)} = g_c. \tag{85}$$

Also, the growth rate of consumption depends on the ratio of public capital to private capital, u_c.

3.2.1. Steady State Balanced Growth for the Command Economy

In steady state balanced growth, the growth rate of consumption, $\dot{c}/c$ must be constant. In steady state $\dot{k}_g/k_g = \dot{k}/k = \dot{c}/c = g_c$. Now equating $\dot{k}_g/k_g = \dot{k}/k$, we obtain the equilibrium tax rate in the command economy, which is the optimal tax rate,

$$\tau_c = \frac{A\eta^\eta(1-\eta)^{1-\eta} + \delta_g(1-\gamma) - \delta_k - \beta}{(1-\gamma)A\,\eta^\eta(1-\eta)^{1-\eta}\frac{(1+u_c)}{u_c}}. \tag{86}$$

The optimal tax rate τ_c, which maximizes the welfare of the economy, must lie between 0 and 1 (i.e, $0 < \tau_c < 1$). Since, u_c is positive, $1 + u_c/u_c > 0$. Also, since we have assumed $-\infty \leq \gamma \leq 1$ and $0 < \eta < 1$ therefore, the denominator of equation

(86) is positive.

So the sufficient condition for$\tau_c \geq 0$, the numerator of τ_c must also be ≥ 0. Therefore, $A\eta^{\eta}(1-\eta)^{1-\eta} + \delta_g(1-\gamma) - \delta_k - \beta \geq 0$ holds if $A\eta^{\eta}(1-\eta)^{1-\eta} \geq \delta_k + \beta - \delta_g(1-\gamma)$.

The condition for τ_c to be less than one is shown in the Appendix A2. The steady-state balanced growth rate of the command economy is,

$$g_c = \frac{A\eta^{\eta}(1-\eta)^{1-\eta} - \beta - \delta_k}{1-\gamma}. \tag{87}$$

For steady state growth rate to be positive or $g_c \geq 0$, we require the condition $A\eta^{\eta}(1-\eta)^{1-\eta} \geq \beta + \delta_k$. Note that, if this condition is satisfied, the condition for $\tau_c \geq 0$ is also satisfied.

Proposition 8. *There exists a unique growth rate in command economy when public and private capital are treated as perfect substitutes and also there exists a positive optimal tax rate to be imposed on output for financing infrastructure service.*

From (64) and (85) we find that steady-state command economy growth rate is higher than the competitive economy growth rate. When no tax is imposed on the competitive economy, (which implies complete financing by privatization) then competitive economy growth rate is equal to command economy growth rate.

Proposition 9. *Command economy growth rate is higher than the competitive economy growth rate.*

Barro (1990) finds a similar result where command economy grows faster. In command economy, the social planner is able to internalize the social productivity of infrastructure (public capital) and determines tax rate optimally. Whereas, in a market economy the private marginal product of capital is only taken into account and tax rate is considered exogenously given while optimizing the present discounted value of utility. Therefore, in the present paper, the competitive economy growth rate is lesser than the command economy growth rate for all positive tax rates.

Now, let us assume that depreciation rate of both public capital and private capitals are zero. In the decentralized economy, when $\delta_k = \delta_g = 0$, then, $u_d = (\tau_d A\eta^{\eta}(1-\eta)^{1-\eta}(1-\gamma))/(A\eta^{\eta}(1-\eta)^{1-\eta}(1-2\tau_d+\tau_d\gamma)-\beta)$.

However, in the command economy, when $\delta_k = \delta_g = 0$, then, any solution of positive optimal u_c is not obtained. Therefore, command economy solution suggests that no public capital should be used if public capital and private capital are perfect substitutes in the infrastructure production and the depreciation rates of both types of capital are same. This is simply because an accumulation of public capital is financed by tax revenue and an imposition of taxation creates distortion.

