JOURNAL OF ECONOMIC DEVELOPMENT
Volume 39, Number 4, December 2014


# PUBLIC CERTIFICATION AND OPTIMAL LEVEL OF QUALITY STANDARD

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This paper analyzes public certification program where the government sets a quality standard and provides labels to sellers whose qualities are above the standard. Buyers are uninformed of sellers' qualities without the program. We study an optimal level of quality standard or optimal ratio of labeling that maximizes the social welfare. We find that the optimal quality standard becomes higher or the optimal ratio of labeling decreases as the unit certification cost increases and the quality spread grows. We also examine two public certification programs. One is mandatory and the other is voluntary. We find that the social welfare under the mandatory program is higher than under the voluntary program if and only if the unit certification cost is sufficiently small and the quality spread is sufficiently large.

*Keywords*: Quality Uncertainty, Certification, Label, Optimal Quality Standard, Quality Spread
*JEL classification*: D82, L1

## 1. INTRODUCTION

The program of certification and labeling is widely used in situations where buyers have imperfect information on the qualities provided by sellers. They help buyers to at least partially resolve asymmetric information and alleviate the problems of market failures that could arise without them. There are numerous examples in various countries. Many importing countries adopt mandatory labeling of foods that contain genetically modified organisms (Seldon, 2002; Roe and Seldon, 2007). The U.S. Environmental Protection Agency's Energy Star Program provides a label to goods that surpass energy efficiency standards (Roe and Seldon, 2007). In Korea, the government provides KC

* This paper is a revised part of a research report "A Strategy Proposal to Improve Korea's Certification Program on Tourists' Shops", which will be published at the end of 2014 by KCTI (Korea Culture and Tourism Institute). The author submitted a part of chapter 3 of the report as a consultant. The author is also most grateful to an anonymous referee for valuable suggestions.

(Korea Certification) mark to goods which meet certain quality standards in hundreds of manufactured products. Independently of KC mark program, manufacturers can voluntarily apply for Q mark certifying that they passed quality or safety tests after being evaluated by the government's or private agencies. Also in tourism industries, the government provides certification labels to the shops who offer high quality services.

When there are a number of sellers who each provide a good of different qualities and buyers do not know the qualities of sellers, the buyers evaluate their qualities by considering the average quality offered in the market. Thus some sellers with the qualities above the average will not sell the good and leave the market, because the price based on the average quality is too low for their qualities. The buyers, in turn, downgrade the average quality. This causes sellers with the next highest qualities to leave the market. In the end, there will remain only 'lemons' in the market. This is the typical example of the market failure suggested by Akerlof (1970). But as Viscusi (1978) and Grossman and Hart (1980) point out, sellers will reveal their true qualities if they have the chance to do so and the ability to make buyers believe their statements without any costs. The result of truthful disclosure is however very difficult to achieve when it is costly to *certify* the qualities, i.e., to investigate and verify the qualities and to make the buyers believe the revealed qualities. It gets even worse when the spectrum of qualities gets more diverse and the information to be revealed requires more specialized knowledge from buyers. There is an extensive literature on the factors that could limit the incentives of quality disclosure. Viscusi (1978), Grossman and Hart (1980), and Jovanovic (1982) show that if disclosure is costly, sellers will disclose only if their quality exceeds some threshold level. Fishman and Hagerty (2003) show that sellers' incentive to disclose can be restricted if some buyers cannot understand the exact contents or meanings of revealed information. Board (2009) provides another explanation for failure of information disclosure. He shows that in a duopoly setting, a firm may not choose to reveal its quality if doing so would result in fiercer competition with the rival.[1]

Reputation can be another solution for the market failure caused by asymmetric information. When repeated purchase is possible, buyers who have experienced various sellers and their qualities can identify the quality of a particular seller. However, there are many situations where repeated purchase is not possible or ineffective, if possible, for verifying the qualities. For instance, in a particular country's tourism market targeted for foreign tourists, reputation of a particular tourist's shop or restaurant is not easily obtainable. Foreign tourists do not usually visit the same country and the same location several times. Also in many consumer goods such as credence goods, quality is not easily revealed even after purchasing. Consumers, who already experienced the products such as Free-range meat and poultry, organically produced food, various health foods, and etc., have difficulty in evaluating or verifying their specific characteristics although

[1] Dranove and Jin (2010) provide an excellent survey on the literature.

these characteristics crucially affect their preferences (Crespi and Marette, 2001; Roe and Seldon, 2007). It takes long time for consumers to realize the effects of consuming high or low quality.

In such situations where incentives for voluntary quality disclosure are limited or reputation mechanism does not work effectively, the government's certification program is a useful and widespread method to deal with asymmetric information. At least two important policy issues among many could arise in the certification program; how many sellers should be provided with labels and who should pay the certification costs. This paper addresses these issues. In this paper we consider a situation where there is a continuum of sellers who each offer a good of different qualities, and there is also a continuum of buyers who each are randomly matched with sellers. The buyers are not informed of the qualities. We then analyze the government's certification program where it sets a quality standard and provides labels to the sellers whose qualities are above the standard. Notice that there is a tradeoff in choosing the quality standard. If the standard is set at a very high level, the average quality and the price for labeled sellers will be so high that they will earn handsome profits and will not leave the market. But there might be too many unlabeled sellers. The price for unlabeled sellers might be too low for the sellers of intermediate qualities without labels to earn positive profits, forcing them out of the market. On the other hand, the quality standard, if set at too a low level, will not provide proper information to buyers. The average quality of labeled sellers is not high enough for the highest quality seller to be active in the market. Our objective is to characterize an optimal level of the quality standard that maximizes the social welfare. We find that the optimal quality standard becomes higher or the optimal ratio of labeled sellers decreases as the unit certification cost increases and the variance of quality, which we will interpret as the quality spread, grows.

We examine two government's certification programs, according to the method of financing the certification costs. One is the mandatory program where the government investigates all the active sellers in the market and pays all the certification costs. The other is the voluntary program where sellers voluntarily apply for certification on their own payment of the cost. By comparing the social welfares under both programs, we find that the mandatory program is more preferable if and only if the unit certification cost is sufficiently small and the variance of the quality is sufficiently large.

The first paper that analyzes the effect of minimum quality standard (MQS) under quality uncertainty is Leland (1979). His model is similar to ours in many respects. There are continuum of buyers and sellers in both models, although sellers are price-takers in his model while they are price-setters in our model. In both models sellers' qualities are exogenously given so that market failure arises because of adverse selection instead of moral hazard. However, trade of the qualities below MQS are *legally* prohibited in his model, contrary to our model where the sellers of qualities below the quality standard can be active in the market without labels as long as they earn positive profits. In our model, the government sets the quality standard to provide labels to high qualities, but not to expel low qualities out of the market.[2] Further, Leland did not explicitly characterize the optimal level of MQS, nor address the problem of how to finance the cost of certifying qualities, both of which are the main concerns in this paper. Crespi and Marette (2001) directly deal with the financing method of certification costs and show that the social welfare is always higher under voluntary certification with a per-unit fee than mandatory certification. Their result is different from ours in that the relative ranking can be reversed in our model, depending on the parameters such as the variance of qualities and the per-unit certification cost. Furthermore, mainly focusing on the comparison among various financing method, they sidestepped the issue of the optimal level of quality standard. In their model, there are a large number of sellers who each can choose the type between only two qualities, high and low. Therefore it is not an issue for the government on which level it should set the quality standard. In the same vein with Crespi and Marette (2001), Roe and Seldon (2007) analyze the effects of various certification programs, including private agency certification as an additional option. They build on a model of vertical product differentiation where two firms of distinct qualities compete[3] and show that the high quality firm has incentive to hire a private certifier if the government's quality standard substantially deviates from its quality.

