A SIMPLE METHOD FOR MEASURING SYSTEMIC RISK USING CREDIT DEFAULT SWAP MARKET DATA^{*}

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This paper proposes a simple method that employs credit default swap (CDS) data for analyzing systemic risk. The proposed method overcomes inconsistency problems in existing methods and can produce various indicators of systemic risk in a consistent manner. In addition, this method can measure systemic risk contributions. In particular, the method measures systemic risk contributions in both directions, that is, the overall effect of systemic risk on individual credit risks and vice versa. Using CDS data, we employ the proposed method to measure systemic risk for a group of large financial institutions in the U.S. In addition, we provide empirical results for systemic risk contributions as well as various measures of the overall level of systemic risk and verify the applicability of the proposed method.

Keywords: Systemic Risk, Financial Stability, Systemic Risk Contribution, Credit Default Swap *JEL classification*: C15, E53, G21

1. INTRODUCTION

Severe financial instability can directly and indirectly entail high costs for the economy (see, for example, Hoggarth *et al.*, 2002). To maintain financial stability, financial regulators use various policy tools not only to prevent individual financial institutions from defaulting but also to control the riskiness of the financial system as a

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whole (i.e., systemic risk). This system-wide macroprudential perspective has become widely accepted through the 2007-2009 global financial crisis.¹

Policy efforts to maintain financial stability first require accurate and timely information on systemic risk. However, measuring systemic risk is not a simple task. Employing informative data is indispensable for measuring systemic risk. Historical credit event data are typically provided with a considerable time lag. In addition, defaults by financial institutions are relatively rare events, and thus, it may be difficult to predict such events by using historical data. By contrast, equity return data can convey market participants' expectations of financial institutions in a timely manner. Because of their availability and informativeness, equity return data have been widely used to measure systemic risk.² By employing structural models, we can infer the default probability from equity prices. However, such models require restrictive assumptions. For example, Merton's (1974) model regards a firm's equity as a call option written on its unobservable asset with some predetermined maturity date and debt (plus interest payments) amount. Because financial institutions are continuously intermediating depositors and borrowers, it may be difficult to determine a precise maturity date, and moreover, the amount of debt at maturity should change as a result of the intermediation.

Data on credit default swaps (CDSs) may be used for measuring systemic risk. Under a CDS contract, the protection buyer pays periodic premiums to the protection seller and can be compensated for financial losses from some designated credit event. Such data on CDS premiums may provide high-quality information on credit risk. In particular, the time horizon for credit risk can be considered the CDS contract's maturity date. In addition, market participants determine CDS premiums based mainly on their expectations of well-defined credit events. Indeed, the default probability can be accurately inferred from CDS premiums. CDS products are relative new, but recent years have witnessed the rapid growth of the CDS market. As a result, data on CDS premiums are available for an increasing number of financial institutions.

Despite these desirable properties of CDS data, few studies have employed them to measure systemic risk. For example, Huang, Zhou, and Zhu (2009, 2010, 2011) inferred default probabilities from CDS premiums and then combined them with equity return correlations to simulate credit events. They measured systemic risk by the price of insurance against defaults for a hypothetical asset portfolio in the financial system. Their method is simple and easy to implement, but it has several drawbacks. First, the default probability inferred from CDS premiums is not the actual probability but the risk-neutral

¹ For a macroprudential perspective of financial supervision, see, for example, Crocket (2000), Borio (2003), Acharya (2009), Brunnermeier *et al.* (2009), the Financial Stability Forum (2009a, b), and the BCBS (2009).

² Examples include Adrian and Brunnermeier (2010), Acharya *et al.* (2010), Brownlees and Engle (2012), and Suh (2012), among others. More broadly, Bisias *et al.* (2012) surveyed various methods of systemic risk measurements.

probability, which reflects not only the actual probability but also the risk premium component. These risk-neutral probabilities are combined with equity return correlations estimated under the physical measure. Because their measure of systemic risk is defined as the insurance premium calculated under the risk-neutral measure, methodological consistency requires the use of risk-neutral correlations, not physical correlations. As Driessen et al. (2009) and Tarashev and Zhu (2008) pointed out, investors require correlation-risk premiums because of uncertainty over correlations, implying that physical correlations, not risk-neutral correlations, yield erroneous results. Moreover, because correlations often exhibit a time-varying feature, the correlation-risk premium is also time-varying. Therefore, combining physical correlations with risk-neutral default probabilities may result not only in an inconsistency problem but also in erroneous measurements of systemic risk. Second, previous studies have proposed several alternative measures of systemic risk for various purposes. Ideally, a good method is expected to appropriately produce all relevant measures. However, the method proposed by Huang, Zhou, and Zhu (2009, 2010, 2011) has been applied only to cases of risk-neutral probabilities. For example, we may be interested in the physical probability of multiple financial institutions defaulting simultaneously within a certain time horizon. However, their method cannot address this physical probability question. Third, a simulation for systemic risk measurements requires unobservable asset return correlations, instead of which equity return correlations are estimated from stock market data and then used as proxies. Noteworthy is that another inconsistency problem may arise from this substitution.

In this paper, we propose a simple method that employs CDS data for analyzing systemic risk. This method aims to overcome several drawbacks inherent in existing methods. Specifically, we adopt a model proposed by Collin-Dufresne and Goldstein (2001) for individual financial institutions. This model is a structural model, thus it enables us to analyze credit risk events in a correct way. Then, we utilize a method to combine individual credit risk events to measure system-wide credit risks. This method is flexible enough to capture possible time-varying correlations among individual financial institutions. Our method also allows for a direct estimation of asset return correlations, and thus, it is free from inconsistency problems that may arise from the use of equity return correlations. In addition to its theoretical advantages, the proposed method can also produce various indicators of systemic risk under the physical probability measure. Not only systemic risk measures but also systemic risk contributions are defined mainly under the physical probability measure. The proposed method allows for analyses of such systemic risk contributions.