The comparative static effects on growth rates and optimal tax rates of the decentralized economy with public and private goods being complementary/substitute are compared and summarized in a table given below:

Table 1. Comparison between results under complementary and substitute relations between private capital and public capital

	When private capital and public capital are complements in the PPP model $\left[K_I^s = B(\theta K)^{\alpha} K_g^{1-\alpha}\right]$	When private capital and public capital are substitutes in the PPP model $\left[K_I^s = \theta K + K_g\right]$
Decentralized Economy (where, d denotes decentralized economy.)	(1) There exists unique steady state growth rate (2) There exists a unique, interior growth-maximizing tax rate given by, $\tau_d = (1-\eta)(1-\alpha)$	(1) There exists unique steady state growth rate. There also exists a range of feasible tax rates for which growth rate is positive. (2) The impact of atax rate on growth is negative. Hence,the growth-maximizing tax rate is zero. It suggests that the complete privatization rather than PPP would maximize the growth rate.
Command Economy (where, c denotes command economy.)	(1) There exists unique steady state growth rate (2) The unique, optimal tax rate is found out. (3) The command economy growth rate may not be higher than the competitive economy growth rate.	(1) There exists unique steady state growth rate. (2) The unique, optimal tax rate is found out. (3) The command economy growth rate is higher than the competitive economy growth rate.

4. CONCLUSION

In this paper, we develop an endogenous growth model with an infrastructure service that is an impure public good. Infrastructure sector uses private and public capital as factors of production. Private investment and public investment are not independent of each other. There is crowding out effect in case of substitute relationship between private capital and public capital and crowding in effect in case of a complementary relationship between the two. This paper studies both the substitutes and complementary relationship in a public-private partnership model and finds the equilibrium as well as the optimal public policy in this context. The transitional dynamics result shows saddle-path stability of the dynamic system for both the complementary and substitute case. The government is assumed to impose output tax to finance the expenditure on public capital and if the optimal tax rate is found to be zero, Public-Private Partnership (PPP) is not desirable, otherwise, it is. The main objective of this paper is to inquire whether PPP in infrastructure is feasible and optimal. The results obtained in this paper suggests that public-private partnership (PPP) model is optimal in the provision of infrastructure because we obtain an interior optimal solution of the tax rate in a command economy, no matter public capital and private capital are a substitute or complementary to each other. When depreciation rates of physical capital are assumed to be zero, we find that PPP is an equilibrium outcome for the complementary case and complete privatization emerges as an equilibrium solution when public capital and private capital are perfect substitutes.

While comparing the command economy growth rate and competitive economy growth rate, we find that in the case of substitute, command economy growth rate is higher than the competitive economy growth rate but in the case of complementary relationship, command economy growth rate may not be higher than the competitive economy growth rate. This implies that the public-private partnership is always an optimal solution for financing infrastructure no matter the relationship between private capital and public capital be substitute or complementary. But, PPP solution is not growth maximizing in case of substitute relationship between the public capital and private capital.

This paper is subject to some limitations. We have assumed that the government runs a balanced budget. But, most of the time, the government of an economy, especially a developing economy, faces a deficit budget. We have not considered that possibility in this model. It would be interesting to analyse how debt financing of public investment affects growth rates and welfare in comparison to the tax-financed one. We abstract from any kind of subsidization by the government which is an important variable affecting the investment decision of the private sector in the real world. Further, For simplicity, apart from physical capital, we do not consider other factors of production like labour in this model which is a limitation of our study. Also, we have considered perfectly competitive product and factor market and consequently constant returns to scale in the production of infrastructure which may not be too realistic. But, considering market imperfection and more general production functions (with non-constant returns to scale) are beyond the time and scope of this present paper. We admit this is another limitation of our study and we intend to include more general production function and imperfect competition in our future work. Though usually, infrastructure service generates a positive external effect on other sectors of the economy, we have ignored the presence of an external effect in the present paper. This paper attempts to seek whether PPP is an optimal policy when the government runs a balanced budget and finds out that it is an optimal policy in the provision of infrastructure, though may not be a growth-maximizing one.

