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PHYSICAL AND HUMAN CAPITAL ACCUMULATION AND THE EVOLUTION OF INCOME AND INEQUALITY

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We study how financial and educational institutions affect the evolution of income and income inequality in an overlapping generations model with heterogenous agents. While the literature mostly focuses on either physical or human capital, we make an attempt to study the joint evolution of these variables. In our model, we find that better educational institutions increase income of the individuals and are associated with lower income inequality. Better financial institutions also foster economic growth, but are associated with higher income inequality. Our model also demonstrates that focusing on aggregate measures of financial and educational institutions provides misleading results if one neglects the possibility of unequal access to these institutions.

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JEL classification: O15, O16, O43

1. INTRODUCTION

How do financial and educational institutions impact on income and inequality? We construct an overlapping generations model with heterogenous agents to study the effects of these two types of institutions on both physical and human capital accumulation. This allows us to draw conclusions for the evolution of income and inequality. In our model, individuals are heterogeneous in the sense that they are endowed with different initial physical and human capital. We assume that there are educational institutions that accelerate human capital accumulation, and financial institutions that enable higher investment in physical capital. The modeling strategy of physical and human capital institutions is standard. Our model aims at combining the modeling of these two institutions and studying the joint development of physical and human capital.

* I would like to thank an anonymous referee and the editor for very useful comments. All remaining errors are my own.

Our model first considers the case where every individual has equal access to public education and to financial services. We show that equal access leads to convergence in capital holdings and thereby income across individuals. The path to convergence is associated with higher inequality for better financial institutions and lower inequality for better educational institutions. Better institutions of one type also promote growth of the other factor, which implies that there is a spillover effect. For example, better financial institutions will not only lead to a higher stock of physical capital, but also increase aggregate human capital and vice versa. In an extension of the model, we study the effects of unequal access to financial and educational institutions. If one observes that institutions improve, the effects on income and income inequality are therefore ambiguous if one does not know whether individual access to these institutions at the same time has become more or less equal. We thus argue that one needs to make a distinction between the aggregate quality of institutions and the degree of inequality in individual access to these institutions.

The empirical literature finds positive correlations between education and income per capita (see e.g., Ahituv and Moav, 2007) as well as between financial development and income per capita (see e.g., King and Levine, 1993). However, one should note that the establishment of causality has been much more difficult (see e.g., Esso, 2010 and Pan and Wang, 2013). The empirical relationship between both types of capital and inequality is less obvious, but seems to indicate that income inequality can be reduced with institutional development (see Claessens and Perotti, 2007; Mejia and St-Pierre, 2008 and Demirguc-Kunt and Beck, 2008). However, as pointed out by Demirguc-Kunt and Beck (2008), this is subject to ample qualifications. One important qualification is that the degree of inequality in the access to institutions is often neglected (Beck, Demirguc-Kunt and Martinez Peria, 2006). In theory, institutional development can benefit either poor or rich individuals. For example, the development of institutions could primarily improve institutional access for individuals who previously had no access to the related services. This would tend to reduce economic inequality. However, institutional development can also primarily benefit individuals who already have access to these institutions. This would cause a rise in income inequality.

Inequality in the access to financial or educational institutions may arise because an individual’s stock of physical and human capital is too small to benefit fully from the institutions of a country. For example, existing physical capital can be too small to be appropriate as a collateral for borrowing. It can also be that property rights are not well defined for poor people, as emphasized by de Soto (2003). In the case of human capital, a child may not “possess” enough human capital to be eligible for higher education. Empirical research on unequal access to institutions is mostly based on case studies (see e.g., Claessens, 2005). Demirguc-Kunt and Beck (2008) create indicators to measure the degree of inequality in the access to financial institutions. They find that unequal access to either borrowing or saving has a negative impact on growth and a positive impact on inequality.

Our model is related to Galor and Moav (2004), who argue that physical and human capital accumulation are associated with opposite developments of inequality. In their model, physical capital accumulation goes with with higher ecconomic growth and higher inequality. A higher degree of human capital accumulation also fosters economic growth, but is associated with lower inequality. Our model exhibits the same relationship, namely a negative relationship between educational institutions and inequality as well as a positive relationship between financial institutions and inequality. In addition to these findings, we argue that this pattern depends on the degree of inequality in individual access to these institutions.