This study employs the proposed method to measure systemic risk for a group of 22 large financial institutions in the U.S. for the period from March 2006 to August 2010. The main empirical results are summarized as follows. First, there were considerable variations in asset correlations over time. This time-varying feature implies that investors may require correlation-risk premiums and that such premiums can be time-varying. Thus, disregarding correlation-risk premiums when measuring risk-neutral

systemic risk may produce misleading results. Second, the level of systemic risk measured under subjective probability measure was higher than that of systemic risk measured under objective probability measure. In addition, the difference between the level of systemic risk under two different probability measures changed over time, widening during the 2007-2009 global financial crisis. This indicates that the heightened credit and liquidity concerns during the crisis increased the subjective probability of defaults more than the objective probability of defaults. Third, the substitution of equity correlations for asset correlations had little effect on the measurement of systemic risk. Fourth, systemic risk contributions also exhibited a time-varying feature. In particular, systemic risk contributions defined as the extent to which a default by a particular institution influences systemic risk tended to increase during the crisis period compared to the pre-crisis period. Finally, systemic risk contributions defined as the extent to which a systemic risk event affects the level of individual default risk were closely related to the realized risk represented by equity returns during the crisis period. In addition, systemic risk contributions also had the ex-ante ability to predict future risks, although not very precisely.

The rest of this paper is organized as follows: Section 2 explains the methodology, and Section 3 describes the data. Section 4 presents the empirical results for asset correlations and measurements of systemic risk. Sections 5 employs the proposed method to analyze systemic risk contributions, and Section 6 concludes.

2. METHODOLOGY

2.1. Model

2.1.1. Asset Dynamics and the Default Probability

We consider a continuous-time economy with N financial institutions indexed by j(=1,...,N). We model the value of the *j* th financial institution's asset, $V_{j,t}$, to follow the geometric Brownian motion under the physical measure **P**, that is,

$$\frac{dV_{j,t}}{V_{j,t}} = \mu_j dt + \sigma_j dW_{j,t}, \qquad (1)$$

where $dW_{j,t}$ denotes the increment of the standard Wiener process, μ_j denotes the expected drift, and σ_i denotes the volatility.

We assume that there is a default when the asset value falls below a particular threshold. Merton (1974), Black and Cox (1976), and Longstaff and Schwartz (1995) exogenously specified this threshold while Collin-Dufresne and Goldstein (2001)

modeled it to change dynamically over time. Following Collin-Dufresne and Goldstein (2001), we specify the dynamics of the default threshold $K_{j,t}$ as

$$d \log K_{j,t} = \lambda_j (\log V_{j,t} - v_j - \log K_{j,t}) dt .$$
(2)

This specification parsimoniously captures the notion that more debt tends to be issued when the leverage ratio is less than some target and vice versa.

We define the log leverage $l_{j,t} \equiv \log K_{j,t} - \log V_{j,t}$. Then, a default is formally defined as the event that $l_{j,t}$ is positive. Applying Ito's lemma, we can derive the dynamics of the log leverage as follows:

$$dl_{j,t} = \lambda_j (\bar{l}_j - l_{j,t}) dt - \sigma_j dW_{j,t}, \qquad (3)$$

$$\bar{l}_j \equiv \frac{-\mu_j + 0.5\sigma_j^2}{\lambda_j} - \nu_j \,. \tag{4}$$

Now, under the risk-neutral measure Q, we have the following asset value dynamics:

$$\frac{dV_{j,t}}{V_{j,t}} = rdt + \sigma_j d\hat{W}_{j,t}, \qquad (5)$$

where r denotes the constant riskless interest rate. The log leverage dynamics are:

$$dl_{j,t} = \lambda_j (\bar{l}_j^{\ Q} - l_{j,t}) dt - \sigma_j d\hat{W}_{j,t}, \qquad (6)$$

$$\bar{l}_j^{\ Q} \equiv \frac{-r_j + 0.5\sigma_j^2}{\lambda_j} - v_j \,. \tag{7}$$

With the above risk-neutral dynamics, Collin-Dufresne and Goldstein (2001) derived the risk-neutral probability $Q(l_t, T)$ that a default occurs before time *T*, given that the leverage ratio is l_t at time *t*:

$$Q(l_t,T) = \sum_{i=1}^n q_i , \qquad (8)$$

$$q_1 = \frac{\Phi(a_1)}{\Phi(b_{1/2})},$$
(9)

$$q_{i} = \frac{1}{\Phi(b_{1/2})} \left[\Phi(a_{i}) - \sum_{j=1}^{i-1} q_{j} \Phi(b_{i-j+1/2}) \right], \quad i = 2, ..., n ,$$
(10)

$$a_i = \frac{M(i\Delta)}{S(i\Delta)},\tag{11}$$

$$b_i = \frac{L(i\Delta)}{S(i\Delta)},\tag{12}$$

$$M(\tau) = l_t e^{-\lambda \tau} + \bar{l}^{\mathcal{Q}} \left(1 - e^{-\lambda \tau} \right), \tag{13}$$

$$L(\tau) = \bar{l}^{Q} \left(1 - e^{-\lambda \tau} \right), \tag{14}$$

$$S(\tau)^2 = \frac{\sigma^2}{2\lambda} \left(1 - e^{-2\lambda\tau} \right),\tag{15}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function and $\Delta \equiv (T-t)/n$.

