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MULTIDIMENSIONAL NATION WELLBEING, MORE EQUAL YET MORE POLARIZED: AN ANALYSIS OF THE PROGRESS OF HUMAN DEVELOPMENT SINCE 1990

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Mounting concern regarding inadequacies of per capita GDP or GNI as a source of nation wellbeing classification and comparison lead to the employment of multidimensional approaches with attendant concerns regarding their arbitrary and complex nature. Here, based upon commonalities in multidimensional behavior of nations, feasible, less arbitrary, classification methodologies and techniques for assessing wellbeing within and between groups are proposed. Implementation in a three dimensional study of 164 countries from 1990 to 2014 in a Human Development Index (HDI) framework reveals substantive multi-dimensional growth in a slowly evolving, relatively immobile three group world exhibiting simultaneous increases in equality and polarization with a growing Lower HD class and shrinking Middle and High HD classes.

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JEL Classification: C14, I32, O1

1. INTRODUCTION

For most of the 20th Century real GDP or GNI per capita were used as univariate measures of societal productive or consumptive capacity as a proxy for the capacity to generate "Wellbeing". After suitable exchange rate or purchasing power adjustments, they proved useful instruments for international wellness classification and comparison.[1] As such they have been employed extensively in nation growth and wellbeing debates to identify both β and σ convergence or divergence (dependent on whether nation

[1] The World Bank still uses three GNI per capita ($ US equivalent) thresholds for determining a fourfold classification of nation status (for a review, see Fantom and Serajuddin, 2016) and has used an updated $1 a day per capita poverty measure (World Bank, 1990).

observations are population weighted or not) and have been the basic instrument in the "twin peaks" polarization literature.[2] Sala-i-Martin (2006), by integrating nation income distributions for some 138 countries anchored on their respective GDP per capita, brought some resolutions to these debates concluding that world poverty and inequality were both diminishing (though note this does not preclude intensified polarization).[3]

However, as instruments reflecting wellbeing, these measures have met with increasing dissatisfaction in recent years. Aside from issues of measurement (Coyle, 2014; Deaton, 2010), the idea of equating "wellbeing" solely with "consumption utility" (see for example Fleurbaey, 2009; Fleurbaey and Blanchet, 2013) was problematic and expanding the dimensions of wellbeing measurement became a mantra of The 2008 Commission on the Measurement of Economic Performance and Social Progress (Stiglitz, Sen and Fitoussi, 2010). The Human Development Index (UNPD, 2016) is typical of this approach, first published in 1990, the HDI is basically an equally weighted geometric mean aggregation of the three bounded dimensions of Education (a combination of literacy and school enrollment rates), Life Expectancy (essentially a proxy for health status), and GNI per capita of a nation.

A common concern with these multidimensional indices is robustness issues surrounding alternative parameterization and nation classification assumptions (Ravallion, 2010). In the one dimensional paradigm there is a long established practice of using "hard" boundaries to classify nations into groups, a recent example is the 2013 Gross National Income per capita categories published by the World Bank (see Appendix 1 for details). Determination of the cut-offs is somewhat arbitrary and thus contentious, (see for example Atkinson and Brandolini, 2011; Banerjee and Duflo, 2008; Citro and Michael, 1995; Easterly, 2001; Quah, 1993, 1997; and Ravallion, 2012). Not only does the practice categorize poorness and wellness in a fairly arbitrary fashion (boundaries established many years ago are simply updated on a US dollar equivalent basis without reference to current conditions), but it ultimately affects the way transition and class mobility behavior is evaluated, that is, specific choices can be prejudicial to other aspects of analysis. For example, defining classes by quantiles fixes class sizes over time precluding analysis of poverty reduction strategies, tying class boundaries to some proportion of a location measure ties movement of classes to movements in the overall distribution and assumes away the possibility of independent class variation (incidentally contravening the focus axiom frequently invoked in poverty analysis). These issues are compounded in the multi-dimensional paradigm. Alkire and Foster (2011, 2011a) have proposed a many dimensioned poverty/deprivation measure and the

[2] This extensive literature (Baumol, 1986; De Long, 1988; Barro and Sala-i-Martin, 1992; Mankiw Roemer and Weil, 1992; Quah, 1996, 1997; Sala-i-Martin, 1996; Pritchett, 1997; Jones, 1997; Kremer, Onatski and Stock, 2001) is reviewed in Sala-i-Martin (2006).

[3] Subgroup decomposition of the Gini coefficient (Mookerjee and Shorrocks, 1982; Yitzhaki, 1994) illustrates that nation groups or "clubs" can simultaneously become more equal and yet more polarized. Polarized empirical evidence may be found in Anderson (2004) and Pittau, Zelli and Johnson (2010).

Hu- man Development Report (United Nations Development Programme, 2016) proposed cut-off points for the categories of human development index. Both cases require specification of "hard" boundaries in each dimension, the problems here are much like those of the univariate approach (determining boundaries in a particular fashion also determines the nature of the group in a way that is often prejudicial and precludes the notion of "trade-offs" between dimensions) however now the classification problem is compounded as it is "many dimensioned".[4]

Classification problems led Anderson, Pittau and Zelli (2014, 2016) to propose a univariate semi-parametric method for determining poorness and wellness status where the classification basis is the commonality of behaviors reflected in the components of an overall mixture distribution. Determining the number and size of the classes and their distributional parameters on a "goodness of fit" criterion, facilitates free and independent variation in the number, size, distributional location and spread of subgroups over time. Group membership is determined probabilistically rather than categorically, however this does not inhibit analysis of the progress (or otherwise) of groups. Intuitively progress of poor nations is no longer influenced by the progress of non-poor nations (a fundamental Focus axiom in poverty analysis), the magnitude and wellness of subgroups can be studied independently and groupings can emerge, disappear, converge or polarize. It is of interest to see if such phenomena prevails in a multivariate paradigm. An objective of this study is the extension of this univariate technique to a multidimensional mixture distribution framework of the HDI components namely the GNI per capita, Life Expectancy, Education triplet, and to examine the classification and progress of groups of nations in the modern era in that context.

The triplet is modelled as a process of latent states identified as different sub-populations of countries sharing inherent but fundamentally unobservable circumstances of human development (similar functioning and capability sets in the terminology of Sen, 1985, 1993; and Nussbaum, 1997, 2011). Countries belonging to a specific state or category share in each period a common multivariate distribution of the (observable) outcome variables with the overall distribution being a mixture of these sub-population distributions. The latent states are linked through time by a Markov process in what is termed a Hidden Markov Model (HMM), the properties of which will reveal trends in various aspects of world wellbeing. Because the focus of attention is wellbeing the analysis is performed in population weighted terms so that each countries triplet can be interpreted as that of the representative agent of that country. In addition, tools will be

[4] Responding to this, Jones and Klenow (2016) provide national uni-dimensional consumption equivalent wellbeing measures incorporating aspects of consumption, leisure, life expectancy and in- equality in these variates in a rigorously defined, parametrically homogeneous across nations formulation for a collection of countries. They find that, while GDP per capita is highly correlated with this measure, the differences can be substantive and important. Note that in this formulation the separate influences of the various aspects are parametrically tied to each other in the unidimensional consumption equivalent that is homogeneously constant across all possible groupings of countries.

proposed and implemented for measuring the poverty, inequality, polarization, convergence and mobility of the latent groups in the context of the many dimensions of the HDI.