The paper is organized as follows. In Section 2, we start by a typical model of information asymmetry as a benchmark and extend the model to incorporate the government's certification program. In Section 3, we consider the mandatory certification program and find the optimal level of quality standard. We also compare the social welfare under this program with the social welfare of the benchmark case. In Section 4, we derive the optimal level of quality standard under the voluntary program. We also compare the social welfares under both programs. We conclude in Section 5.

## 2. THE MODEL

### 2.1. Before Introduction of the Government's Certification Program

There is a continuum of potential sellers, who are each willing to sell one unit of a good of different qualities indexed by $q$. $q$ also refers to the seller of quality $q$. We

[2] Since low qualities are expelled from the market in Leland (1979), introduction of MQS has not only a positive effect, but also a negative effect on social welfare and consumer surplus because trade surplus from low qualities disappears and the price is raised. This particular type of negative effect does not arise in our model.

[3] There are more than two firms in their model, but the parameters are set in such a way that only two firms remain in the market because of fixed costs.

assume that $q$ is uniformly distributed over the interval $[a-b, a+b]$, where $a>0$, $0<b\leq a$. The total population of sellers is 1. Therefore, the density of quality is $g(q)=\frac{1}{2b}$, the average quality is $E(q)=a$, and the variance of quality is $Var(q)=\frac{b^2}{3}$. As $b$ increases, the spread of quality between sellers increases without changing average quality. We interpret $b$ as the quality spread. Seller $q$'s cost is $c(q)=3q$, so a seller with higher quality bears higher costs.[4] There is also a continuum of buyers of population 1. Each buyer is randomly matched with a seller. The seller sets the price and the buyer decides whether to buy from the seller or not. Each buyer's willingness to pay for quality $q$ is $4q$, and his utility for purchasing from seller $q$ at price $p$ is $u(q)=4q-p$.[5] His utility is 0 if he decides not to buy. If the buyer perfectly observes the quality of the seller, seller $q$'s optimal price is $p=4q$ and her profit is $\pi(q)=4q-3q=q$. Observe that all the sellers earn positive profits and thus remain in the market.

However, we assume that buyers cannot observe the quality of the matched seller. They only know the distribution of qualities or sellers in the market. If all the sellers are active in the market, average quality in the market is $E(q)=a$, and buyers would not buy if the price is above $4a$. Since the sellers set the price, the price becomes $4a$ and seller $q$'s profit is $\pi(q)=4a-3q$.[6] Therefore, the sellers with $q\in\left[\frac{4a}{3}, a+b\right]$ would earn negative profits and would leave the market.[7] And the sellers with quality below $\frac{4a}{3}$ are active in the market. Then buyers' expectation over the qualities of those sellers becomes lower than $E(q)=a$, and the price also will decrease, forcing more sellers to leave the market. This process will continue until the highest quality seller among those who eventually remain in the market earns zero profit.

To provide a formal description, denote the highest quality remained in the market by $q_U$. That is, seller $q\in[a-b, q_U]$ remains in the market and all the sellers with quality above $q_U$ leave the market because their profits are less than zero. In this situation,

[4] Alternatively, we can suppose that a seller of higher quality has higher opportunity cost.

[5] These specific forms of sellers' cost functions and buyers' utility functions are taken to simplify calculations. The results would not change with more general forms of cost and utility functions.

[6] Since sellers offer the price and matched buyers decide to take it or leave in our model, the optimal price is set at the level where buyer's expected utility becomes 0. An alternative market structure is perfect competition, where sellers and buyers are price-takers. However, there would be no qualitative change in the results of this paper under perfect competition.

[7] Here we assume that $\frac{a}{3}<b$.

buyers' conditional expectation over the remaining sellers' qualities, i.e., the average quality of the market, is $E(q|q \le q_U) = \dfrac{a-b+q_U}{2}$. The seller's optimal price is then $p = 4E(q|q \le q_U) = 2(a-b+q_U)$ and seller $q$'s profit is $\pi(q) = 2(a-b+q_U) - 3q$. By the definition of $q_U$, $\pi(q) \ge 0$ for $q \in [a-b, q_U]$ and $\pi(q_U) = 2(a-b+q_U) - 3q$ $= 2(a-b) - q_U = 0$. Therefore, we have $q_U = 2(a-b)$. That is, in the equilibrium of our benchmark model, only sellers with quality $q \in [a-b, 2(a-b)]$ remain in the market and those with quality higher than $q_U = 2(a-b)$ have to leave the market because they earn negative profits. This is the situation of Akerlof's (1970) 'lemons market'. There is a market failure under asymmetric information while all the qualities are sold and Pareto efficiency is achieved under perfect information. Notice that we need a requirement of $q_U = 2(a-b) < a+b$ or $b > \dfrac{a}{3}$ for this argument. Otherwise, even the seller with the highest quality, $q = a+b$, earns a positive profit so that all the sellers remain in the market. There is no market failure despite asymmetric information. We will also assume that $a \ge b$ to guarantee $q_U = 2(a-b) \ge 0$. Without it, no seller will earn a positive profit and there will be no trade at all in the equilibrium. This is the case of extreme market failure.

Summarizing these, we have

**Proposition 1**: Suppose that buyers cannot observe the qualities of the sellers, but only know the distribution of qualities in the market. Then the equilibrium is as follows:

(1) If $b \le \dfrac{a}{3}$, all the qualities are provided. Thus there is no market failure.

(2) If $b \ge a$, no qualities are provided in the market.

(3) If $\dfrac{a}{3} < b < a$, only qualities lower than $2(a-b)$ are provided in the market.

Proposition 1 provides a simple lesson. If $\dfrac{a}{3} < b < a$, the ratio of the sellers active in the market is $\dfrac{a-b}{2b}$ and decreases as $b$ increases. This, along with (1) and (3), implies that market failure becomes serious as the quality spread grows. For the further analysis, we will assume throughout the paper that $\dfrac{a}{3} < b < a$ so that only lower qualities are provided in the market under asymmetric information.

### 2.2. Two Certification Programs

Suppose now that the government introduces a certification and labeling program. That is, the government sets a quality standard and provides labels to the sellers whose qualities are above the standard. Of course the sellers who do not satisfy the standard cannot receive the label, but they are allowed to sell without labels. A buyer, if matched with a seller with the label, expects the quality to surpass the standard and is willing to pay more than when he meets a seller without label. Notice that there is a tradeoff in choosing the quality standard. As the standard is set at a higher level, the average quality for labeled sellers becomes higher, raising their price enough for them to earn positive profits. But there might be too many unlabeled sellers. The price for unlabeled sellers might be too low for the sellers of intermediate qualities without labels to earn positive profits, forcing them out of the market. On the other hand, the quality standard, if set at too a low level, will not provide proper information to buyers. The average quality of labeled sellers is not high enough for the highest quality seller to be active in the market. Our objective is to find an optimal level of the quality standard to maximize social welfare.

Certification accompanies costs as it requires investigating and verifying whether sellers' qualities meet the standard. We assume that certification incurs a per-unit cost for every unit certified and there is no fixed cost. The certification cost per a seller is $f$. We consider two possible scenarios of the government's certification program, according to the method of financing the certification costs. One is the mandatory program where the government investigates *all the active* sellers in the market and pays all the certification costs.[8] If no seller leaves the market, the government has to pay the total cost of $f$ since the population of the active sellers is 1. If the population of active sellers is $\alpha(\leq 1)$, the total cost is $\alpha f$. The other scenario is the voluntary program. After the quality standard being announced by the government, a seller voluntarily applies for certification on her own payment of the certification cost $f$. The government investigates *only the applicant's quality* and provides the label if it satisfies the standard. Under this program, the sellers of qualities lower than the standard do not have incentive to participate in the program because doing so only costs them the fee without receiving the labels. Our concern is to induce all the sellers of qualities higher than the standard to apply for certification and obtain the labels. Throughout the paper, we assume that certification and labeling by a private agency is not available. It is also impossible for the government to provide information on each seller's exact quality level because of prohibitively high verification costs.