2.1.2. Risk-neutral Default Probability Implied from CDS Premiums

In a typical single-name CDS contract, the protection buyer pays periodic premiums whose fixed amount is determined by the CDS premiums to the protection seller until the contract matures or there is a default, whichever comes first. If there is a default before the maturity date, then the protection seller compensates the protection buyer for the loss given default (LGD).

Following the framework of Duffie (1999) and applying the no-arbitrage condition, we require that the present value of CDS premium payments be equal to the present value of protection payments, that is,

$$s_{j,t} \int_{t}^{t+T} e^{-r\tau} \left(1 - \int_{0}^{\tau} \psi_{j,u} du \right) d\tau = LGD_{j,t} \int_{t}^{t+T} e^{-r\tau} \psi_{j,\tau} d\tau , \qquad (16)$$

where $\psi_{j,\tau}$ denotes the annualized unconditional risk-neutral default intensity of *j* th financial institution. Under the standard simplifying assumptions that $\psi_{j,\tau}$ does not change between *t* and *t*+*T*, we can derive the one-year risk-neutral default probability:

$$\psi_{j,\tau} = \frac{s_{j,t} \int_{t}^{t+T} e^{-r\tau} d\tau}{LGD_{j,t} \int_{t}^{t+T} e^{-r\tau} d\tau + s_{j,t} \int_{t}^{t+T} \tau e^{-r\tau} d\tau}.$$
(17)

2.2. Estimation

For each individual financial institution (j = 1,...,N), given the CDS premium $s_{j,t}$, and assumed parameter values σ_j, λ_j, v_j (and a one-year horizon), we can determine the log leverage ratio $l_{j,t}$ from Eqs. (8)-(15). Suppose that data observations are discretely sampled with a fixed time interval Δ . Then, under the **P** measure, we have

$$l_{t+\Delta|t} \sim N \Biggl[l_t e^{-\lambda \Delta} + \bar{l} (1 - e^{-\lambda \Delta}), \ \frac{\sigma^2}{2\lambda} e^{-\lambda \Delta} (e^{2\lambda \Delta} - 1) \Biggr], \tag{18}$$

where the index for the financial institution is suppressed for simplicity. Given the estimated data $\{\hat{l}_1,...,\hat{l}_m\}$ with the time interval Δ and applying the results in Duan (1994, 2000), we derive the following log-likelihood function:

$$\log L = -\frac{m-1}{2}\log(2\pi) - \frac{m-1}{2}\log\left[\frac{\sigma^2}{2\lambda}e^{-\lambda\Delta}\left(e^{2\lambda\Delta} - 1\right)\right] - \frac{1}{2}\sum_{i=2}^{m}\frac{\left[l_i - l_{i-1}e^{-\lambda\Delta} - \bar{l}\left(1 - e^{-\lambda\Delta}\right)\right]^2}{\frac{\sigma^2}{2\lambda}e^{-\lambda\Delta}\left(e^{2\lambda\Delta} - 1\right)} - \sum_{i=2}^{m}\log\frac{\partial Q(l_i, T)}{\partial l_i}.$$
(19)

Here,

$$\frac{\partial Q(l,T)}{\partial l} = \sum_{i=1}^{n} \frac{\partial q_i}{\partial l},$$
(20)

$$\frac{\partial q_1}{\partial l} = \frac{\varphi(a_1)}{\Phi(b_{1/2})} \frac{e^{-\lambda\Delta}}{S(\Delta)},\tag{21}$$

$$\frac{\partial q_i}{\partial l} = \frac{1}{\Phi(b_{1/2})} \left[\varphi(a_i) \frac{e^{-\lambda \Delta}}{S(i\Delta)} - \sum_{j=1}^{i-1} \frac{\partial q_j}{\partial l} \Phi(b_{i-j+1/2}) \right], \quad i = 2, \dots, n ,$$
(22)

where $\varphi(\cdot)$ denotes the standard normal probability density function. Using the above log-likelihood function and the maximum likelihood method, we estimate the parameters $\Theta_j = \{\mu_j, \sigma_j, \lambda_j, \nu_j\}$ for one institution at a time.

2.3. Dynamics of Correlations

Not only constituent institutions' credit risk levels but also their correlations are important factors in the determination of systemic risk. We want to model that the de-meaned log leverage terms are flexibly correlated with each other. Define $w_{j,t}$ as the disturbance term of the log leverage, which is formally specified as follows:

$$l_{j,t} = l_{j,t-\Delta} e^{-\lambda_j \Delta} + \bar{l}_j \left(1 - e^{-\lambda_j \Delta} \right) + w_{j,t} , \qquad (23)$$

$$w_{j,t} \sim N \Biggl[0, \frac{\sigma^2}{2\lambda_j} e^{-\lambda_j \Delta} \Biggl(e^{2\lambda_j \Delta} - 1 \Biggr) \Biggr].$$
⁽²⁴⁾

In particular, we assume that the vector consisting of the disturbance terms $w_t \equiv [w_{1,t}, ..., w_{N,t}]'$ follows a multivariate normal distribution with a time-varying covariance matrix, that is,

$$w_t \sim MVN[0, \Delta \cdot \Sigma_t]. \tag{25}$$

Then we employ a diagonal-vech model for the dynamics of Σ_t to allow for flexible correlation. Specifically, the covariance $\sigma_{jk,t}$ at time t between $w_{j,t}$ and $w_{k,t}$ is determined by

$$\sigma_{jk,t} = c_{j,k} + a_{jk} w_{j,t-1} w_{k,t-1} + b_{jk} \sigma_{jk,t-1} , \qquad (26)$$

where $\sigma_{jk,t}$ is the (j,k) element of Σ_t . Under this specification, the conditional correlation between the log leverages for institutions j and k is also time-varying, that is,

$$\rho_{jk,t} = \frac{\sigma_{jk,t}}{\sigma_{j,t}\sigma_{k,t}}.$$
(27)

This can be regarded as a generalized specification in comparison to a simple exponentially weighted moving average method, as in, for example, Lehar (2005).