In the following, Section 2 outlines and discusses the proposed modelling and measurement methods. Section 3 reports the study of the world multivariate distribution of the components of the HDI over the period 1990-2014. Section 4 concludes. Appendix 1 provides a companion analysis of the World Bank categorization with respect to GNI per capita.

To anticipate the results, a relatively immobile three class world in which all groups are improving in a wellbeing sense in all dimensions is revealed. Nonetheless while there was substantial evidence of reduced multidimensional inequalities both within and between groups, which is consistent with the Sala-i-Martin (2006) univariate analysis, it did not inhibit the increased sense of segmentation, differentness or divergence that groups experienced, which is not consistent with the Sala-i-Martin analysis. In essence the groups were simultaneously becoming more equal and more polarized. However, unlike the results in Appendix 1, the relative sizes of high human development and middle human development groups are declining and the low human development group is increasing.

2. EMPIRICAL METHODS

2.1. Model Development and Estimation

The basic model assumes a mixture distribution of k latent classes or subgroups indexed $j = 1, \cdots, k$, each subgroup relates to a jointly normally distribute[5] tri-variate vector y which contains a nations three human development outcomes. Thus, for subgroup j, with mean vector μj and an assumed[6] diagonal covariance matrix Σ_j, $y \sim N\ (\mu_j, \Sigma_j)$, its multivariate normal density is denoted $f_j(y; \mu_j, \Sigma_j)$ and its mixing weight w_j.

Letting Ψ be a vector containing all of the unknown parameters in the model, the $k-1$ mixing weights w_j, $j = 1, \cdots, k-1$, the k mean vectors μ_j and k covariance

[5] Finite mixture models have featured in many fields where heterogeneity of individual types, data contamination, mis-classification or dynamic regime switching are issues (Eckstein and Wolpin, 1990; Keane and Wolpin, 1997; Kim and Nelson, 1999; Lewbel, 2007; Chen et al., 2011). The choice of normal densities is not an overly strong assumption since any continuous distribution can be approximated to some desired degree of accuracy by an appropriate finite Gaussian mixture (Rossi, 2014).

[6] This assumption was removed for a sensitivity analysis but the results did not change significantly, indeed underlying the equally weighted additively separable structure of the HDI is an independent influence interpretation consistent with a diagonal covariance matrix.

matrices Σ_j, $j = 1, \cdots, k$, the mixture model being entertained may be written as:

$$f(y; \Psi) = \sum_{j=1}^{k} w_j f_j(y; \mu_j, \Sigma_j).$$

Ultimately, the progress of y will be viewed as a developmental process and therefore the implicit assumption of time independence can be seen too restrictive. To better understand the evolution of human development, we remove the time-invariance hypothesis by estimating a hidden Markov model (HMM) for panel data in which the system is assumed to be a Markov chain with time-varying unobserved (hidden) states.[7] In this model, each outcome and each time period are considered independent only conditionally on an unobserved discrete latent variable. Based upon Bartolucci et al. (2013, 2014) and Farcomeni (2015), this model relies on similar assumptions but assumes that the number of latent states is not constant over time. Relaxing these assumptions requires estimating the model in a Bayesian context.[8]

Formally, let Y_{itr} denote the measurement for the r-th outcome at time t for country i. Assume there are k_t latent states, where the (unknown) latent state for country i at time t is denoted U_{it}, further assume $Y_{itr} | U_{it} = j$, $k_t = k \sim N(\mu_{jtkr}, \sigma^2_{jtkr})$, that is, when there are k groups and the i-th country belongs to the j-th one, the r-th outcome has mean μ_{jtkr} and variance σ^2_{jtkr}.

Since the number of parameters to be estimated increases with t, the most rational assumptions, especially when we deal with small sample sizes, are that the country-state-variable means follow a linear trend as:

$$\mu_{jtkr} = \mu_{j1kr} + \beta_{jkrt},$$

and the corresponding variances also may vary exponentially over time as:

$$\sigma^2_{jtkr} = \alpha^{2t}_{jkr} \sigma^2_{j1kr}.$$

These latter parameters specify the manifest distributions.

For the latent distribution we assume that U_{it} follows a time-homogeneous Markov chain with variable number of states, which is fully specified by initial distributions $Pr(U_{i1} = j | k_1 = k) = \pi_{jk}$ and (possibly rectangular) transition matrices $Pr(U_{it} = j | k_t = k, k_{t-1} = l, U_{i,t-1} = h) = \pi_{hjlk}$.

In summary, the latent variable follows a variable-support time-homogeneous

[7] As in Hobijn and Franses (2001) the issue of convergence is examined by looking at the dynamics of the whole distribution of the indicators but, unlike them, the evolution of the joint distribution of the indicators is considered rather than the dynamics of the distribution of each indicator separately.

[8] A frequentist version of a simplified model, based on time-constant manifest parameters, has been introduced in Anderson et al. (2019).

Markov Chain so that the joint distribution of the three outcomes is modelled simultaneously over time, taking into account dependence due to correlation and unobserved heterogeneity. The discrete latent distribution provides a natural way to cluster nations with respect to their measurements. Not only are transitions between groups allowed, but also year-specific number of clusters (components of the mixture). The (possibly rectangular) hidden transition matrices link the group compositions across years.

In order to fit this complex model, Bayesian techniques are employed. Transdimensional moves are obtained through a birth-and-death reversible approach, while full conditionals are available for all parameters except σ, α and β. For these parameters Adaptive Rejection Metropolis Sampling is used. To assess evidence for specific parameter configurations, the encompassing prior approach (Klugkist et al., 2005; Bartolucci et al., 2012) is used for dealing with discrete parameters, and Schwarz criterion for continuous ones.

2.2. Measuring Aspects of World Well-being

In order to measure various aspects of HDI wellbeing the following indices are proposed and implemented.