To be more precise, we describe the procedure of the game as follows. At stage 1, the government chooses one of the two certification program and announces the quality

[8] The government needs to monitor and investigate all the qualities in the market when the program is mandatory (Crespi and Marette, 2003).

standard $q_L$ to the sellers and the buyers. At stage 2, each seller $q \in [a-b, a+b]$ decides whether to stay in the market or to leave. Sellers who expect negative profits will leave the market. Stage 3 is the stage of certification. The government verifies the qualities of the sellers following the method it chooses at stage 1 and provides the label to the sellers who meet the standard. At stage 4, each buyer is randomly matched with a seller who decided to remain in the market at stage 2.[9] Buyers do not know the exact qualities of matched sellers, but are informed of the distributions of labeled and unlabeled sellers. They of course recognize whether the matched seller has a label or not. At stage 5, each active seller sets her price and the matched buyer decides whether to buy or not.

## 3. THE MANDATORY CERTIFICATION PROGRAM

### 3.1. Equilibrium

Consider the first scenario that the government investigates all the active sellers in the market and pays all the costs. Our main task is to find the optimal level of quality standard that maximizes the social welfare. To do this, we need to characterize the equilibrium behavior from stage 2 for given quality standard level. Suppose that the level of quality standard set by the government at stage 1 is $q_L$. Suppose also that all the sellers of qualities higher than $q_L$ decided to remain in the market at stage 2 and thus they all obtained labels at stage 3.[10] Then the average quality for labeled sellers is $E(q|q \ge q_L) = \dfrac{a+b+q_L}{2}$, and labeled sellers will charge the price of $4E(q|q \ge q_L) = 2(a+b+q_L)$. The profit of labeled seller $q$ is $\pi^L(q) = 2(a+b+q_L) - 3q$. In order for all the labeled sellers to remain in the market, we should have $\pi^L(q) \ge 0$ for all $q \in [q_L, a+b]$. In particular, the highest quality seller's profit should be nonnegative.

$$\pi^L(a+b) = 2(a+b+q_L) - 3(a+b) = 2q_L - (a+b) \ge 0 \text{, or } q_L \ge \frac{a+b}{2}.$$

If the quality standard level $q_L$ were set below $\dfrac{a+b}{2}$ at stage 1, the sellers of the

[9] If the measure of the sellers who are active in the market is $\alpha$, the buyers of measure $1-\alpha$ cannot find a match.

[10] For this supposition, all the sellers of qualities above $q_L$, the highest quality seller in particular, should earn nonnegative profits, which we will mention shortly.

highest qualities, i.e., the sellers of $q \in [2q_L, a+b]$ would leave the market at stage 2, and only the sellers of $q \in [q_L, 2q_L]$ would obtain the labels. However, we can verify that the social welfare in this case is lower than when $q_L$ is set at or above $\frac{a+b}{2}$. Therefore we take it for granted that the government sets the quality standard at $q_L \geq \frac{a+b}{2}$ so that all the sellers of qualities above $q_L$ decide to remain in the market at stage 2 and obtain labels at stage 3.

Now consider the sellers without labels, i.e., the sellers of qualities below $q_L$. Notice that not all the sellers of qualities below $q_L$ are active in the market. Recall from Proposition 1 that only sellers of qualities $q \in [a-b, 2(a-b)]$ remain in the market without the government's certification program. Therefore if $q_L > 2(a-b)$, the sellers of qualities $q \in [2(a-b), q_L]$ will leave the market at stage 2 and the sellers of qualities $q \in [a-b, 2(a-b)]$ are active without labels in the market. If $q_L \leq 2(a-b)$, no sellers would have left the market at stage 2 and thus all the unlabeled sellers are active in the market. In the former case, the average quality of unlabeled sellers and their price is $\frac{3(a-b)}{2}$ and $6(a-b)$, respectively. And unlabeled seller $q$'s profit is $\pi^U(q) = 6(a-b) - 3q$. In the latter case, unlabeled seller $q$'s profit becomes $\pi^U(q) = 2(a-b+q_L) - 3q$ with the average quality of unlabeled sellers and their price being $\frac{a-b+q_L}{2}$ and $2(a-b+q_L)$, respectively.

From the discussion, the equilibrium for given $q_L$ can be summarized as in Lemma 1.

**Lemma 1**: Consider the mandatory certification program. Suppose that $q_L \geq \frac{a+b}{2}$. Then all the sellers of qualities above $q_L$ decide to remain in the market at stage 2 and obtain labels at stage 3. At stage 5, they set the price of $2(a+b+q_L)$. Labeled seller $q$'s profit is $\pi^L(q) = 2(a+b+q_L) - 3q$. The distribution, the price, and the profit of unlabeled sellers differ, depending on the level of $q_L$.

(1) If $q_L > Max\left[2(a-b), \frac{a+b}{2}\right]$, sellers of qualities $q \in [2(a-b), q_L]$ leave the market at stage 2 and those of qualities $q \in [a-b, 2(a-b)]$ remain in the market without labels. The price of unlabeled sellers is $6(a-b)$ and unlabeled seller $q$'s profit is $\pi^U(q) = 6(a-b) - 3q$.

(2) If $\frac{a+b}{2} \le q_L \le 2(a-b)$, all the sellers of qualities below $q_L$ remain in the market without labels. The price of unlabeled sellers is $2(a-b+q_L)$ and unlabeled seller $q$ 's profit is $\pi^U(q) = 2(a-b+q_L) - 3q$.

### 3.2. The Optimal Level of Quality Standard

Let us find the social welfare when the quality standard level set by the government is $q_L$. Notice first that the 'individual' social welfare generated by an active seller $q$, whether she has a label or not, is the sum of her profit and the matched buyer's consumer surplus, net of the certification cost $f$. Therefore it can be simply written as $sw(q) = q - f = u(q) + \pi(q) - f = 4q - p + p - 3q - f$, where $p$ is the seller $q$ 's price. As the social welfare generated by inactive sellers are 0, the (total) social welfare with the quality standard of $q_L$, $SW(q_L)$, is the sum of the individual social welfare over the all active sellers and can be written as follows:

(1) When $q_L \ge Max\left[2(a-b), \frac{a+b}{2}\right]$,

$$SW(q_L) = \int_{q_L}^{a+b} [q-f]\frac{1}{2b}dq + \int_{a-b}^{2(a-b)} [q-f]\frac{1}{2b}dq = \frac{4(a^2 - ab + b^2) - 4af + 2fq_L - q_L^2}{4b}.$$

(2) When $\frac{a+b}{2} \le q_L \le 2(a-b)$,

$$SW(q_L) = \int_{q_L}^{a+b} [q-f]\frac{1}{2b}dq + \int_{a-b}^{q_L} [q-f]\frac{1}{2b}dq = a - f.$$

A few remarks are in order. First of all, we need a requirement of $q_L \ge \frac{a+b}{2}$ to ensure all the sellers of qualities above $q_L$ to be active as labeled sellers. Secondly, observe that the distinction between the case (1) and (2) follows Lemma 1 and is based on the existence of inactive sellers. If $q_L > Max\left[2(a-b), \frac{a+b}{2}\right]$, the sellers of qualities $q \in [2(a-b), q_L]$ will leave the market and thus the social welfare can be expressed as in (1). If $\frac{a+b}{2} \le q_L \le 2(a-b)$, on the other hand, all the sellers are active.[11] The social welfare is then $SW(q_L) = \int_{a-b}^{a+b} \frac{q-f}{2b} dq = a-f$ as expressed in (2), and does not depend on $q_L$. Lastly, we will assume throughout the paper that $f < a+b$ so that the certification cost is not too high. Recall that the 'individual' social welfare generated by an active seller $q$ is $sw(q) = q - f$. Therefore, if $f > a+b$, the individual social welfare generated by even the highest quality seller $q = a+b$ becomes negative. In this case, introduction of the certification program definitely reduces the social welfare and is therefore out of our concern.