APPENDIX

Appendix A1. The second order condition is,

$$
\begin{aligned}
\left(\frac{\partial^2 u_d}{\partial \tau_d^2}\right) = {} & \frac{1}{[\{\eta+\alpha(1-\eta)\}[(1-\gamma)\frac{\tau_d}{u_d}+(1-\tau)(1-\eta)(1-\alpha)]]^2}[\{\eta+\alpha(1-\eta)\}\big((1-\gamma)+\{\eta+\alpha(1-\eta)\}u_d\big) \\
& -\left(\frac{(1-\gamma)}{u_d}+\{\eta+\alpha(1-\eta)\}\right)\{\eta+\alpha(1-\eta)\}(1-\gamma) \\
& [1-\frac{(1-\gamma)+\{\eta+\alpha(1-\eta)\}u_d}{\{\eta+\alpha(1-\eta)\}[(1-\gamma)\frac{\tau_d}{u_d}+(1-\tau_d)(1-\eta)(1-\alpha)]}\frac{\tau_d}{u_d}-\frac{(1-\eta)(1-\alpha)u_d}{(1-\gamma)}]],
\end{aligned}
$$

For $\left(\partial^2 \mathrm{u_d}/\partial \tau_d^2\right)$ to be positive, $[1-((1-\gamma)+\{\eta+\alpha(1-\eta)\}u_d)/(\{\eta+\alpha(1-\eta)\}[(1-\gamma)\tau_d/u_d+(1-\tau_d)(1-\eta)(1-\alpha)])(\tau_d/u_d)-((1-\eta)(1-\alpha)u_d)/(1-\gamma)]$ must be less than zero. Therefore,

$$[1-\frac{(1-\gamma)+\{\eta+\alpha(1-\eta)\}u_d}{\{\eta+\alpha(1-\eta)\}[(1-\gamma)\frac{\tau_d}{u_d}+(1-\tau_d)(1-\eta)(1-\alpha)]}\frac{\tau_d}{u_d}-\frac{(1-\eta)(1-\alpha)u_d}{(1-\gamma)}]<0,$$

Since, $((1-\gamma)+\{\eta+\alpha(1-\eta)\}u_d)/(\{\eta+\alpha(1-\eta)\}[(1-\gamma)\tau_d/u_d+(1-\tau_d)(1-\eta)(1-\alpha)])=\partial u_d/\partial \tau_d$.Therefore, substituting it in the above equation, we obtain,

$$\frac{\partial u_d}{\partial \tau_d}\frac{\tau_d}{u_d}+\frac{(1-\eta)(1-\alpha)u_d}{(1-\gamma)}>1. \tag{30}$$

Equation is a sufficient condition for $(\partial^2 u_d/\partial \tau_d^2)$ to be positive and $(\partial^2 g_d/\partial \tau_d^2)$ to be negative. In other words, equation (30) is a sufficient condition for the existence of growth maximizing tax rate.

Appendix A2. For τ_c to be less than one,In equation (64), $\left[A\eta^{\eta}(1-\eta)^{1-\eta}+\delta_g(1-\gamma)-\beta-\delta_k\right]/[A(1-\gamma)\eta^{\eta}(1-\eta)^{1-\eta}]\times\left[\delta_k-\delta_g-A\eta^{\eta}(1-\eta)^{2-\eta}\right]/\left(\delta_k-\delta_g\right)<1,\ u_c/(1+u_c)<1$. Therefore, $\left[\delta_k-\delta_g-A\eta^{\eta+1}(1-\eta)^{2-\eta}\right]/\left(\delta_k-\delta_g\right)<1$.

So the sufficient condition for τ_c to be less than 1 is $\left[A\eta^{\eta}(1-\eta)^{1-\eta}+\delta_g(1-\gamma)-\beta-\delta_k\right]/[A(1-\gamma)\eta^{\eta}(1-\eta)^{1-\eta}]$ must be less than 1.