2.2.1. Between Group Differences and Distributional Inequality

In the equality of opportunity literature, the extent of differences in distributions conditioned on circumstances based upon stochastic dominance comparisons have long been used to measure the lack of equality of opportunity (see for example Lefranc, Pistolesi, and Trannoy, 2008, 2009). A problem with this approach is that it does not quantify the extent of differences; it merely examines what type (i.e. order of dominance) and whether or not a difference exists. One way of considering the extent to which the world has become more unequal is to look at inequality or differences in the group joint densities via a multivariate generalization of Gini's transvariation measure (Gini, 1916; 1959; Pittau and Zelli, 2017; Anderson, Linton and Thomas, 2017). Suppose three different groups of countries have been identified, say Low, Medium and High human development groups. Then, the multidimensional transvariation measure is of the form:

$$3 \cdot Trans = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(max\big(f_L(x,y,z), f_M(x,y,z), f_H(x,y,z)\big) - min\big(f_L(x,y,z), f_M(x,y,z), f_H(x,y,z)\big)\Big)\, dxdydz,$$

where x, y and z are respectively relative per capita GNI, Life expectancy and Education and $fL()$, $fM()$ and $fH()$ are the corresponding Low, Medium, and High human development distributions. The measure corresponds to an index between 0 and 1 of inequality of distribution which will be 0 when all distributions are identical and 1 when there is no overlap between distributions.[9] This formulation treats all nation groupings as equally important, in attaching the same weight to each group distribution it can be interpreted as measuring the extent of distributional differences of the prospects for a representative low, medium and high developed nation. As such it focuses on the between group differences across the three dimensions of the Human Development Index. Increases in the measure signal diverging distributions, reductions correspond to increasing similarities or sigma convergence overall. It is also possible to construct a statistic, which weights the comparison distributions by their relative importance in the mixture. This is of the form:

$$3 \cdot WTrans = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\big(max\big(s_L f_L(x,y,z), s_M f_M(x,y,z), s_H f_H(x,y,z)\big) - max(s_L f_L(x,y,z), s_M f_M(x,y,z), s_H f_H(x,y,z))\big)\, dxdydz,$$

where s_k, $k = L$, M, H is given by $w_k/\Sigma w_k$. Together, $Trans$ and $WTrans$ can be considered a multidimensional measure of world inequality.

2.2.2. Within Group Inequality

Of itself, the extent of within group inequality is of interest (Foster, Greer and Thorbeke (1984) intensity of poverty measures account for this with respect to the poor group in a univariate paradigm) but, in reflecting the degree of within group disassociation, it is also an important component of the polarization measure to be outline below.

In order to assess within class inequality and convergence in the context of the triple x, y and z, note that for a given class in a given time period the distribution of the triple may be written as:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \nu \sim \frac{1}{\sqrt{2\pi|\Sigma|}}(\nu - \mu_\nu)'\Sigma^{-1}(\nu - \mu_\nu), \text{ where } \Sigma = \begin{pmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{pmatrix}.$$

It follows that:

$$\sqrt{|\Sigma|} = \sigma_x \sigma_y \sigma_z,$$

is a measure of the overall relative variation in the class at that time and diminutions (increases) in it correspond to sigma convergence (divergence). Given that x, y and z

[9] For year-by-year comparison purposes, under some strong assumptions, the Transvariation statistic p can be considered asymptotically normal with a standard error $\sqrt{p(1-p)/3n}$ where n is the sample size.

are "base year" relative measures, this measure corresponds to a multivariate "coefficient of variation" where the base year mean is the standardizing factor.

2.2.3. A Measure of between Group Polarization

The extent to which the classes are polarizing or converging can be studied using a multi-dimensional bi-polarization measure (Anderson, Linton and Leo, 2012) based upon kernel estimates, between two unimodal group distributions i, j, with relative population sizes w_i, w_j, given by:

$$POL_{i,j} = \frac{0.5}{w_i+w_j}(w_i f_i(x_{mi}, y_{mi}, z_{mi}) + w_j f_j(x_{mj}, y_{mj}, z_{mj}))|(x_{mi}, y_{mi}, z_{mi}) - (x_{mj}, y_{mj}, z_{mj})|,$$

where $|(x_{mi}, y_{mi}, z_{mi}) - (x_{mj}, y_{mj}, z_{mj})|$ is the Euclidian distance between the modal points (x_{mi}, y_{mi}, z_{mi}) and (x_{mj}, y_{mj}, z_{mj}). In the present context with independent multivariate normal distributions in a mixture distribution this may be written as:

$$POL_{i,j} = \frac{0.5}{\sqrt{2\pi}^3(w_i+w_j)}(w_i \frac{1}{\sigma_{xi}\sigma_{yi}\sigma_{zi}} + w_j \frac{1}{\sigma_{xj}\sigma_{yj}\sigma_{zj}})|(x_{mi}, y_{mi}, z_{mi}) - (x_{mj}, y_{mj}, z_{mj})|.$$

2.2.4. Measures of Transitional Polarization Convergence and Mobility

Anderson (2018) demonstrates how transition matrices can be employed to develop indices of directional mobility, polarization and convergence. In essence the elements of the transition matrix facilitate a "balance of probabilities" measure of whether populations are transiting from the center to the peripheries or from the peripheries to the center of the world distribution of Human Development which has a polarizing/converging interpretation. For a 3×3 transition matrix $||T_{ij}||$, where T_{ij} is the probability of arriving in state j given departure from state i, $PCONV$, the balance of probabilities of converging to the center is:

$$PCONV = wT_{12} + (1-w)T_{32} - (T_{21} + T_{23}),$$

where w is the probability of being in the initial state low class given they are not in the initial state middle class. The balance of probabilities of an upward transition (PUT) is:

$$PUT = w_1(1-T_{11}) - w_2(T_{21} - T_{23}) - w_3(1-T_{33}),$$

where w_i is the probability of being in initial state i. Consider the transformation $PTN = 0.5 + PT/2$, so that when net transfers are balanced the index would return to 0.5. As a probability measure, on the null hypothesis that $PTN = 0.5$, it can be shown (Anderson, Ge and Leo, 2009), that $PTN \sim N(0.5, 0.25/n)$ where n is the sample size, thus facilitating hypothesis testing and confidence interval interpretations. Finally, another application of Gini's Transvariation to the Transition matrix yields a measure of mobility in the system.

3. MEASURING THE WELL-BEING OF NATIONS: CATEGORIZATION, CONVERGENCE, MOBILITY

3.1. Data and Model Choice

The analysis is carried out on a panel of 164 countries over a period spanning from 1990 to 2014. Data are taken from the Human Development Reports web-site (hdr.undp.org/en/data) and have been analyzed every five years, all estimates and comparisons are population weighted. Table 1 reports (weighted) means and standard deviations of the three variables involved in the HDI construction: per capita GNI, life expectancy at birth and years of education. There is one slight deviation from the HDI index, only one education variable (expected years of schooling) is used since including mean years of schooling would have involved too great a loss of data points. Per capita GNI are estimated in 2011 purchasing power parity.