For the estimation of the time-varying covariance matrix Σ_t , we first use the estimates $\hat{\Theta}_j$ for institution j to construct the log leverage time series $\{\hat{i}_{j,t}\}$ and then obtain the residuals $\hat{w}_{j,t}$ from (23). We adopt the estimation algorithm proposed by Ledoit *et al.* (2003), which produces a positive semidefinite conditional covariance matrix.

2.4. Measures of Systemic Risk

A systemic risk can be regarded as an event in which a substantial portion of financial institutions default simultaneously. Several measures of systemic risk have been specified. Instead of inventing new measures, we adopt existing measures and show that these measures can be successfully calculated with CDS data.

In particular, our measures of systemic risk include (i) the index based on the number of defaults (the ND index), which is defined as the probability that the ratio of the number of defaulting financial institutions to the total number of financial institutions exceeds a prescribed threshold, ζ ; (ii) the index based on the weighted assets (the WA index), which is defined as the probability that the ratio of assets of defaulting financial institutions to total assets of all financial institutions exceeds a prescribed threshold, ζ ; and (iii) the index based on the conditional expected losses (the CEL index), which is defined as the expected loss ratio for a hypothetical portfolio conditional on the event that the loss ratio exceeds a prescribed threshold, ζ . The hypothetical portfolio is constructed with weights proportional to total assets of financial institutions in the system. Lehar (2005) calculated the ND index, the WA index, and a measure of expected short fall with data on stock prices. The CEL index and the expected short fall index are based on the same concept. Huang, Zhou, and Zhu (2009, 2010, 2011) employed CDS data but provided only one measure of systemic risk similar to the CEL index but calculated under the risk-neutral probability measure.

By using these three measures, which allow for clear economic interpretations, we illustrate the temporal trend in the overall level of systemic risk. Obviously, these three measures are not exhaustive, and other measures may be devised according to the objectives.³ Because the proposed model specifies the joint log-leverage dynamics of individual institutions, it permits a wider range of analyses of systemic risk.

³ See, for example, Chan-Lau and Gravelle (2005), Avesani, Pascual, and Li (2006), Cont (2010), and

2.5. Simulation

We set one year as the time horizon for measuring systemic risk. One year is the shortest maturity in CDS data. Longer-maturity data are available, but longer horizons are less relevant from the perspective of financial regulators. We employ Monte Carlo simulations to calculate the measures of systemic risk because no analytical solutions are available for these measures over a multi-period time horizon. For simulating correlated disturbances, we draw multivariate normal random variates as specified by (25). We substitute these simulated disturbances into Eq. (23) to simulate hypothetical log leverage paths and perform 500,000 repetitions by using the variance reduction technique.

3. DATA

We obtained data on CDS premiums from Bloomberg. The sample included 22 financial institutions in the U.S. We obtained the corresponding total assets as well as equity prices from the Compustat and CRSP databases. The data on CDS premiums and equity prices were obtained on a weekly basis from January 2004 to August 2010, and the data on total assets and liabilities (book values) were obtained on a quarterly basis and then linearly interpolated. To control for survivorship bias, we included inactive financial institutions in the sample.

Figure 1 shows the CDS premiums for the 22 institutions over the sample period. The premiums remained at low levels until 2007. However, they increased sharply during the 2007-2009 financial crisis and declined afterward, although not to pre-crisis levels until August 2010 (the end of the sample period). In addition, the gaps between individual institutions widened and fluctuated wildly during the crisis period.

Table 1 provides the summary statistics for the data. The sample included 8 banks, 11 insurance firms, and 3 other financial institutions. The banks were much larger than the insurance firms: The average total assets of the banks were USD 1,034.9 billion, whereas those of the insurance firms were USD 157.0 billion. CDS premiums increased sharply from 28.7 basis points (between January 2004 and July 2007) to 373.6 during the crisis period (between August 2007 and December 2008) and then declined to 249.5 (between January 2009 and August 2010). The banks showed a lower level of credit risk than the insurance firms: Average CDS premiums for the banks increased from 14.7 basis points to 215.6 and then declined to 131.2, whereas those for the insurance firms increased from 45.2 basis points to 466.6 and then declined to 324.0.

Billio, Getmansky, Lo, and Pelizzon (2010).



Figure 1. CDS Premiums

	Table 1.Summary	Statistics	of Sample D	ata			
FI.	Sector Total CDS Premiums						
No		Asset	Period 1	Period 2	Period 3		
1	Insurance	139.3	6.4	138.0	70.0		
2	Diversified Financial Services	140.4	6.2	317.2	120.6		
3	Banks	1649.9	8.1	103.7	129.9		
4	Banks	170.6	59.1	435.6	125.3		
5	Insurance	50.3	n.a.	52.7	35.2		
6	Healthcare Services	47.8	14.0	89.1	75.5		
7	Banks	1814.5	7.0	208.5	207.4		
8	Insurance	55.3	53.2	307.2	217.0		
9	Banks	823.8	11.7	218.8	125.1		
10	Insurance	311.3	n.a.	465.6	234.4		
11	Banks	1570.3	9.0	71.7	64.1		
12	Insurance	15.5	n.a.	45.3	56.4		
13	Insurance	502.6	9.0	358.8	214.7		
14	Insurance	9.2	n.a.	1436.9	881.7		
15	Banks	892.8	11.5	414.3	161.2		
16	Insurance	5.0	132.9	1484.6	1363.8		
17	Insurance	475.4	n.a.	607.3	209.2		
18	Diversified Financial Services	158.1	94.7	954.3	680.6		
19	Insurance	110.2	n.a.	60.2	57.4		
20	Insurance	52.9	24.6	176.4	223.7		
21	Banks	617.2	5.9	183.3	147.9		
22	Banks	740.0	5.2	88.8	88.6		
Mean		470.6	28 7	373 6	249.5		