Investigating the shape of $SW(q_L)$, we can find the optimal level of quality standard that maximizes $SW(q_L)$ as follows.

**Proposition 2**: Consider the mandatory certification program. Assume that $f < a+b$. The optimal level of quality standard $q_L^*$ and the maximized social welfare $SW(q_L^*)$ are given as follows:

(1) When $Max\left[2(a-b), \frac{a+b}{2}\right] = \frac{a+b}{2}$ or $\frac{3a}{5} < b < a$,

(1-i) if $f \le \frac{a+b}{2}$, $q_L^* = \frac{a+b}{2}$ and $SW(q_L^*) = \frac{3(5a^2 - 6ab + 5b^2) - 4f(3a-b)}{16b}$,

(1-ii) if $\frac{a+b}{2} \le f < a+b$, $q_L^* = f$ and $SW(q_L^*) = \frac{4(a^2 - ab + b^2) - 4af + f^2}{4b}$.

(2) When $Max\left[2(a-b), \frac{a+b}{2}\right] = 2(a-b)$ or $\frac{a}{3} < b \le \frac{3a}{5}$,

(2-i) if $f \le 2(a-b)$, $q_L^* \in \left[\frac{a+b}{2}, 2(a-b)\right]$ and $SW(q_L^*) = a - f$,

(2-ii) if $2(a-b) \le f < a+b$, $q_L^* = f$ and $SW(q_L^*) = \frac{4(a^2 - ab + b^2) - 4af + f^2}{4b}$.

**Proof**: See the appendix.

The intuition for this result can be explained as follows. The effect of reducing the level of the quality standard $q_L$ on the social welfare is different, depending on the

[11] Observe that the inequalities are valid only if $\frac{a+b}{2} \le 2(a-b)$ or $\frac{3a}{5} \ge b$.

existence of inactive sellers in the market. When $q_L > Max\left[2(a-b), \frac{a+b}{2}\right]$, so that the sellers of qualities $q \in [2(a-b), q_L]$ do not exist in the market, reduction in the quality standard has two countervailing effects. The first is the switch of some inactive sellers to labelled sellers, which will obviously increase the social welfare. But this switch has also a negative impact on the social welfare, because of the additional certification costs. More precisely, the marginal social welfare of reducing $q_L$ is $q_L - f$, where $q_L$ is the marginal benefit generated by the marginal seller $q_L$ and $f$ is the marginal certification cost.[12] That is, the marginal social welfare of reducing $q_L$ is exactly the same as the individual social welfare generated by seller $q_L$. When $\frac{a+b}{2} \le q_L \le 2(a-b)$, however, all the sellers are active either as labeled sellers or as unlabeled ones. In this case, the marginal effect of reducing the quality standard on the social welfare is 0 because there is neither switch from the inactive sellers to active sellers nor additional certification costs. There is only the switch from the unlabeled sellers to labelled sellers, which does not affect the social welfare. Recall that the individual social welfare generated by an active seller $q$ is $(q) = q - f$. It is the same across all the active sellers and does not depend on whether seller $q$ is labeled or unlabeled.

The optimal level of the quality standard can be found by the following logic. Consider first the case of $2(a-b) < \frac{a+b}{2}$, or $\frac{3a}{5} < b < a$. Since the marginal social welfare of (increasing) $q_L$ is $f - q_L$ and the lower bound of $q_L$ is $\frac{a+b}{2}$, the social welfare is maximized at $q_L = f$ if $f \ge \frac{a+b}{2}$. But if $f < \frac{a+b}{2}$, the social welfare is maximized by reducing $q_L$ to the lower bound, $\frac{a+b}{2}$. This is the result of (1-i) and (1-ii) of Proposition 2. Consider now the case of $\frac{a+b}{2} \le 2(a-b)$, or $\frac{a}{3} < b \le \frac{3a}{5}$. Notice that the marginal social welfare of (increasing) $q_L$ has a jump at $q_L = 2(a-b)$ because it is 0 when $\frac{a+b}{2} \le q_L \le 2(a-b)$, but becomes $f - q_L$ when $q_L > 2(a-b)$. Therefore the social welfare is maximized at any level $q_L \in \left[\frac{a+b}{2}, 2(a-b)\right]$ if $f \le 2(a-b)$,

[12] The marginal effect of increasing $q_L$ is $f - q_L$.

while it is maximized at $q_L = f$ if $f > 2(a-b)$, as specified in (2-i) and (2-ii) of Proposition 2. Combining all these results, we can summarize Proposition 2 as follows. When $f$ is small, it is optimal to minimize the size of inactive sellers either by making it 0 (i.e., when $\frac{a+b}{2} \le 2(a-b)$) or by setting $q_L$ at the lower bound (i.e., when $2(a-b) < \frac{a+b}{2}$). However, as $f$ becomes larger and thus the certification cost outweighs the marginal benefit of reducing the quality standard, the optimal quality standard should be set at a higher level although the size of inactive sellers could have been reduced at a lower level of $q_L$.

Since the sellers of $q \in [q_L^*, a+b]$ have labels at the optimal level of quality standard, the optimal ratio of certification or the optimal ratio of labeled sellers is $\mu^* = \frac{a+b-q_L^*}{a+b-(a-b)} = \frac{a+b-q_L^*}{2b}$. From Proposition 2, the optimal ratio of certification is easily obtained.

**Corollary 1**: $\mu^*$, the optimal ratio of certification or the optimal ratio of labeled sellers, is as follows:

(1-i) if $\frac{3a}{5} < b < a$ and $f \le \frac{a+b}{2}$, $\mu^* = \frac{a+b}{4b}$, (1-ii) if $\frac{3a}{5} < b < a$ and $\frac{a+b}{2} \le f < a+b$, $\mu^* = \frac{a+b-f}{2b}$,

(2-i) if $\frac{a}{3} < b \le \frac{3a}{5}$ and $f \le 2(a-b)$, $\mu^*$ is any number in $\left[\frac{3b-a}{2b}, \frac{a+b}{4}\right]$, and

(2-ii) if $\frac{a}{3} < b \le \frac{3a}{5}$ and $2(a-b) < f < a+b$, $\mu^* = \frac{a+b-f}{2b}$.

Corollary 1 shows that $\mu^*$ decreases as $b$ increases except for the case of (2-ii) where the optimal quality standard $q_L^*$ and thus $\mu^*$ are not uniquely determined. This implies that the optimal ratio of certification should be smaller as the quality spread becomes larger.

A properly designed certification program, say the program with the quality standard set at the optimal level, can partially resolve buyers' information asymmetry because it enables buyers matched with labeled sellers to upgrade their expectations over the qualities and to buy from them, which would have been impossible without the program. However, the program accompanies the certification costs. Therefore we need to compare the social welfare after introduction of the certification program with the social welfare before the introduction. The social welfare with the quality standard being set at the optimal level, $SW(q_L^*)$, is given in Proposition 2. The social welfare without the certification program, denoted by $SW^o$, can be also easily obtained as $SW^o = \int_{a-b}^{2(a-b)} \frac{q}{2b} dq = \frac{3(a-b)^2}{4b}$. Comparing these two, we have the following result.