Table 1. Means and Standard Deviations of per Capita GNI, Life Expectancy and Years of Schooling over Time for the World Population (164 Countries)

	Means			Standard deviations		
Year	GNI	Life_exp	Yrs_Educ	GNI	Life_exp	Yrs_Educ
1990	8 661.10	65.19	9.58	11 918.8	8.45	3.07
1995	8 920.82	66.17	9.93	12 307.4	8.36	3.20
2000	9 949.40	67.43	10.42	13 713.3	8.48	3.16
2005	11 329.30	68.88	11.34	14 473.3	8.30	2.80
2010	12 915.70	70.35	12.34	14 143.8	7.68	2.54
2014	14 169.22	71.34	12.70	14 551.4	7.31	2.53

In implementing the modeling process, per capita GNI has been log-transformed[10] and all variables have been standardized with respect to the initial year 1990. Thus all analyses are performed relative to the base year weighted average. Initially, assuming time-independent multidimensional mixtures for each year, determination of the optimal number of components was based upon a Bayesian Information Criterion (BIC). Models were estimated separately using a standard finite mixture model with Gaussian components (Fraley and Raftery, 2002) in a classical frequentist framework with a standard Expectation-Maximization (EM) algorithm. In all cases, BIC favored a well

[10] Income is taken in logarithms "in order to reflect the diminishing returns to transforming income to human capabilities" (Anand and Sen, 1994; Brandolini, 2008).

separated 3 group model which can be considered as representing Low, Medium and High levels of Human Development (HD).

Table 2a. Estimated Means (Relative to the Base Year), Standard Deviations of the Components in the Year-by-year Mixture Model

	Means			Std deviations		
	Low	Medium	High	Low	Medium	High
GNI pc						
1990	-1.23	-0.02	1.15	0.287	0.253	0.308
1995	-1.18	0.02	1.31	0.431	0.187	0.172
2000	-1.11	0.09	1.36	0.417	0.172	0.186
2005	-1.00	0.24	1.41	0.402	0.157	0.174
2010	-0.91	0.39	1.44	0.328	0.149	0.137
2014	-0.81	0.46	1.46	0.332	0.138	0.133
Life Exp.						
1990	-1.41	0.23	0.91	0.219	0.194	0.236
1995	-1.25	0.41	1.09	0.299	0.130	0.119
2000	-1.16	0.52	1.18	0.295	0.121	0.132
2005	-0.92	0.65	1.28	0.317	0.124	0.137
2010	-0.60	0.75	1.41	0.268	0.121	0.112
2014	-0.38	0.84	1.50	0.261	0.108	0.105
Education						
1990	-1.40	0.25	0.87	0.167	0.147	0.180
1995	-1.04	0.39	1.22	0.536	0.232	0.214
2000	-0.78	0.59	1.47	0.549	0.226	0.245
2005	-0.43	0.82	1.61	0.459	0.179	0.198
2010	-0.12	0.97	1.74	0.383	0.173	0.160
2014	-0.03	1.03	1.81	0.385	0.160	0.154

Table 2b. Relative Group Sizes of the Components in the Year-by-year Mixture Model

	Low HD	Medium HD	High HD
1990	0.26	0.45	0.29
1995	0.30	0.45	0.25
2000	0.31	0.41	0.27
2005	0.32	0.40	0.29
2010	0.31	0.41	0.28
2014	0.32	0.40	0.28

Table 2a and 2b report the year-by-year means, standard deviations and relative group sizes of the subgroups. Group GNI per capita, life expectancy and education status means, perhaps best illustrated in Figure 1, have been improving steadily throughout the period in all groups. For the medium and high HD groups, GNI standard deviations have been falling steadily over the period, an indication of within group convergence whereas the Low HD groups' standard deviation has taken an "inverted U"

Kuznets (1955) curve like profile over the period. With respect to life expectancy there appears to be convergence (shrinking standard deviations in all three classes over time). Perhaps most interestingly the Education dimension exhibits a Kuznets curve profile with a peak around 2000 in all three groups.

Table 3 reports the group membership weighted and un-weighted transvariation measures, both of which are diminishing over time indicative of significant overall convergence over the period as is the case with the middle and high HD groups.

Table 3. Transvariations and within Group Inequality Measures of the Year-by-year Mixture Model

Year	Transvar	WTrans	Within group inequality		
			Low HD	Medium HD	High HD
1990	0.9884	0.9938	0.1050	0.0072	0.0131
1995	0.8578	0.9206	0.0691	0.0056	0.0044
2000	0.9018	0.9491	0.0675	0.0047	0.0060
2005	0.8570	0.9268	0.0585	0.0035	0.0047
2010	0.6705	0.8218	0.0337	0.0031	0.0025
2014	0.6658	0.8090	0.0334	0.0024	0.0022

Note: Computation of Transvar and WTrans is facilitated by noting that, given the present case of diagonal covariance matrices, these are differences of integrals of products of independent normal distributions which can be calculated by standard methods.

Table 4. Polarization Measures between Components from the Year-by-year Model (In Brackets the Approximated Standard Errors)

	Low vs. Medium	Low vs. High	Medium vs. High
1990	20.43	21.81	10.80
	(2.88)	(2.32)	(2.74)
1995	17.80	29.04	20.83
	(2.76)	(2.65)	(3.65)
2000	20.05	22.09	20.71
	(2.85)	(2.32)	(3.56)
2005	25.02	26.80	25.07
	(3.25)	(2.63)	(4.05)
2010	26.89	47.86	32.97
	(3.53)	(3.62)	(4.80)
2014	32.14	51.39	39.77
	(3.95)	(3.83)	(5.32)

Polarization measures between components, together with approximate standard errors, are reported in Table 4. Note the trending polarization between all groups especially post 2005. Thus, a picture of a world in which between and within group inequalities are diminishing and yet the world is polarizing in the sense that the groups are becoming increasingly different. In essence, the extent to which the groups overlap is diminishing, having less in common as time proceeds they are becoming increasingly segmented with greater perceived differences.

In terms of relative class sizes, the Low HD class has grown over the period with a shrinking Medium HD Class and a relatively stable High HD class. Their respective annual GNI growth rates are 1.75%, 2% and 1.29%. With respect to life expectancy, the Low class has experienced some catch-up with the Medium and High classes (whose Relative Life Expectancy gap persists) with an annualized growth rate of 4.3% compared to 2.5% for the two upper classes. Relative education levels have also seen a big advance for the Low HD with annualized growth rates of 5.7%, 3.2% and 3.9% respectively for Low, Medium and High classes respectively.