Our modeling of human and physical capital accumulation takes into account the previous findings in the literature. As to human capital, several papers show that family background and parental education matter for human capital accumulation and children's earnings (see e.g., Heckman and Hotz, 1986; Johnson and Neal, 1996 and Cornelissen, Jirjahn and Tsertsvadze, 2008). In addition, time allocated to learning and the quality of educational institutions is found to be crucial (e.g., Glomm and Ravikumar, 1992 and Mejia and St-Pierre, 2008). Hence, human capital accumulation depends both on private and public elements. In our model, we attempt to take this into account, assuming that the human capital stock of each individual depends on time allocated to learning, the stock of human capital of the individual's parents and public education expenditures.

Similar to the case of human capital, physical capital accumulation can also be seen as depending on private elements comprising the existing capital stock and investment decisions and an element depending on institutional development. Usually, financial development is characterized by the degree to which firms are credit constrained (see e.g., Aghion and Howitt, 2009). Firms are credit constrained because it is difficult for the financial system to assess the quality of a project and to channel its funds towards private investment projects. An efficient financial system can alleviate borrowing constraints and attract more savings. It can also induce individuals, who have invested their savings abroad, to repatriate these savings and invest in their home country.

This paper is organized as follows. In Section 2, we describe a model with financial and educational institutions. The properties of the model are then analyzed in Section 3. Section 4 describes the choice of the parameter values, which are used to illustrate the characteristics of the model. In Section 5, we illustrate these basic properties by calibrating and simulating the model. An extension of the model for the case, where individuals have unequal access to institutions, is presented in Section 6. Finally, Section 7 contains the conclusion.

2. THE MODEL

2.1. The Individual

We consider an overlapping generation model, where every generation lives for two periods. Young individuals receive income from physical and human capital that they provide to firms. In addition, they invest in physical and human capital, because they derive utility from their future capital holdings of their children. When individuals are old, they consume what they have saved when they were young and pay back loans. Such a modelling framework is common in the literature on growth and inequality (see e.g., Viaene and Zilcha, 2001 and Azariadis and de la Croix, 2002).

Following this reasoning, an individual's utility function formally depends on four elements. The first two elements are consumption when young and when old. The third and forth element are related to future capital holdings and can be motivated by some form of altruism for the next generation. Every individual has exactly one child and is concerned about human and physical capital of this child. Formally, the utility function $U(\cdot)$ of any individual j born in period t is given by:

$$U_t^j(c_{t,t}^j, c_{t,t+1}^j, h_{t+1,t+1}^j, k_{t+1,t+1}^j) = \alpha_1 \ln c_{t,t}^j + \alpha_2 \ln c_{t,t+1}^j + \alpha_3 \ln h_{t+1,t+1}^j + \alpha_4 \ln k_{t+1,t+1}^j \,, \tag{1}$$

where $c_{t,t}^j$ and $c_{t,t+1}^j$ are consumption of generation t when young and when old, and $h_{t+1,t+1}^j$ and $k_{t+1,t+1}^j$ are physical and human capital holdings of the generation $t+1$. For the remainder of this paper, we drop the index for a generation to simplify the notation. The weighting parameters α_i determine the importance of each element in the utility function. This form of "altruism" or concern about the future directly modeled with human or physical capital rather than income or consumption is common in models of intergenerational analysis and often called a "warm-glow" bequest function (see, for instance, Galor and Moav, 2004). However, as these capital holdings indirectly determine future income and consumption, such a utility function can also be seen as the utility function of an infinitely lived individual.[1] Human capital accumulation is modeled in a way similar to that in other related models (see e.g., Glomm and Ravikumar, 1992 and Glomm and Kaganovich, 2008) and depends on school quality, parents' human capital and time devoted to learning. In this basic version of the model, all agents use the same learning technology and have equal access to public education:

$$h_{t+1}^j = G^{1-\theta}(h_t^j n_t^j)^\theta \,, \tag{2}$$

where θ denotes the elasticity of future human capital with respect to current investment in human capital. G stands for the quality of educational institutions. In this

[1] The functional form of a log-linear utility function simplifies the analysis considerably. Clearly, it might lead to corner solutions as the first-order conditions do not necessarily hold at the maximum for any given level of the exogenous variables. In the following, we will only consider those cases where an interior solution exists.

basic model, G is assumed to be a pure public good. It does not only capture government spending, but also comprises the quality of educational institutions. In Section 6, we investigate what happens if access to G is not equal for every individual. Since we assume $0 < \theta < 1$, the production function exhibits decreasing returns to scale in the privately provided inputs. G is the public component of future human capital, and $h_t^j n_t^j$ is the private component. If G is small relative to $h_t^j n_t^j$, the level of the parents' human capital holdings and their investment decisions determine future human capital primarily. If G is high, this is reversed and the private background of an individual plays a smaller role.