**Proposition 3**: Consider the mandatory certification program. Suppose that the government sets the quality standard for certification at the optimal level, i.e., at $q_L^*$. Assume that $f < a+b$. The social welfare increases with introduction of the certification program only if $f$ is sufficiently small. More precisely,

(1) When $\frac{3a}{5} < b < a$, $SW(q_L^*) > SW^o$ if and only if $f < 2a - \sqrt{(a-b)(3a+b)}$.

(2) When $\frac{a}{3} < b \le \frac{3a}{5}$, $SW(q_L^*) > SW^o$ if and only if $f < \frac{(3a-b)(3b-a)}{4b}$.

**Proof**: See the appendix.

Since the government should pay the certification costs of all the active sellers including unlabeled ones, the result is not surprising that introduction of the certification program decreases the social welfare when the certification cost is high.

A less obvious feature in Proposition 3 is that introduction of the certification program is more likely to increase the social welfare when the quality spread $b$ becomes larger. To see this, observe that

$$\frac{\partial}{\partial b}\left(2a - \sqrt{(a-b)(3a+b)}\right) = \frac{a+b}{\sqrt{(a-b)(3a+b)}} > 0,$$

$$\frac{\partial}{\partial b}\left(\frac{(3a-b)(3b-a)}{4b}\right) = \frac{3(a-b)(a+b)}{4b^2} > 0.$$

That is, both the critical values of $f$, below which we have $SW(q_L^*) - SW^o > 0$, are increasing functions of $b$. We can also verify that the critical value in case of $\frac{3a}{5} < b < a$ is larger than that in case of $\frac{a}{3} < b \le \frac{3a}{5}$, i.e., $2a - \sqrt{(a-b)(3a+b)} > \frac{(3a-b)(3b-a)}{4b}$. These facts mean that the range of $f$ in which $SW(q_L^*) - SW^o > 0$ increases as $b$ grows.

**Corollary 2**: The mandatory certification program is more likely to increase the social welfare when the quality spread $b$ is larger. More precisely, the range of $f$ in which $SW(q_L^*) - SW^o > 0$ increases as $b$ grows.

The intuition for this result will be provided at the end of Section 4, in a slightly different context.

## 4. THE VOLUNTARY CERTIFICATION PROGRAM

### 4.1. Equilibrium

Consider next the voluntary certification program. Under this program, sellers of qualities below the government's standard will not apply for labels because of the application fee of $f$. Our concern is to induce sellers of qualities above the quality standard to remain in the market at stage 2 *and* to apply for certification and obtain the labels at stage 3. First of all, their profits should be nonnegative to ensure them to remain in the market. Furthermore, in order for them to apply for certification, their profits should be higher with labels than the profits they would get without labels. Notice that the second incentive problem does not arise under the mandatory program where the government investigates *all* the sellers' qualities as long as they decided to remain in the market.

To satisfy the first incentive, we need a lower bound of the quality standard as we analyzed in the previous section. To derive the lower bound, let $q_L$ be the quality standard level set by the government at stage 1. Suppose that all the sellers of qualities above $q_H$ decided to remain in the market at stage 2 and applied for labels (and obtained labels) at stage 3. Recall from Proposition 2 that the profit of labeled seller $q$ is $\pi^L(q) = 2(a+b+q_L) - 3q$ under the mandatory program. Under the voluntary program, her profit changes to $\pi^L(q) = 2(a+b+q_L) - 3q - f$ because of the application fee. Since the highest quality seller's profit should be nonnegative, we should have

$$\pi^U(a+b) = 2(a+b+q_L) - 3(a+b) - f = 2q_L - (a+b+f) \geq 0 \text{, or } q_L \geq \frac{a+b+f}{2}.$$

Observe that the lower bound of $q_L$ under the current program, $\frac{a+b+f}{2}$, is higher than $\frac{a+b}{2}$, the lower bound under the mandatory program. This is because labeled sellers' profits are lower under the current program by the application fee. To ensure all the sellers of qualities above $q_L$ to be active in the market, the reduced profit has to be recouped and the price of labeled sellers should become higher. The only way to raise the price of labeled sellers is to raise the average quality of labeled sellers by increasing the quality standard.

The second incentive is also satisfied as long as $q_L \geq \frac{a+b+f}{2}$. We will show this in the proof of Lemma 2. With both the first and the second incentive problems being resolved, unlabeled sellers' incentives are the same as under the mandatory program because they do not have incentive to apply for labels and do not pay the certification costs. The distribution of active unlabeled sellers, their prices, and their profits are the same as specified in Lemma 1. But for expositional purpose, we summarize the equilibrium for given $q_L$ in Lemma 2.

**Lemma 2**: Consider a voluntary certification program. Suppose that $q_L \geq \frac{a+b+f}{2}$. Then all the sellers of qualities above $q_L$ decide to remain in the market at stage 2. They all apply for and obtain labels at stage 3. At stage 5, they set the price of $2(a+b+q_L)$. Labeled seller $q$'s profit is $\pi^L(q) = 2(a+b+q_L) - 3q - f$. Other sellers' behaviors are as follows.

(1) If $q_L > Max\left[2(a-b), \frac{a+b+f}{2}\right]$, the sellers of qualities $q \in [2(a-b), q_L]$ leave the market at stage 2 and those of qualities $q \in [a-b, 2(a-b)]$ remain in the market without labels. The price of unlabeled sellers is $6(a-b)$ and unlabeled seller $q$'s profit is $\pi^U(q) = 6(a-b) - 3q$.

(2) If $\frac{a+b+f}{2} \leq q_L \leq 2(a-b)$, all the sellers of qualities below $q_L$ remain in the market without labels. The price of unlabeled sellers is $2(a-b+q_L)$ and unlabeled seller $q$'s profit is $\pi^U(q) = 2(a-b+q_L) - 3q$.

**Proof**: See the appendix.

### 4.2. The Optimal Level of Quality Standard

$SW(q_L)$, the social welfare when the quality standard is set at $q_L$, can be found in a very similar fashion as in Section 3. Following the discussion of Section 3 and Lemma 2, we have

(1) When $q_L \geq Max\left[2(a-b), \dfrac{a+b+f}{2}\right]$,

$$SW(q_L) = \int_{q_L}^{a+b}[q-f]\frac{1}{2b}dq + \int_{a-b}^{2(a-b)} q\frac{1}{2b}dq = \frac{4(a^2-ab+b^2)-2f(a+b)+2fq_L-q_L^2}{4b}.$$

(2) When $\dfrac{a+b+f}{2} \leq q_L \leq 2(a-b)$,

$$SW(q_L) = \int_{q_L}^{a+b}[q-f]\frac{1}{2b}dq + \int_{a-b}^{q_L} q\frac{1}{2b}dq = \frac{2ab-f(a+b)+fq_L}{2b}.$$

Two changes are made from the previous section. One is the lower bound of $q_L$. It is raised to $\dfrac{a+b+f}{2}$ from $\dfrac{a+b}{2}$. The other is the change in the individual social welfare generated by unlabeled sellers. An unlabeled seller $q$ generates an individual social welfare of $q$ instead of $q-f$, because there is no certification cost incurred for unlabeled sellers. Notice that we still need the assumption of $f < a+b$. Otherwise the lower bound of $\dfrac{a+b+f}{2}$ exceeds $a+b$, in which case the highest quality seller will leave the market.