The smoothly trending processes illustrated in the foregoing suggests a model in which the progress of the classes is systematically linked with past class structure informing the present. To reflect this, the model now entertained is the hidden Markov model with time-varying number of latent states described in Section 2.[11]

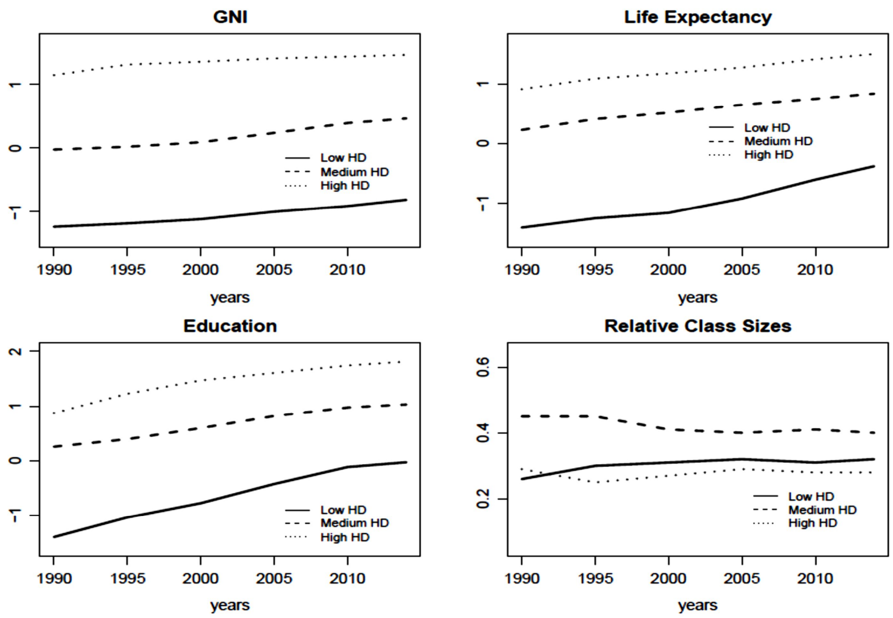

Figure 1. Evolution of the Estimated Means of the Components in the Year-by-year Model: Per Capita GNI (Panel A), Life Expectancy (Panel B), Education (Panel C) and Estimated Weights of the Components (Panel D)

[11] For simplicity purposes in the modelling process the Kuznets curve nature of Low HD GNI per capita and the Education standard deviations revealed in the inter-temporally independent model has been ignored here.

The first hypothesis to be tested is that the number of components k of the multivariate distribution remains fixed (the alternative being that the number varies over time). The null hypothesis of k fixed is strongly not rejected (the estimated probability of rejecting the null is 0.002). Conditionally fixed k over time its actual value has to be assessed and there was overwhelming evidence in favor of $k = 3$ with respect to $k = 1;\ 2;\ 4;\ 5$, as assessed by practically any measure. For instance, testing $k = 3$ against $k = 2$ (which is the second most likely), the probability of rejecting the null smaller than 0.001. Incidentally, this endogenously determined clustering contrasts the four categories proposed by the 2014 Human Development Report for country grouping the HDI.

In accord with the time independent model results, the final results are based on the assumption that the component means are systematically independently trending over time with each component variance varying exponentially over time.

Table 5. Estimated parameters of the HMM model

	log(GNI)	Life exp	Yrs Educ
		means (1990)	
Low HD	-1.274	-1.298	-1.348
Medium HD	0.314	0.273	0.278
High HD	1.123	1.007	1.107
		standard deviations (1990)	
Low HD	0.547	0.756	0.794
Medium HD	0.742	0.363	0.486
High HD	0.369	0.256	0.642
		parameter β	
Low HD	0.143*	0.139*	0.144*
Medium HD	0.112	0.108	0.106
High HD	0.097*	0.108	0.087*
		parameter α	
Low HD	1.072	0.937#	0.972
Medium HD	0.966#	0.902#	1.003
High HD	0.879#	0.714#	0.964

Note: Components are labeled: Low HD, Medium HD and High HD. The base year is 1990. For all the other years, the mean can be calculated as: $\mu_{j,t,kr} = \mu_{j(1990)kr} + \beta_{jkrt}$, where j is the generic component, r the generic variable and t is time. $t = 1$ stands for 1995, $t = 2$ stands for $2000, \cdots, t = 5$ for 2014. Similarly, the standard deviation can be calculated as: $\sigma_{jtr} = \alpha_t \sigma_{j(1990)r}$, where j is the generic component, r the generic variable and t is time. $t = 1$ stands for 1995, $t = 2$ stands for $2000, \cdots, t = 5$ for 2014. Asterisk * means significantly different from the world slope $\beta = 0.11$ at least at 5% level. Hashtag # means significantly different from $\alpha = 1$ at least at 5% level. Interestingly, for the GNI Low HD and all classes in the Education variate, the classes exhibiting the Kuznets curve standard deviation pattern, the α parameter is not significantly different from 1.

3.2. Characteristics of the Components

The estimated parameters of the hidden Markov model in which category means have a constant independent growth process and the category variances also may vary over time are reported in Table 5.

The components are well separated and reflect the different stages of human development of the three groups. Annual growth rates in group means of the three indicators, obtained by dividing the Beta coefficients by 5, indicate growth rates of around 3% for the Low Development group and growth rates of around 2% for the Middle and High Development groups suggesting some catch-up or diminished polarizing alienation patterns for the Low Development group with respect to the other groups. However, another interesting feature is, with the exception of the income variable for the poorest group, the reduction of variability for all three variables in all three groups which is increasing in size, indicating a substantial process of increased within group association which is probably driving increases in the polarization measures.

Table 6 reports the relative group sizes estimated with the hidden Markov model. Over the period the relative size of the groups has changed considerably with the poor group membership increasing somewhat (interpretable as an increase in the relative poverty rate) with a corresponding reduction in the middle and rich group relative size.

Table 6. Relative Group Size of the Components of the HMM Model

Year	Relative group		Size
	Low HD	Medium HD	High HD
1990	0.328	0.509	0.163
1995	0.342	0.501	0.157
2000	0.362	0.487	0.151
2005	0.375	0.478	0.147
2010	0.388	0.469	0.143
2014	0.399	0.462	0.139

Table 7. The Estimated 5-year and 25-year (hidden) Transition Matrices

Final year	Initial Year		
	Low HD class	Medium HD class	High HD class
	5-year		
Low HD class	0.990	0.006	0.004
Medium HD class	0.014	0.983	0.003
High HD class	0.007	0.010	0.983
	25-year		
Low HD class	0.954	0.028	0.018
Medium HD class	0.066	0.918	0.016
High HD class	0.032	0.048	0.920

3.3. Mobility and Polarization

Following Anderson (2018), for the 25-year transitions this yields a Mobility Index of 0.095 which corresponds to a slowly evolving long run process with a considerable lack of mobility between the classes. What mobility there is tends to be downward, though the upward advancement index of 0.499 is insignificantly smaller than 0.5, similarly the polarization index favored, but did not indicate significant, polarization (0.476). All of this corresponds to a fairly rigid and very slowly evolving class structure.