Future capital depends on an individuals's own investment plus the loans he gets from the financial system. The amount of loans an individual receives depends on the individual's investment plans (as in Kunieda, 2008) and not on the stock of capital, because, in this model, capital fully depreciates and only newly invested capital can be used as a collateral in the next period. In addition, investment determines future performance of firms. Similar to human capital, an individual invests a fraction x_t^j of his physical capital in future capital and supplies a fraction $(1 - x_t^j)$ of physical capital to firms. Physical capital is assumed to evolve according to:

$$k_{t+1}^j = x_t^j k_t^j + \lambda_t k_t^j x_t^j \,. \tag{3}$$

λ_t tells us to what extent the individual is credit rationed and is determined by the aggregate efficiency of the financial sector and aggregate savings. Since aggregate savings are not known to an individual, we assume that individuals form expectations of $E_t[\lambda_t]$ further specified below when deciding on their behavior. For simplicity, we assume that physical capital completely depreciates over a generation.

In period t, an individual uses his factor income to consume and save s_t^j for the next period. The current period budget constraint of a young individual is thus given by:

$$c_t^j + s_t^j = wh_t^j (1 - n_t^j) + rk_t^j (1 - x_t^j) \,. \tag{4}$$

w is the return to human capital and r is the return to physical capital. Both returns are assumed to be exogenous and constant. Holding these returns constant can be justified either by assuming a small open economy or by limiting the analysis to the partial equilibrium case where the production sector is not explicitly modeled. Since a young individual invests the fractions n_t^j and x_t^j of his capital holdings, the remaining fractions $(1 - n_t^j)$ and $(1 - x_t^j)$ are supplied to firms in order to generate income. In the next period, an individual of generation t does not work any more. He consumes his income from savings and he has to pay back his loans plus accrued interest payments:

$$c_{t+1}^{j} = (1+r)s_t^j - \lambda_t (1+r) k_t^j x_t^j \; . \tag{5}$$

Note that the interest rates on savings and loans are assumed to be equal to each other. The maximization problem is then:

$$\max_{c_t, c_{t+1}, h_{t+1}, k_{t+1}, s_t, n_t, x_t} \alpha_1 \ln c_t^j + \alpha_2 \ln c_{t+1}^j + \alpha_3 \ln h_{t+1}^j + \alpha_4 \ln k_{t+1}^j \, ,$$

$$s.t. \quad c_t^j + s_t^j = wh_t^j (1 - n_t^j) + rk_t^j (1 - x_t^j) \, ,$$

$$c_{t+1}^j = (1+r)s_t^j - E_t\{\lambda_t\}(1+r)k_t^j x_t^j \, ,$$

$$h_{t+1}^j = G^{1-\theta} (h_t^j n_t^j)^{\theta} \, ,$$

$$k_{t+1}^j = x_t^j k_t^j + E_t\{\lambda_t\} x_t^j k_t^j \; .$$

As explained below, the individual does not know λ_t at the time when he makes his choices. Thus, he needs to take expectations of the amount of loans he gets, which is formally written as $E_t\{\lambda_t\}$. We can substitute the constraints into the objective and derive the first-order conditions given by:

$$\text{for } s_t^j : \quad \frac{\alpha_2 (1+r)}{s_t^j (1+r) - (1+r) E_t\{\lambda_t\} x_t^j k_t^j} = \frac{\alpha_1}{wh_t^j (1 - n_t^j) + rk_t^j (1 - x_t^j) - s_t^j} \, , \tag{6}$$