The optimal level of $q_L$ that maximizes $SW(q_L)$ is easy to find and is specified in Proposition 4.

**Proposition 4**: Consider the voluntary certification program. Assume that $f < a+b$. The optimal level of quality standard $q_L^{**}$ and the maximized social welfare $SW(q_L^{**})$ are given as follows:

(1) When $Max\left[2(a-b), \dfrac{a+b+f}{2}\right] = 2(a-b)$ or $f \leq 3a-5b$,

$$q_L^{**} = 2(a-b) \text{ and } SW(q_L^{**}) = \frac{2ab-f(3b-a)}{2b}.$$

(2) When $Max\left[2(a-b), \dfrac{a+b+f}{2}\right] = \dfrac{a+b+f}{2}$ or $f > 3a-5b$,

$$q_L^{**} = \frac{a+b+f}{2} \text{ and } SW(q_L^{**}) = \frac{3[5a^2-6ab+5b^2-2f(a+b)+f^2]}{16b}.$$

**Proof**: See the appendix.

Summarizing (1) and (2) of Proposition 4, we have $q_L^{**} = Max\left[2(a-b), \frac{a+b+f}{2}\right]$. Thus it is always optimal to minimize the size of inactive sellers, regardless of the size of $f$. When $f$ is small, the optimal quality standard should be set at the level just enough to get rid of the inactive sellers ((1) of Proposition 4). When $f$ is large, the optimal level is set at the lower bound ((2) of Proposition 4). Compared with Proposition 2, the optimal level becomes higher under the current program than under the mandatory program. That is, $q_L^{**} \geq q_L^{*}$. This is mainly because the lower bound for quality standard becomes higher due to the certification fee that individual sellers have to pay. Accordingly, the optimal ratio of labeled sellers, $\mu^{**} = \frac{a+b-q_L^{**}}{a+b-(a-b)} = \frac{a+b-q_L^{**}}{2b}$, will be also lower than the ratio under the previous program.

From Proposition 4, it is easy to verify that the social welfare, if the quality standard is set at the optimal level, is always higher than the social welfare without the certification program. That is, we have $SW(q_L^{**}) > SW^o = \frac{3(a-b)^2}{4b}$. The reason is obvious. It is because each labeled seller under the current program generates a positive individual social welfare of $SW(q) = q - f$, which is positive for $q \geq q_L^{**}$, while labeled sellers would not have been active without the certification program. Now let us compare the social welfare under the voluntary program with the one under the mandatory program. We of course evaluate both social welfares at the corresponding optimal quality standard levels. As can be seen in Proposition 2 and Proposition 4, the expressions of $SW(q_L^{*})$ and $SW(q_L^{**})$ differ from case to case depending on the sizes of $a$, $b$ and $f$. Thus the comparison is rather complicated, but the result can be summarized in Proposition 5.

**Proposition 5:** Suppose that the government sets the quality standard at the optimal level. Assume that $f < a+b$. The social welfare under the mandatory program is higher than under the voluntary program if and only if $b$ is sufficiently large and $f$ is sufficiently small. More precisely,

(1) When $\frac{a}{3} < b < \frac{3a}{5}$, $SW(q_L^{*}) < SW(q_L^{**})$.

(2) When $\frac{3a}{5} \leq b \leq \frac{15a}{17}$, $SW(q_L^{*}) > SW(q_L^{**})$ if and only if $f < \frac{2(5b-3a)}{3}$.

(3) When $\frac{15a}{17} < b < a$, $SW(q_L^{*}) > SW(q_L^{**})$ if and

only if $f < 5a - 3b - 2\sqrt{2(a-b)(3a-b)}$ .

**Proof:** See the appendix.

The intuition for Proposition 5 can be explained as follows. Since the optimal quality standard is higher under the voluntary program, there will be less labeled sellers and more inactive sellers. This is the disadvantage of the voluntary program in terms of the social welfare. But this program is less costly since only labeled sellers pay the certification costs, while the government should pay the costs for unlabeled sellers as well as labeled sellers under the mandatory program. Therefore the social welfare is higher under the mandatory program if the individual certification cost $f$ is small, but the ranking will be reversed if $f$ becomes larger.

A more interesting result is that the social welfare is more likely to be higher under the mandatory program as the quality spread $b$ increases. Observe from (1) of Proposition 5 that the social welfare is always lower under the mandatory program when $b$ is small. Observe also from (2) and (3) that both the two critical values of $f$ at which the relative social welfare ranking is reversed, $\frac{2(5b-3a)}{3}$ and $5a - 3b - 2\sqrt{2(a-b)(3a-b)}$, are increasing in $b$. Furthermore, it can be verified that the latter critical value is larger than the former. All these facts imply that the range of $f$ where the mandatory program has the social welfare advantage over the other increases as $b$ grows. To see the reason for this, suppose $\frac{3a}{5} < b < a$ and $f < \frac{a+b}{2}$ for instance so that the optimal quality standards are set at the lower bounds under both programs. That is, $q_L^* = \frac{a+b}{2}$ and $q_L^{**} = \frac{a+b+f}{2}$ . In this case the sellers $q \in [a-b, 2(a-b)]$ are unlabeled active sellers under both programs and thus their population is $\frac{a-b}{2b}$, where $\frac{1}{2b}$ is the density of $q$. The difference in the social welfare is $SW(q_L^*) - SW(q_L^{**}) = \int_{\frac{a+b}{2}}^{\frac{a+b+f}{2}} \frac{q-f}{2b} dq - f\frac{a-b}{2b}$ . The first term of the right-hand side of the equality is the social welfare generated by the sellers who obtain labels under the mandatory program, but turn to be inactive under the voluntary program. The second term is the certification costs required for unlabeled active sellers under the mandatory program. Increase in $b$ definitely decreases these certification costs, because it will decrease the size of unlabeled active sellers $\frac{a-b}{2b}$. However, the effect of $b$ on the first term is ambiguous. This is because increase in $b$ affects the first term in two opposite directions. One is that the qualities of the labeled sellers under the mandatory program who turn to become inactive under the voluntary program are raised as the quality spread grows. That is, high qualities become higher as $b$ grows. This definitely increases the social welfare advantage of the mandatory program. The opposing effect is the reduction in the population of this group of sellers, which decreases the social welfare advantage of the mandatory program. Our result is because of dominance of the sum of the cost reduction effect and the quality increasing effect over the last effect. The intuition for Corollary 2 can be found in a very similar way.[13]

## 5. CONCLUSION

When buyers have asymmetric information on sellers' qualities, high qualities disappear and only low qualities remain in the market. Public certification and labeling is a useful method to alleviate the market failure. Most widespread among various public certification programs is simple labeling where the government sets a quality standard and provides labels to sellers whose qualities are above the standard. We address two policy issues concerning this simple labeling program; what proportion of sellers should be provided with labels and how the program should be financed. We examine two public certification programs, according as who pays the certification costs. One is mandatory where the government investigates all the active sellers in the market and pays all the costs. The other is voluntary where sellers voluntarily apply for labels with paying application fee and the government investigates only the qualities of the applicants.

Our results are summarized as follows. First, in both programs, the optimal level of quality standard that maximizes social welfare becomes higher as the unit certification cost increases and the quality spread grows in most cases. Second, the mandatory program improves social welfare only if the unit certification cost is sufficiently small and the quality spread is sufficiently large. Third, the voluntary certification program always improves social welfare. Lastly, the social welfare under the mandatory program is higher than under the voluntary program if and only if the unit certification cost is sufficiently small and the quality spread is sufficiently large.