Therefore, $A\eta^{\eta}(1-\eta)^{1-\eta}+\delta_g(1-\gamma)-\beta-\delta_k<A(1-\gamma)\eta^{\eta}(1-\eta)^{1-\eta}\delta_g(1-\gamma)-\beta-\delta_k<A\eta^{\eta}(1-\eta)^{1-\eta}[(1-\gamma)-1],\ A\eta^{\eta}(1-\eta)^{1-\eta}\gamma<\delta_k+\beta-\delta_g(1-\gamma)$.

REFERENCES

Barro, R.J. (1990), "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, 98(5), 103-125.

Barro, R.J. and X. Sala-i-Martin (1992), "Public Finance in Models of Economic Growth," *The Review of Economic Studies*, 59(4), 645-661.

Bucci, A. and C.D. Bo (2012), "On the Interaction between Public and Private Capital in Economic Growth," *Journal of Economics*, 106(2), 133-152.

Bom, P.R.D. andJ. Ligthart (2014), "Public Infrastructure Investment, Output Dynamics, and Balanced Budget Fiscal Rules," *Journal of Economic Dynamics and Control*, 40, 334-354.

Besley, T. and M. Ghatak (2017), "Public-Private Partnership for the Provision of Public

Goods: Theory and an Application to NGOs," *Research in Economics*, 71, 356-371.

Chatterjee, S. and A.K.M.M. Morshed (2011), "Reprint to: Infrastructure Provision and Macroeconomic Performance," *Journal of Economic Dynamics and Control*, 35(9), 1405-1423.

Chatterjee,S. and S. Ghosh (2011), "The Dual Nature of Public Goods and Congestion: The Role of Fiscal Policy Revisited," *Canadian Journal of Economics*, 44(4), 1471-1496.

Chen, S. and J. Guo (2016), "Progressive Taxation, Endogenous Growth, and Macroeconomic (In)stability," *Bulletin of Economic Research*, 68(S1), 3307-3378.

Devarajan, S., D. Xie and H. Zou (1999), "Should Public Capital be Subsidized or Provided?" *Journal of Monetary Economics*, 41(2), 319-331.

Dasgupta, D. (1999), "Growth versus Welfare in a Model of Nonrival Infrastructure," *Journal of Development Economics*, 58(2), 359-385.

_____ (2001), "Lindahl Pricing, Nonrival Infrastructure and Endogenous Growth," *Journal of Public Economic Theory*. 3(4), 413-430.

Erden. L. and R.G. Holcombe (2006), "The Linkage between Public and Private Investment: A Co-integration Analysis of a Panel of Developing Countries," *Eastern Economic Journal*, 32(3), 479-492.

Futagami, K., Y. Morita and A. Shibata (1993), "Dynamic Analysis of an Endogenous Growth Model with Public Capital," *Scandinavian Journal of Economics*, 95, 607-625.

Fisher, W.H. and S.J. Turnovsky (1998), "Public Investment, Congestion and Private Capital Accumulation," *Economic Journal*, 108, 399-413.

Fisher, W.H. and F.X. Hof (2000), "Relative Consumption, Economic Growth and Taxation," *Journal of Economics*, 72(3), 241-262.

Greiner, A. and H. Hanusch (1998), "Growth and Welfare Effects of Fiscal Policy in an Endogenous Growth Model with Public Investment," *International Tax and Public Finance*, 5(3), 249-261.

Kateja, A. (2012), "Building Infrastructure: Private Participation in Emerging Economies," *Procedia-Social and Behavioural Sciences*, 37, 368-378.

Mourmouras, I.A. and J.E. Lee (1999), "Government Spending on Infrastructure in an Endogenous Growth Model with Finite Horizon," *Journal of Economics and Business*, 51(5), 395-407.

Rashid, A. and I. Ahmad (2005), "Public Private Investment Linkages: A Multivariate Cointegration Analysis," *Pakistan Development Review*, 44(4), 805-817.

Tsoukis, C. and N.J. Miller (2003), "Public Services and Endogenous Growth," *Journal of Policy Modeling*, 25(3), 297-307.

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