Notes: The sample period was from January 2004 to August 2010. Period 1 was from January 2004 to July 2007; Period 2 was from August 2007 to December 2008; and Period 3 was from January 2009 to August 2010. Total asset indicates the average total asset in billion USD over the whole sample period, and CDS premiums indicate the average weekly premiums (in basis points) for CDS contracts with one year to maturity for each period.

4. EMPIRICAL RESULTS

4.1. Correlations Between Asset Returns

Not only individual default probabilities but also asset correlations are important factors in the determination of systemic risk. In particular, if asset correlations are time-varying, then the asset correlation channel for systemic risk is also time-varying.



Figure 2. Correlations between the Log Leverage of Individual Institutions

Figure 2 shows the 25th, 50th (median), and 75th percentile correlations over all possible bilateral correlation coefficients between log leverages for individual institutions. We obtained the correlation coefficient between log leverages for institutions j and k according to (27).

Most bilateral pairs exhibited positive correlations over the sample period. This implies the importance of properly taking correlations into account in measuring systemic risk. The median correlations were positive but moderate, fluctuating between 0.2 and 0.5 (mean=0.36), whereas the 75th percentile correlations fluctuated between 0.3 and 0.6 (mean=0.48).

This significant time-varying feature implies that investors demand correlation-risk premiums for taking a risk from the uncertainty over correlations and that such risk premiums can be substantial. Insurance premiums for a portfolio, as suggested by Huang, Zhou, and Zhu (2009, 2010, 2011), do not include these correlation-risk premiums, and thus, this indicator of systemic risk is subject to an underestimation problem.

4.2. Systemic Risk Measurements

We now present the results for the three measures of systemic risk: the ND index, the WA index, and the CEL index. The ND and WA indices are presented according to two thresholds for systemic risk -10% and 20%- whereas the CEL index is presented only for 10%. We restricted the sample period from March 2006, after which the total number of financial institutions in the sample exceeded 10. Because the total number of institutions in the sample varied over time, the ND index with $\zeta = 10\%$ corresponded to the event in which the number of defaulting institutions exceeded 1 until May 2008 but exceeded 2 afterward.



Notes: Here systemic risk was defined as the probability that the ratio of the number of defaulting financial institutions to the total number of financial institutions would exceed a prescribed threshold, that is, 10% or 20%. Systemic risk was measured under physical and risk-neutral probability measures, and the difference was compared with the credit risk (defined as the difference between Baa-rated and Aaa-rated yields) and liquidity risk (defined as the difference between three-month LIBOR and the overnight indexed swap rate) indicators. Both the credit-risk and the liquidity-risk indicators are measured in 25% points.

Figure 3. Systemic Risk Index Based on the Number of Defaults (The ND Index)

Figure 3 shows the measure of systemic risk based on the number of distressed financial institutions (the ND index). The temporal trend indicates that the level of systemic risk increased sharply during the global financial crisis and declined sharply afterward. As shown in Panel A, the ND index with a low threshold (10%) was higher

than that with a high threshold (20%). Panels B and C provide a comparison of these two ND indices under both the risk-neutral and physical measures. The risk-neutral ND index was higher than the physical ND index for both thresholds. This implies that risk-averse investors require premiums for the risk they take and that such risk premiums are included in CDS premiums. CDS premiums are determined based on risk-neutral default probabilities, and thus, these probabilities exceed physical default probabilities for individual institutions. This discrepancy between these two types of default probabilities for individual institutions induces a positive gap between risk-neutral and physical measures of systemic risk. Noteworthy is that the gap between the two ND indices under both probability measures changed over time. In particular, it increased sharply during the crisis period. Indeed, the gap between the two ND indices was related to risk premiums. As shown in Panel D, the gap was closely related to an indicator of credit risk (the credit yield spread between Baa-rated and Aaa-rated corporate bonds) and an indicator of liquidity risk (the spread between three-month LIBOR and the overnight index swap.)



Notes: Here systemic risk was defined as the probability that the ratio of the number of defaulting financial institutions to the total number of financial institutions would exceed a prescribed threshold, that is, 10% or 20%. Systemic risk was measured under physical and risk-neutral probability measures, and the difference was compared with the credit risk (defined as the difference between Baa-rated and Aaa-rated yields) and liquidity risk (defined as the difference between three-month LIBOR and the overnight indexed swap rate) indicators. Both the credit-risk and the liquidity-risk indicators are measured in 25% points.

Figure 4. Systemic Risk Index Based on Weighted Assets (The WA Index)

A SIMPLE METHOD FOR MEASURING SYSTEMIC RISK

Figure 4 shows the measure of systemic risk based on weighted assets of defaulting financial institutions (the WA index). The WA index increased sharply during the same periods as the ND index. The WA index indicated a lower level of risk than the ND index.⁴ This may be explained by the size effect, that is, the banks in the sample, which were larger than the insurance firms, reflected a lower level of risk than the insurance firms. The other results for the WA index are generally consistent with those for the ND index.