From the above results, we can suggest two policy implications. From the first result, the government should set more stringent quality standard and reduce the ratio of

[13] Suppose for instance that $\frac{3a}{5} < b < a$ and $f < \frac{a+b}{2}$. Then the difference in the social welfare under the mandatory program and the social welfare without the certification program is given by $\int_{\frac{a+b}{2}}^{a+b} \frac{q-f}{2b} dq - f\frac{a-b}{2b}$. The only change is that the labeled sellers who would have been inactive without the program are those of $q \in \left[\frac{a+b}{2}, a+b\right]$ instead of $q \in \left[\frac{a+b}{2}, \frac{a+b+f}{2}\right]$.

labeling as the unit certification cost increases and the quality spread grows. Secondly, from the results on the social welfare comparison, the government should adopt the voluntary program when the unit certification cost is large or the quality spread is not so severe. But the mandatory program is best otherwise.

We have adopted several assumptions to simplify the analysis. In particular we have only considered public certification. We can think of two possible scenarios when there is a private certifying agency. One is the case where the private agency replaces public certification. The social welfare in this case will be lower than the social welfare under the voluntary program in our model because the monopolistic private agency will charge a higher fee than the unit certification cost and therefore fewer labels will be provided. The other is the case where private certification is available in addition to the public certification. When the government's program is mandatory, a labeled seller with quality far higher than the government's standard has incentive to apply for the private label in order to signal her quality more accurately and raise the price. Also an unlabeled seller with intermediate quality may apply for the private label if the private certification fee is not too high. The private agency then has to decide the seller group it targets by appropriately setting the fee. The decision of course depends on the government's quality standard. It is an interesting task to characterize the optimal quality standard. We have a similar issue when the government's program is voluntary. Depending on the government's quality standard, sellers opt for the public label with a lower fee or the private label with a higher fee. Another interesting question to ask is which of the two public programs is better when private certification is available. We leave these issues for future research.

# APPENDIX

## 1. Proof of Proposition 2

Suppose first that $Max\left[2(a-b), \frac{a+b}{2}\right] = \frac{a+b}{2}$ or $\frac{3a}{5} < b < a$. Since $\frac{dSW(q_L)}{dq_L} = \frac{f-q_L}{2b}$ and $q_L \geq \frac{a+b}{2}$, $\frac{dSW(q_L)}{dq_L} \leq 0$ and $SW(q_L)$ is thus decreasing in $q_L$ and is maximized at $q_L = \frac{a+b}{2}$ if $f \leq \frac{a+b}{2}$. But, if $\frac{a+b}{2} \leq f < a+b$, $SW(q_L)$ is $\cap$-shaped with respect to $q_L$ and is maximized at $q_L = f$. This completes the proof of (1).

Consider now the case of $Max\left[2(a-b), \frac{a+b}{2}\right] = 2(a-b)$ or $\frac{a}{3} < b \leq \frac{3a}{5}$. In this case, $SW(q_L) = a - f$ if $\frac{a+b}{2} \le q_L \le 2(a-b)$ and, $SW(q_L) = \frac{4(a^2 - ab + b^2) - 4af + 2fq_L - q_L^2}{4b}$ if $2(a-b) \le q_L \le a+b$. Observe that $SW(q_L)$ is continuous, but is not differentiable at $q_L = 2(a-b)$. Since $\frac{dSW(q_L)}{dq_L} = \frac{f - q_L}{2b}$ for $q_L \in [2(a-b), a+b]$, $\frac{dSW(q_L)}{dq_L} < 0$ and thus $SW(q_L)$ is decreasing in $q_L$ over this interval if $f < 2(a-b)$. Therefore $SW(q_L)$ is maximized at any $q_L \in \left[\frac{a+b}{2}, 2(a-b)\right]$, where $SW(q_L)$ is constant at $a - f$. This is the proof of (2-i) of the proposition. If $2(a-b) \le f < a+b$, $\frac{dSW(q_L)}{dq_L} = \frac{f - q_L}{2b} = 0$ at $q_L = f$. Therefore, $SW(q_L)$ is constant if $q_L \in \left[\frac{a+b}{2}, 2(a-b)\right]$, and $\cap$-shaped thereafter with the peak achieved at $q_L = f$. Q.E.D.

## 2. Proof of Proposition 3

Using $SW(q_L^*)$ specified in Proposition 2,

(1-i) when $\frac{3a}{5} < b < a$ and $f \le \frac{a+b}{2}$, we have

$$SW(q_L^*) - SW^o = \frac{3(5a^2 - 6ab + 5b^2) - 4f(3a-b)}{16b} - \frac{3(a-b)^2}{4b} = \frac{3(a+b)^2 - 4f(3a-b)}{16b}$$

$$\ge \frac{3(a+b)^2 - 4\left(\frac{a+b}{2}\right)(3a-b)}{16b} = \frac{(a+b)(5b-3a)}{16b} > 0.$$

The first inequality of the above expression comes from the supposition of $f \le \frac{a+b}{2}$, and the second inequality also from the supposition of $\frac{3a}{5} < b$.

(1-ii) When $\frac{3a}{5} < b < a$ and $\frac{a+b}{2} \le f < a+b$, we have

$$SW(q_L^*) - SW^o = \frac{4(a^2 - ab + b^2) - 4af + f^2}{4b} - \frac{3(a-b)^2}{4b} = \frac{(a+b)^2 - 4af + f^2}{4b}.$$

It is easy to check that $SW(q_L^*) - SW^o > 0$ if and only if $< 2a - \sqrt{(a-b)(3a+b)}$. It is also straightforward to verify that $\frac{a+b}{2} < 2a - \sqrt{(a-b)(3a+b)} < a+b$ under our supposition of $\frac{3a}{5} < b < a$.

(2-i) If $\frac{a}{3} < b \le \frac{3a}{5}$ and $f \le 2(a-b)$,

$$SW(q_L^*) - SW^o = a - f - \frac{3(a-b)^2}{4b} = \frac{(3a-b)(3b-a)}{4b} - f .$$

Therefore $SW(q_L^*) - SW^o > 0$ if and only if $< \frac{(3a-b)(3b-a)}{4b}$. Observe also that $\frac{(3a-b)(3b-a)}{4b} \le 2(a-b)$ under the supposition of $\le \frac{3a}{5}$.

Lastly, (2-ii) when $\frac{a}{3} < b \le \frac{3a}{5}$ and $2(a-b) < f < a+b$, we have

$$SW(q_L^*) - SW^o = \frac{4(a^2 - ab + b^2) - 4af + f^2}{4b} - \frac{3(a-b)^2}{4b} = \frac{(a+b)^2 - 4af + f^2}{4b}.$$

Observe that $(a+b)^2 - 4af + f^2$, the numerator of the right-hand side of the last equality, has a negative value if $f \in [2(a-b), a+b]$ under our supposition of $b \le \frac{3a}{5}$. Therefore, we have $SW(q_L^*) < SW^o$ in this case. This completes the proof. Q.E.D.