The 5-year and implicit 25-year transition matrices (obtained as the 5-year transition matrix to the power of 5) are given in Table 7.

Looking at country specific results in detail[12] few changes in classes are observed in the vast majority of cases, which accords with the rigidity of the transition matrix. In tune with the suggestion of some downward mobility, increasing probability of poor class membership and decreasing probability of middle and rich class membership the changes that were detected were downward. Notable movers were Botswana, Gabon and South Africa[13] who all moved into the Low Human Development Class from the Middle class. One notable upward mover from the Middle to the Rich class was Chile. Unlike with the single dimensioned GNI per capita World Bank criteria (see Appendix 1) China stayed resolutely in the Middle Human Development grouping and India stayed in the Low Human Development Group.

4. CONCLUSIONS

Recent concerns about the measurement of wellbeing have led to the progress of nations to be classified and studied in a multidimensional context. Perhaps the most popular multi-dimensioned measure is the Human Development index. Unfortunately, increasing dimensionality, whilst better reflecting wellbeing, compounds the difficulties encountered in categorizing groups largely with regard to the arbitrary choice of boundaries (Ravallion, 2010). In a one dimensional setting Anderson, Pittau and Zelli (2014, 2016) circumvented this problem by defining classes in terms of the commonality of behaviors of the actors. The downside of this approach is that nations can no longer be definitively placed in a class, all that be discerned is the probability that a nation is in a particular class. However, this was shown not to hinder analysis and it did circumvent the problems associated with arbitrarily determined boundaries by classifying groups according to the commonalities of their behaviors.

Here a feasible methodology for performing a similar analysis in a multidimensional

[12] Details of ex post group membership probabilities for all years are available from the authors upon request.

[13] Jones and Klenow (2016) documented this demise of Botswana and South Africa attributing it to the AIDs induced fall in life expectancy.

setting has been presented and the progress of 164 nations has been examined over the period 1990-2014. In that context, measures of relative poverty, inequality, polarization and mobility have also been proposed and implemented. Contrary the usual four group classification reported in World Bank (2017), three groups, Low HD, Medium HD and High HD, each with a commonality of behaviors were established. While the mean group characteristics (mean log GNI, Life Expectancy and Education) improved systematically over the period for all groups the transition analysis detected a slowly evolving, relatively immobile world, very different from the World Banks income based univariate analysis. Over the period, reflective of some downward mobility, the poor group increased in size, which may be interpreted as an increase in the multi-dimensional relative poverty rate. In concert with univariate analyses (Sala-i-Martin 2006), there was substantial evidence of reduced inequalities both within and between groups over the period (though this was not universal the low HD group experience an inverted U shaped inequality profile over the period), the transition structure and the year-by-year analysis revealed substantive polarizing patterns. Increasing within and between group equality did not inhibit the groups increased sense of segmentation or "differentness". In essence, groups were simultaneously becoming more equal and more polarized. For the most part countries stayed within their groupings though some deterioration was seen for some African nations.

APPENDIX

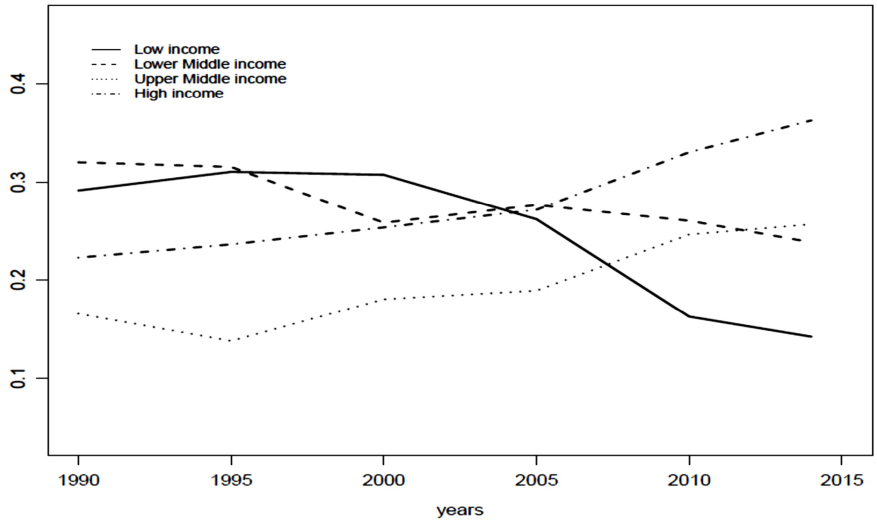

Figure A1. Evolution of the Relative Class Sizes of the World Bank Classification according to per Capita GNI–Unweighted

Details of the World Bank classification methodology is available in World Bank (2017). Its three GNI per capita ($ US equivalent) thresholds for determining a fourfold classification of nation status (Low, Lower Middle, Upper Middle, High income) are updated annually at the beginning of the bank's fiscal year with an inflation adjustment. The thresholds were established in 1989 "based largely on operational thresholds that had previously been established". In 1990 they were 545, 2200, and 6000 and in 2014 they had risen to 1045, 4125 and 12735 respectively. Based upon an un-weighted country count, the following diagram (Figure A.1) indicates how the class sizes have changed.

As can be observed, the Low and Lower Middle income classes have diminished substantially while the Upper Middle and High income classes have increased in size. If one were to aggregate the Lower and Upper Middle classes into one class, it would be seen to have grown in size slightly from 0.486 to 0.495. Table A1 reports the 1990–2014 unweighted transition matrix associated with this model.

Table A1. The Estimated 1990–2014 Unweighted Transition Matrix

1990	2014			
	Low	Lower middle	Upper middle	High
Low	0.437	0.527	0.036	0.000
Lower middle	0.000	0.355	0.578	0.067
Upper middle	0.000	0.000	0.451	0.549
High	0.000	0.000	0.000	1.000

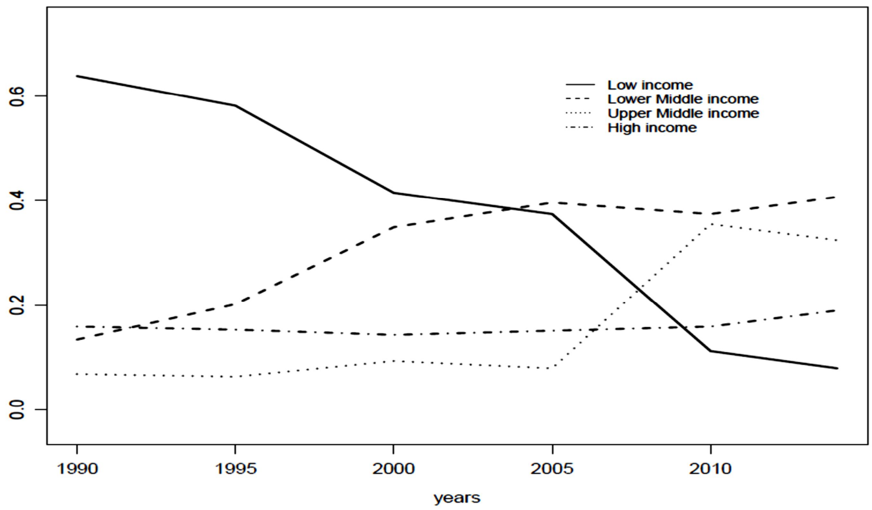

Figure A2. Evolution of the Relative Class Sizes of the World Bank Classification According to per Capita GNI-Weighted