Notes: Here systemic risk was defined as the amount of expected losses conditional on the systemic risk event that the ratio of assets of defaulting financial institutions to total assets in the system would exceed a prescribed threshold (10%). Systemic risk was measured under physical and risk-neutral probability measures, and the difference was compared with the credit risk (defined as the difference between Baa-rated and Aaa-rated yields) and liquidity risk (defined as the difference between three-month LIBOR and the overnight indexed swap rate). Both the credit-risk and the liquidity-risk indicators are measured in 100% points.

Figure 5. Systemic Risk Index Based on Conditional Expected Losses (The CEL Index)

⁴ The ND and WA indices showed similar levels of systemic risk for a given threshold. For the ND index, the systemic risk event corresponded to simultaneous defaults by 2 (3) or more institutions when the number of institutions in the sample increased from 11 to 20 (exceeded 20). By contrast, the corresponding systemic risk event for the WA index changed from 18.2% (i.e., 2 defaults among 11 institutions in the sample) to 10% (i.e., 2 defaults among 20 institutions) and from 14.3% (i.e., 3 defaults among 21 institutions) and 13.6% (i.e., 3 defaults among 22 institutions). Therefore, adjusting this threshold discrepancy would indicate that the WA index shows a lower level of systemic risk than the ND index.

Figure 5 shows the CEL index. The CEL index increased sharply during the crisis but to a lesser extent than the ND and WA indices. This may be because the ND and WA indices measure the probability of a systemic risk event, whereas the CEL index gauges the severity of such an event when it occurs. Consistent with the ND and WA indices, the risk-neutral CEL index was higher than the physical CEL index. However, this difference was not closely related to the usual indicators of credit risk and liquidity risk.



Note: The three measures of systemic risk (i.e., the ND, WA, and CEL indices) were calculated and compared with both asset correlations and corresponding equity correlations.

Figure 6. Asset Correlations versus Equity Correlations

4.3. Asset Return Correlations versus Equity Return Correlations

Despite some methodological inconsistencies, a number of studies have substituted equity return correlations for asset return correlations because of the observability and availability of the former. For example, Huang, Zhou, and Zhu (2009, 2010, 2011) combined risk-neutral default probabilities for individual institutions with corresponding equity return correlations. In theory, equity prices and asset prices are non-linearly

related. Therefore, the question of whether this substitution is a good approximation remains an empirical issue which we investigate in this subsection.⁵

Figure 6 shows the three measures of systemic risk for asset return correlations and equity return correlations. For the ND and WA indices, substituting equity return correlations for asset return correlations produced no significant differences. For the CEL index, this substitution produced a larger but not substantial difference: the mean difference was 0.23%p, and both the mean and standard deviation of the absolute difference were 0.78%p.

5. SYSTEMIC RISK CONTRIBUTIONS

In the previous sections, we have proposed a model for measuring of the overall level of systemic risk and presented the results. Once the overall level of systemic risk is identified, financial regulators are naturally interested in determining the most important institutions from the perspective of systemic risk or the most vulnerable institutions in the case of a systemic risk event. In this regard, an increasing number of studies have analyzed systemic risk contributions. Recently, Adrian and Brunnermeier (2010) proposed a measure of systemic risk called CoVaR, which is the value at risk (VaR) of the financial system conditional on institutions being in distress. They defined an institution's contribution to systemic risk as the difference between CoVaR conditional on the institution being in distress and CoVaR at the median state of the institution. They also proposed another measure in the reverse direction, namely exposure CoVaR, which measures the extent to which an individual institution is affected by a systemic financial event. Acharya et al. (2010) presented a simple model of systemic risk and showed that a financial institution's contribution to systemic risk can be measured as its systemic expected shortfall (SES), that is, its propensity to be undercapitalized when the system as a whole is undercapitalized.⁶ Huang, Zhou, and Zhu (2010, 2011) used the additivity of their systemic risk measure and employed the individual components as marginal risk contributions.

In this section, we devise several measures of systemic risk contributions that are matched with the corresponding measures of the overall level of systemic risk. We also provide the measures of systemic risk contributions in both directions, that is, the effect of the overall systemic risk event on an individual credit event and vice versa.

⁵ In this study, although the log leverage correlation is a more precise expression than the asset return correlation, we used the latter in line wih previous research.

⁶ It should be noted that these two studies did not explicitly define default events, which makes it difficult to measure the overall level of systemic risk. In fact, they loosely defined systemic risk by using worse market outcomes. In this regard, the present study's proposed model has an advantage of providing the overall level of systemic risk based on an explicit definition of default events.

Starting from the ND index, we propose two measures of systemic risk contributions: SysND and IndND. The ND-based systemic risk effect $(SysND_j)$ index is defined as the probability of a default by institution j conditional on the occurrence of a systemic risk event, which is defined in the same way as that for the ND index. On the other hand, the ND-based individual causation $(IndND_j)$ index is defined as the probability of a systemic risk event conditional on a default by institution j, where systemic risk is defined in the same way as that for the ND index.

Similarly, we propose two additional measures of systemic risk contributions, SysWA and IndWA, for the WA index. Finally, for the CEL index, we propose the marginal CEL (MCEL) index. $MCEL_j$ is defined as the corresponding portfolio weight times the expected loss ratio for institution j conditional on the occurrence of a systemic risk event, which is defined in the same way as that for the CEL index. We calculate all these measures of systemic risk contributions through simulations.