### 3. Proof of Lemma 2

Here we will only show that the sellers of qualities above $q_L$ will earn more profits by applying for labels than without labels. Recall that with the supposition of $q_L \ge \frac{a+b+f}{2}$, all the sellers whose qualities are above $q_L$ earn positive profits with labels. Notice also that unlabeled sellers' profits are the same as under the previous program, because they do not pay the application fee. As specified in Lemma 1, unlabeled seller $q$'s profit is $\pi^U(q) = 6(a-b) - 3q$ if $q_L > Max\left[2(a-b), \frac{a+b+f}{2}\right]$, while it is $\pi^U(q) = 2(a-b+q_L) - 3q$ if $\frac{a+b+f}{2} \le q_L \le 2(a-b)$. In the former case, there are inactive sellers and $q = 2(a-b)$ is the highest quality among those provided by unlabeled active sellers. Observe that the seller $q = 2(a-b)$'s profit is 0. Therefore, a seller of quality above $q_L$, if she does not apply for label, would get a negative profit of $6(a-b) - 3q$, whereas she earns a positive profit with label. Thus she is better off by applying for label. Consider the latter case of $\frac{a+b+f}{2} \le q_L \le 2(a-b)$. In order for this case to be valid, notice first that we should have $\frac{a+b+f}{2} \le 2(a-b)$ or $f \le 3a - 5b$. And with our assumption of $\frac{a}{3} < b < a$, we have $f \le 3a - 5b < 9b - 5b$ $= 4b$. In other words, we have to assume that $f < 4b$. Now check the incentive problem of a seller whose quality is above $q_L$. She will get a profit of $\pi^L(q) = 2(a+b+q_L) - 3q - f$ if she applies for label, while her profit is $\pi^U(q) = 2(a-b+q_L) - 3q$ if she does not apply. The difference of the profits is $\pi^L(q) - \pi^U(q) = 4b - f$, which is positive given the assumption of $f < 4b$. Therefore she will be better off by applying for label. This completes the proof. Q.E.D.

#### 4. Proof of Proposition 4

Since $SW(q_L) = \frac{2ab - f(a+b) + fq_L}{2b}$ when $\frac{a+b+f}{2} \le q_L \le 2(a-b)$, $SW(q_L)$ is increasing in $q_L \in \left[\frac{a+b+f}{2}, 2(a-b)\right]$. When $q_L \ge Max\left[2(a-b), \frac{a+b+f}{2}\right]$, $\frac{dSW(q_L)}{dq_L} = \frac{f - q_L}{2b}$. Notice that $f < Max\left[2(a-b), \frac{a+b+f}{2}\right]$ by the assumption of $f < a+b$. Therefore $\frac{dSW(q_L)}{dq_L} = \frac{f - q_L}{2b} < 0$ and $SW(q_L)$ is decreasing in $q_L$ if $q_L \ge Max\left[2(a-b), \frac{a+b+f}{2}\right]$. Hence $SW(q_L)$ is maximized at

$q_L = Max\left[2(a-b), \frac{a+b+f}{2}\right]$. Q.E.D.

## 5. Proof of Proposition 5

Observe from Proposition 2 and 4 that both $SW(q_L^*)$ and $SW(q_L^{**})$ are functions of $f$. Let us denote $\Delta(f) \equiv SW(q_L^*) - SW(q_L^{**})$. Consider the first case of $\frac{a}{3} < b < \frac{3a}{5}$. There are three subcases, depending whether $f \le 3a - 5b$, $3a - 5b < f \le 2(a-b)$, or $2(a-b) < f < a+b$.

(Case 1-i) If $\frac{a}{3} < b < \frac{3a}{5}$ and $f \le 3a - 5b$, we have

$$\Delta(f) = a - f - \frac{2ab - f(3b-a)}{2b} = \frac{f(a-b)}{2b} < 0.$$

(Case 1-ii) If $\frac{a}{3} < b < \frac{3a}{5}$ and $3a - 5b < f \le 2(a-b)$, we have

$$\begin{aligned}\Delta(f) &= a - f - \frac{3[5a^2 - 6ab + 5b^2 - 2f(a+b) + f^2]}{16b} \\ &= \frac{-3f^2 + 2f(3a-5b) - (3a-5b)(3a-3b)}{16b},\end{aligned}$$

and $\Delta'(f) = \frac{3a - 5b - f}{8b} < 0$. Therefore $\Delta(f)$ is decreasing in $f \in [3a-5b, 2(a-b)]$ and thus $\Delta(f) < \Delta(3a-5b) = -\frac{(a-b)(3a-5b)}{2} < 0$.

(Case 1-iii) If $\frac{a}{3} < b < \frac{3a}{5}$ and $2(a-b) < f < a+b$,

$$\begin{aligned}\Delta(f) &= \frac{4(a^2 - ab + b^2) - 4af + f^2}{4b} - \frac{3[5a^2 - 6ab + 5b^2 - 2f(a+b) + f^2]}{16b} \\ &= \frac{f^2 - 2f(5a-3b) + (a+b)^2}{16b},\end{aligned}$$

and $\Delta'(f) = \frac{f - (5a-3b)}{8b} < \frac{a + b - (5a-3b)}{8b} = -\frac{(a-b)}{2b} < 0$. Therefore $\Delta(f)$ is decreasing in $f \in [2(a-b), a+b]$ and thus we have $\Delta(f) < \Delta(2(a-b)) = -\frac{(3a-b)(5a-7b)}{16b} < 0$. The last inequality comes from the assumption of $\frac{a}{3} < b < a$.

This completes the proof of (1) of the proposition.

Consider the second case of $\frac{3a}{5} \le b < a$. In this case, $SW(q_L^{**}) = \frac{3[5a^2 - 6ab + 5b^2 - 2f(a+b) + f^2]}{16b}$, but $SW(q_L^{*})$ has two different forms of $\frac{3(5a^2 - 6ab + 5b^2) - 4f(3a-b)}{16b}$ or $\frac{4(a^2 - ab + b^2) - 4af + f^2}{4b}$, depending on whether $f < \frac{a+b}{2}$ or $\frac{a+b}{2} \le f < a+b$.

(Case 2-i) If $\frac{3a}{5} \le b < a$ and $f \le \frac{a+b}{2}$, we have $\Delta(f) = f\frac{2(5b-3a) - 3f}{16b}$. Therefore when $\frac{2(5b-3a)}{3} < \frac{a+b}{2}$ or $b < \frac{15a}{17}$, $\Delta(f) > 0$ if $f < \frac{2(5b-3a)}{3}$ and $\Delta(f) < 0$ if $\frac{2(5b-3a)}{3} < f < \frac{a+b}{2}$. However, when $\frac{a+b}{2} \le \frac{2(5b-3a)}{3}$ or $\frac{15a}{17} \le b < a$, $\Delta(f) > 0$ for all $f \in \left[0, \frac{a+b}{2}\right]$.

(Case 2-ii) Lastly, if $\frac{3a}{5} \le b < a$ and $\frac{a+b}{2} \le f < a+b$, we have $\Delta(f) = \frac{f^2 - 2f(5a-3b) + (a+b)^2}{16b}$ and $\Delta'(f) = \frac{f - (5a-3b)}{8b} < \frac{a+b-(5a-3b)}{8b} = -\frac{(a-b)}{2b} < 0$. Therefore $\Delta(f)$ is decreasing in $f \in \left[\frac{a+b}{2}, a+b\right]$ and is maximized at $f = \frac{a+b}{2}$. And we have $\Delta(f) < \Delta\left(\frac{a+b}{2}\right) = -\frac{(15a-17b)(a+b)}{64b}$. Thus $\Delta(f) < 0$ for all $f \in \left(\frac{a+b}{2}, a+b\right)$ when $b < \frac{15a}{17}$. But when $\frac{15a}{17} \le b < a$, $\Delta(f) > 0$ if $\frac{a+b}{2} \le f < 5a - 3b - 2\sqrt{2(a-b)(3a-b)}$ and $\Delta(f) < 0$ if $5a - 3b - 2\sqrt{2(a-b)(3a-b)} < f < a+b$. Combining (Case 2-i) and (Case 2-ii), we have (2) and (3) of the proposition.

**Q.E.D.**

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*Received November 15, 2014, Revised December 1, 2014, Accepted December 12, 2014.*