Its upper triangular nature reflects the fact that no country dropped to a lower class over the period. The mobility statistic is 0.365 reflecting a moderate amount of mobility. +The standardized PCONV statistic is 0.541 with a standard error of 0.038 clearly fails to reject a convergence hypothesis but also fails to reject a polarization hypothesis. The standardized upward mobility statistic is 0.614 with a standard error of 0.038 indicating a significant degree of upward mobility. Over one third of countries in the sample were in a higher income category at the end of the period then they were at the beginning.

Turning to a population weighted representation generates a substantially different story. As evident from Figure A2, now there appears to be a precipitous decline in the size of the Low income group, from over 60% of the worlds population to less than 10%. The size of the High income group has barely changed, the Lower Middle income group expanded substantially in the early part of the period and the Upper Middle income group expanded greatly in the latter part of the period (largely the result of China emerging from the poor group and passing through to the upper middle income group at the latter part of the observation period). Table A2 reports the 1990–2014 weighted transition matrix associated with this model.

Table A2. The Estimated 1990–2014 Weighted Transition Matrix

1990	2014			
	Low	Lower Middle	Upper middle	High
Low	0.154	0.837	0.009	0.000
Lower Middle	0.000	0.121	0.805	0.074
Upper Middle	0.000	0.000	0.596	0.404
High	0.000	0.000	0.000	1.000

Again its upper triangular nature reflects the fact that no country dropped to a lower class over the period. The mobility statistic is 0.3011 reflecting somewhat less mobility than the unweighted model. The standardized PCONV statistic is 0.7429 with a standard error of 0.038 clearly failing to reject a convergence hypothesis but now clearly rejecting a polarization hypothesis. The standardized upward mobility statistic is 0.7075 with a standard error of 0.038 indicating a significant degree of upward mobility (greater than the unweighted version). The primary movers for these population weighted results are China which moved from a Low income country at the beginning of the period to an Upper Middle income country at the end of the period and India which moved from a Low to Lower Middle income country by the end of the period.

REFERENCES

Alkire, S. and J.E. Foster (2011), "Understandings and Misunderstandings of Multi-Dimensional Poverty Measurement," *Journal of Economic Inequality*, 9, 289-314.

_____ (2011a), "Counting and Multidimensional Poverty Measurement," *Journal of Public Economics*, 95, 476-487.

Anand, S. and A. Sen (1994), "Sustainable Human Development: Concepts and Priorities," UNDP Human Development Report Office, Occasional Paper No. 12, July.

Anderson, G. (2004), "Making Inferences About the Polarization, Welfare and Poverty of Nations: A Study of 101 Countries 1970-1995," *Journal of Applied Econometrics*, 19, 537-550.

_____ (2018), "Measuring Aspects of Mobility, Polarization and Convergence in the Absence of Cardinality: Indices Based Upon Transitional Typology," *Social Indicators Research*, 139(3), 887-907.

Anderson, G., A. Farcomeni, M.G. Pittau and R. Zelli (2019), "Rectangular Latent Markov Models for Time-specific Clustering, with an Analysis of the Well being of Nations," *Journal of the Royal Statistical Society - Series C*, 68(3), 603-621.

Anderson, G., Y. Ge and T.W. Leo (2009), "Distributional Overlap: Simple, Multivariate, Parametric and Nonparametric Tests for Alienation, Convergence and General Distributional Difference Issues," *Econometric Reviews*, 29, 247-275.

Anderson, G., O. Linton and T.W. Leo (2012), "A Polarization-cohesion Perspective on Cross-country Convergence," *Journal of Economic Growth*, 17(1), 49-69.

Anderson, G., O. Linton and J. Thomas (2017), "Similarity, Dissimilarity and Exceptionality: Generalizing Gini's Transvariation to Measure "Differentness" in Many Distributions," *Metron*, 75(2), 161-180.

Anderson, G., M.G. Pittau and R. Zelli (2014), "Poverty Status Probability: A New Approach to Measuring Poverty and the Progress of the Poor," *Journal of Economic Inequality*, 12(4), 469-488.

_____ (2016), "Assessing the Convergence and Mobility of Nations Without Artificially Specified Class Boundaries," *Journal of Economic Growth*, 21(3), 283-304.

Atkinson, A.B. A. and Brandolini (2013), "On the Identification of the Middle Class," in Gornick, J.C. and M. Jantti, eds., *Income Inequality*, Stanford University Press.

Banerjee, A.V. and E. Duflo (2008), "What is Middle Class about the Middle Classes Around the World?" *Journal of Economic Perspectives*, 22(2), 3-28.

Baumol, W. (1986), "Productivity Growth, Convergence, and Welfare: What the Long-Run Data Show," *American Economic Review*, 76, 1072-1085.

Barro, R.J. and X. Sala-i-Martin (1992), "Convergence," *Journal of Political Economy*,100, 223-251.

Bartolucci, F., A. Farcomeni and F. Pennoni (2013), *Latent Markov Models for Longitudinal Data*, Chapman and Hall: CRC Press.

_____ (2014), "Latent Markov Models: A Review of a General Framework for the

Analysis of Longitudinal Data with Covariates," *Test*, 23, 433-465.

Bartolucci, F., L. Scaccia and A. Farcomeni (2012), "Bayesian Inference through Encompassing Priors and Importance Sampling for a Class of Marginal Models for Categorical Data," *Computational Statistics and Data Analysis*, 56, 4067-4080.

Brandolini, A. (2008), "On Applying Synthetic Indices of Multidimensional Well-being: Health and Income Inequalities in selected EU Countries," Banca d'Italia, Temi di discussione Working Papers, No. 668, April.

Chen, X., H. Hong and D. Nekipelov (2011), "Nonlinear Models of Measurement Errors," *Journal of Economic Literature*, 49, 901-937.

Citro, C.F. and R.T. Michael (1995), *Measuring Poverty: A New Approach*, National Academy Press: Washington, DC.

Coyle. D. (2014), *GDP: A Brief but Affectionate History*, Princeton University Press.