FI	SysND		IndND		SysWA		IndWA		MCEL	
No	Prob	Rank								
1	n.a.	n.a.								
2	0.2112	3	0.6248	4	0.0255	7	0.1056	9	0.0003	10
3	0.2322	2	0.5978	6	0.3153	1	1	4	0.0386	2
4	n.a.	n.a.								
5	n.a.	n.a.								
6	0.1532	6	0.5952	7	0.0115	11	0.0562	11	0	11
7	0.1215	8	0.5415	10	0.282	2	1	3	0.0628	1
8	n.a.	n.a.								
9	0.1089	9	0.6765	1	0.0398	5	0.4167	5	0.003	5
10	n.a.	n.a.								
11	0.1353	7	0.6153	5	0.1864	4	1	2	0.0146	3
12	n.a.	n.a.								
13	0.1808	4	0.6331	3	0.0232	9	0.1011	10	0.0011	8
14	n.a.	n.a.								
15	0.1533	5	0.5889	9	0.2245	3	1	1	0.0041	4
16	n.a.	n.a.								
17	n.a.	n.a.								
18	0.3312	1	0.1931	11	0.0352	6	0.1158	8	0.0004	9
19	n.a.	n.a.								
20	0.0605	12	0.0316	12	0.0026	12	0.0054	12	0	12
21	0.0789	11	0.6338	2	0.0204	10	0.2307	6	0.0012	6
22	0.1019	10	0.5906	8	0.0241	8	0.1478	7	0.0012	7

Table 2.Measurements of Systemic Risk ContributionsPeriod 1: March 2006 to July 2007

Period 2: August 2007 to December 2008										
FI	SysND		IndND		SysWA		IndWA		MCEL	
No	Prob	Rank								
1	0.1025	12	0.6403	13	0.0672	16	0.299	14	0.0005	14
2	0.305	5	0.6561	11	0.1944	8	0.3	13	0.0015	11
3	0.0787	13	0.6462	12	0.1564	9	1	3	0.0176	2
4	0.1808	9	0.6976	7	0.1259	10	0.398	11	0.0014	12
5	0.0141	21	0.7263	4	0.0105	20	0.4107	10	0	19
6	0.0497	16	0.7411	2	0.0272	17	0.332	12	0.0001	17
7	0.1165	10	0.4408	17	0.349	3	1	2	0.0709	1
8	0.0207	20	0.1793	22	0.0074	21	0.0537	22	0	20
9	0.0722	15	0.7005	5	0.111	11	0.8933	4	0.0077	6
10	0.371	4	0.7278	3	0.2981	5	0.4772	7	0.004	10
11	0.048	17	0.6918	8	0.0902	13	1	1	0.0122	4
12	0.0254	19	0.7547	1	0.011	19	0.2921	15	0	22
13	0.2453	6	0.7004	6	0.1968	7	0.4196	9	0.0046	7
14	0.4469	2	0.3272	18	0.2437	6	0.1282	18	0.0001	15
15	0.2292	7	0.555	14	0.3457	4	0.6467	5	0.0125	3
16	0.696	1	0.279	19	0.4089	1	0.1252	19	0.0001	16
17	0.4359	3	0.6852	9	0.3754	2	0.4391	8	0.0077	5
18	0.197	8	0.4479	16	0.1079	12	0.186	17	0.0009	13
19	0.003	22	0.2245	20	0.0017	22	0.1107	20	0	21
20	0.0312	18	0.2138	21	0.0121	18	0.0681	21	0	18
21	0.1056	11	0.5438	15	0.087	14	0.2789	16	0.0045	8
22	0.0786	14	0.6807	10	0.0808	15	0.5689	6	0.0044	9

Period 2: August 2007 to December 2008

Period 3: January 2009 to August 2010

Period 3: January 2009 to August 2010										
FI	SysND		IndND		SysWA		IndWA		MCEL	
No	Prob	Rank								
1	0.116	12	0.4592	13	0.0422	15	0.2683	14	0.0003	13
2	0.2163	6	0.5692	6	0.1032	9	0.4194	9	0.0009	10
3	0.1857	8	0.4212	14	0.281	3	1	4	0.0565	1
4	0.0767	15	0.7328	1	0.0443	14	0.6897	6	0.0006	12
5	0.023	19	0.5927	4	0.0082	19	0.3758	11	0	18
6	0.0865	13	0.5896	5	0.0293	16	0.3314	13	0.0001	17
7	0.1993	7	0.2727	15	0.4561	1	1	3	0.0416	2
8	0.0113	20	0.0743	21	0.0045	20	0.0442	21	0	19
9	0.0794	14	0.5963	3	0.0562	13	0.706	5	0.0033	9
10	0.5079	2	0.5557	7	0.2113	4	0.3713	12	0.0035	8
11	0.0761	16	0.545	9	0.0857	11	1	2	0.0152	3
12	0.065	17	0.4647	12	0.0155	18	0.2154	15	0	20

13	0.3499	4	0.5553	8	0.1553	7	0.395	10	0.0041	7
14	0.2949	5	0.1606	17	0.1327	8	0.11	18	0.0001	15
15	0.1714	10	0.4723	11	0.094	10	0.4217	8	0.0043	6
16	0.7046	1	0.1379	18	0.3758	2	0.1133	17	0.0001	14
17	0.4145	3	0.6471	2	0.1972	5	0.4916	7	0.005	5
18	0.1719	9	0.2721	16	0.0646	12	0.1564	16	0.0008	11
19	0.003	21	0.0773	20	0.0013	21	0.0484	20	0	21
20	0.0464	18	0.1093	19	0.0214	17	0.0739	19	0.0001	16
21	n.a.	n.a.								
22	0.1461	11	0.5095	10	0.1766	6	1	1	0.0088	4

Table 2 shows the estimation results for the proposed measures for the three subperiods and for each of the 22 institutions. For a better understanding, Figure 7 plots the average IndND and average IndWA indices for the crisis period (Period 2) against those for the pre-crisis period (Period 1). Figure 8 shows these plots for SysND and SysWA.