Deaton, A. (2010), "Price Indexes, Inequality, and the Measurement of World Poverty," *American Economic Review*, 100, 5-34

De Long, J.B. (1988), "Productivity Growth, Convergence, and Welfare: Comment," *American Economic Review*, 78, 1138-1154.

Easterly, W. (2001), "The Middle Class Consensus and Economic Development," *Journal of Economic Growth*, 6(4), 317-335.

Eckstein, Z. and K. Wolpin (1990), "Estimating a Market Equilibrium Search Model from Panel Data on Individuals," *Econometrica*, 58, 783-808.

Fantom, N. and U. Serajuddin (2016), "The World Bank's Classification of Countries by Income", Policy Research Working Paper 7528, The World Bank Group.

Farcomeni, A. (2015), "Generalized Linear Mixed Models based on Latent Markov Heterogeneity Structures," *Scandinavian Journal of Statistics*, 42, 1127-1135.

Fleurbaey, M. (2009), "Beyond GDP: The Quest for a Measure of Social Welfare," *Journal of Economic Literature*, 47 (4), 1029-1075.

Fluerbaey, M. and D. Blanchet (2013), *Measuring Welfare and Assessing Sustainability*, Oxford University Press.

Foster, J.E., J. Greer and E. Thorbecke (1984), "A Class of Decomposable Poverty Measures," *Econometrica*, 42, 761-766.

Fraley, C. and A.E. Raftery (2002) "Model-based Clustering, Discriminant Analysis and Density Estimation," *Journal of the American Statistical Association*, 97, 611-631.

Gini, C. (1916), "Il concetto di transvariazione e le sue prime applicazioni," Giornale degli Economisti e Rivista di Statistica, in C. Gini (1959): 1-55.

_____ (1959), Transvariazione, A cura di G. Ottaviani, Libreria Goliardica, Roma.

Hobijn, B. and P.H. Franses (2001), "Are Living Standards Converging?" *Structural Change and Economic Dynamics*, 12, 171-200.

Jones, C.I. (1997), "On the Evolution of the World Income Distribution," *Journal of Economic Perspectives*, 11, 19-36.

Jones, C.I. and P.J. Klenow (2016), "Beyond GDP? Welfare across Countries and Time," *American Economic Review*, 106(9), 2426-2457.

Keane, M. and K. Wolpin (1997), "The Career Decisions of Young Men," *Journal of*

Political Economy, 105, 473-522.
Kim, C.J. and C. Nelson (1999), *State-Space Models with Regime-Switching: Classical and Gibbs-Sampling Approaches with Applications*, MIT Press: Cambridge.
Klugkist, I., B. Kato and H. Hoijtink (2005), "Bayesian Model Selection using Encompassing Priors," *Statistica Neederlandica*, 59, 57-69.
Kremer, M., A. Onatski and J. Stock (2001), "Searching for Prosperity," Carnegie-Rochester Conference Series on Public Policy, 55, 275-303.
Kuznets, S. (1955), "Economic Growth and Income Inequality," *American Economic Review*, 45, 1-28.
Lefranc, A., N. Pistolesi and A. Trannoy (2008), "Inequality of Opportunity vs. Inequality of Outcomes: Are Western Societies all Alike?" *Review of Income and Wealth*, 54(4), 513-546.
Lefranc, A., N. Pistolesi and A. Trannoy (2009), "Equality of Opportunity and Luck: Definitions and Testable Conditions, with an Application to Income in France," *Journal of Public Economics*, 93, 1189-1207.
Lewbel, A. (2007), "Estimation of Average Treatment Effects with Misclassification," *Econometrica*, 75, 537-551.
Mankiw, N.G., D. Romer and D. Weil (1992), "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, 107, 407-437.
Mookherjee, D. and A. Shorrocks (1982), "A Decomposition Analysis of the Trend in UK Income Inequality," *Economic Journal*, 92(368), 886-902.
Nussbaum, M.C. (1997), "Capabilities and Human Rights," *Fordham Law Review*, 66, 273-300.
_____ (2011), *Creating Capabilities: The Human Development Approach*, Harvard University Press.
Pittau, M.G., R. Zelli and P.A. Johnson (2010), "Mixture Models, Convergence Clubs and Polarization," *Review of Income and Wealth*, 56, 102-122.
Pittau, M.G. and R. Zelli (2017), "At the Roots of Gini's Transvariation: Extracts from 'Il Concetto di Transvariazione e le sue Prime Applicazioni'," *Metron*, 75(2), 127-140.
Pritchett, L. (1997), "Divergence, Big Time," *Journal of Economics Perspectives*, 11, 3-17.
Quah, D., (1993), "Empirical Cross-Section Dynamics in Economic Growth," *European Economic Review*, 37, 426-434.
_____ (1996), "Convergence Empirics Across Countries with (some) Capital Mobility," *Journal of Economic Growth*, 1, 95-124.
_____ (1997), "Empirics for Growth and Distribution: Polarization, Stratification, and Convergence Clubs," *Journal of Economic Growth*, 2, 27-59.
Ravallion, M. (2010), "Mashup Indices of Development," Policy Research Working Paper, 5432, World Bank, Washington DC.
_____ (2012), "Why Don't We See Poverty Convergence?" *Economic Review*, 102, 504-523.

Rossi. P.E. (2014), *Bayesian Non- and Semi-parametric Methods and Applications*, Princeton University Press.

Sala-i-Martin, X. (1996), "Regional Cohesion: Evidence and Theories of Regional Growth and Convergence," *European Economic Review*, 40, 1325-1352.

_____ (2006), "The World Distribution Of Income: Falling Poverty and⋯Convergence, Period," *Quarterly Journal of Economics*, 121(2), 351-397.

Schultz, T.P. (1998), "Inequality and the Distribution of Personal Income in the World: How It Is Changing and Why," *Journal of Population Economics*, 11, 307-344.

Sen, A.K. (1985), "Wellbeing and Freedom: The Dewey Lectures 1984," *Journal of Philosophy*, 82, 169-221.

_____ (1993), "Capability and Well-Being," in Nussbaum, M. and A.K. Sen, eds., *The Quality of Life*, Oxford: Clarendon Press.

Stiglitz, J.E., A. Sen and J.P. Fitoussi (2010), *Mis-measuring Our Lives: Why GDP Doesn't add up. The Report of the Commission on the Measurement of Economic Performance and Social Progress*, The New Press: New York-London.

UNPD (2016), *Human Development Report 2016, Communications Development Incorporated*, Washington DC.

World Bank (1990), *World Development Report 1990: Poverty*, Washington DC: Oxford University Press.

Yitzahki, S. (1994), "Economic Distance and Overlapping of Distributions," *Journal of Econometrics*, 61, 147-159.

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