Note: The average IndND (circle) and IndWA (rectangle) indices during crisis period (Period 2) were plotted against those during the pre-crisis period (Period 1).

Figure 7. Systemic Risk Contributions Based on Individual Causation: The Pre-Crisis Period vs. The Crisis Period



Note: The average SysND (circle) and SysWA (rectangle) indices during crisis period (Period 2) were plotted against those during the pre-crisis period (Period 1).



The two individual causation indices (i.e., IndND and IndWA) were generally higher during the crisis period than during the pre-crisis period. This indicates that the probability of a systemic risk event was higher during the crisis period than during the pre-crisis period, given the same individual credit event occurrence. Therefore, financial regulators should focus more on individual instability during crisis periods than during normal periods. By contrast, the two systemic risk effect indices (i.e., SysND and SysWA) provided mixed results. For a WA-based systemic risk event, the level of credit risk was generally higher during the crisis period than during the pre-crisis period, whereas for an ND-based systemic risk event, the level of credit risk was higher during the pre-crisis period.



Note: The average MCEL index during crisis period (Period 2) was plotted against that during the pre-crisis period (Period 1).

Figure 9. Systemic Risk Contributions Based on the MCEL Index: The Pre-Crisis Period vs. The Crisis Period



Note: The average relative MCEL index, defined as the ratio of the MCEL index to the CEL index, was compared with corresponding total asset weights during the crisis period (Period 2).

Figure 10. The Relative MCEL Index vs. Total Asset Weights

A SIMPLE METHOD FOR MEASURING SYSTEMIC RISK

For a CEL-based systemic risk event, the MCEL index was generally higher during the crisis period than during the pre-crisis period (Figure 9). To investigate the size effect on the portfolio loss ratio, Figure 10 plots the relative MCEL index, defined as the ratio of the MCEL index to the CEL index, against the corresponding weights of total assets. The relative MCEL index is determined mainly by its size, but the results indicate that the different levels of individual credit risks had a significant effect on the relative MCEL index.



Notes: Realized risks, defined as the equity return (%) for the crisis period (Period 2), were shown on the vertical axis. For the three panels on the left (A, C, and E), the corresponding three indices of systemic risk contributions (SysND, SysWA, and MCEL) for the crisis period (Period 2) were shown on the horizontal axis, whereas those for the pre-crisis period (Period 1) were shown on the horizontal axis in the three panels on the right (B, D, and F). Trend lines are also demonstrated.

Figure 11. Indices of Systemic Risk Contributions: Comovements and Forecastability

We examined the performance of the proposed measures of systemic risk contributions in terms of their comovement and forecastability. We measured the realized risk for individual institutions by the corresponding equity returns, as in Acharya *et al.* (2010). In Figure 11, the three panels on the left (A, C, and E) compare the realized risk with SysND, SysWA, and MCEL for the crisis period. These plots demonstrate how closely these measures were related to the realized risk for the crisis

period. The SysND and SysWA indices were inversely and closely related with the realized risk, whereas the MCEL index was inversely related to the realized risk to a lesser extent. As shown in the three panels on the right (B, D, and F), to investigate the forecastability of the measures of systemic risk contributions, we compared the realized risk with ex-ante systemic risk contributions, which were measured for the pre-crisis period (Period 1). All the ex-ante indices were inversely related to the realized risk but to a lesser extent than the ex-post indices.

6. CONCLUSIONS

This paper proposes a simple method that employs CDS data for analyzing systemic risk. This method overcomes existing methods' inconsistency problems. This method can produce various indicators of systemic risk in a consistent manner and measure systemic risk contributions in both directions, that is, the effect of the overall level of systemic risk on credit risk and vice versa.

Using CDS data, we employed the proposed method to measure systemic risk for a group of 22 large financial institutions in the U.S. for the period from March 2006 to August 2010. The main results are summarized as follows: First, there were considerable variations in asset correlations over time. This time-varying feature implies that investors may require correlation-risk premiums and that such premiums can be timevarying. Thus, disregarding correlation-risk premiums when measuring risk-neutral systemic risk may produce misleading results. Second, the level of systemic risk measured under the risk-neutral probability measure was higher than that of systemic risk measured under the physical probability measure. In addition, the difference between the level of systemic risk between these two probability measures changed over time and widened during the 2007-2009 global financial crisis, reflecting heightened credit and liquidity concerns. Third, the substitution of equity correlations for asset correlations had little effect on the measurement of systemic risk. Fourth, systemic risk contributions also exhibited a time-varying feature. In particular, systemic risk contributions defined as the extent to which a default by a particular institution influences systemic risk were more likely to increase during the crisis period than during the pre-crisis period. Finally, systemic risk contributions defined as the extent to which a systemic risk event influences the level of credit risk for a particular institution were closely related to the realized risk represented by equity returns during the crisis period. Systemic risk contributions also had the ex-ante ability to predict future risks, although not very accurately.

This paper can be extended in several ways. The proposed model can be extended to allow for volatility clustering. Although this study considers financial institutions in the U.S., the proposed method can be applied to other countries. CDS contracts have become available for an increasing number of financial institutions, and thus, such an extension would be more likely. In addition, it would be also beneficial to compare data on equity returns and CDS premiums in terms of the quality of information on credit risk.

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