$$\text{for } n_t^j : \quad \frac{\alpha_3 \theta}{n_t^j} = \frac{\alpha_1 w h_t^j}{wh_t^j (1 - n_t^j) + rk_t^j (1 - x_t^j) - rE_t^j\{\lambda_t^j\}(1 - x_t^j)(k_t^j) - s_t^j} \, , \tag{7}$$

for x_t^j :

$$\frac{\alpha_4 (k_t^j + E_t\{\lambda_t\} k_t^j)}{x_t^j k_t^j + E_t\{\lambda_t\} x_t^j k_t^j} = \frac{\alpha_4 x_t^j k_t^j}{wh_t^j (1 - n_t^j) + rk_t^j (1 - x_t^j) - s_t^j} - \frac{\alpha_2 (1+r) E_t\{\lambda_t\} x_t^j k_t^j}{(1+r)s_t^j - (1+r) E_t\{\lambda_t\} x_t^j k_t^j} \; . \tag{8}$$

Solving these three equations for s_t^j, n_t^j and x_t^j, we get:

$$s_t^j = \frac{(wh_t^j + rk_t^j)((\alpha_2 + \alpha_4) E_t\{\lambda_t\} \alpha_2 r)}{(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_3 \theta)(r + E_t\{\lambda_t\})} \, , \tag{9}$$

$$n_t^j = \frac{\alpha_3 \theta (wh_t^j + rk_t^j)}{wh_t^j (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_3 \theta)}, \tag{10}$$

$$x_t^j = \frac{\alpha_4 (wh_t^j + rk_t^j)}{rk_t^j (r + E_t\{\lambda_t\})(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_3 \theta)}. \tag{11}$$

2.2. The Financial System

The financial system provides loans to individuals who want to invest in physical capital. The amount of loans available is determined by aggregate savings and an aggregate financial efficiency parameter $\mu \in [0,1]$ that determines the efficiency of banks to transform savings into loans. It can also be interpreted as the degree of savings that is transformed into loans and not invested in other (e.g., foreign) assets. A low value of μ would then indicate a high degree of capital flight. Overall, μ is influenced by the institutional environment including e.g., the regulation of the financial system. This modeling strategy is similar to the one presented in Kunieda (2008)). The fraction λ_t an individual can borrow is thus determined by aggregate savings, individual investment plans and aggregate financial efficiency μ. The interest rates on loans and savings are the same and exogenously given. Aggregate loans $\sum_{j=1}^{N} \lambda_t^j k_t^j x_t^j$ are thus given by:

$$\mu \sum_{j=1}^{N} s_t^j = \sum_{j=1}^{N} \lambda_t k_t^j x_t^j . \tag{12}$$

Since λ_t is endogenously determined within the financial system, we can solve for λ_t:

$$\lambda_t = \mu \frac{\sum_{j=1}^{N} s_t^j}{\sum_{j=1}^{N} k_t^j x_t^j} . \tag{13}$$

For the moment, it is assumed that λ_t is the same for every individual. Thus, we do not consider unequal access to financial markets for the moment. However, in Section 6, it is shown in what respect heterogeneous borrowing constraints λ_t can change the results of our model. For the rest of this section, we will also assume that individuals solve their optimization problem based on the assumption that $E_t\{\lambda_t\} = \varsigma\mu$. As becomes clear below, any other constant expected value for λ_t would not affect the qualitative results of our model. For simplicity, we choose a value of $\varsigma = 1$ in the following. The components of Equation (13) can be simplified considerably. Aggregate savings

$S_t = \sum_{j=1}^{N} s_t^j$ are given by:

$$S_t = \sum_{j=1}^{N} s_t^j = \sum_{j=1}^{N} \frac{(wh_t^j + rk_t^j)((\alpha_2 + \alpha_4)\mu + \alpha_2 r)}{(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_3\theta)(r + \mu)} . \tag{14}$$

Aggregate investment plans $\sum_{j=1}^{N} x_t^j k_t^j$ are given by:

$$\sum_{j=1}^{N} x_t^j k_t^j = \sum_{j=1}^{N} \frac{\alpha_4 (wh_t^j + rk_t^j)}{(r(r + \mu))(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_3\theta)} . \tag{15}$$

Inserting (14) and (15) in (13), we get:

$$\lambda_t = \mu \frac{\sum_{j=1}^{N} \frac{(wh_t^j + rk_t^j)((\alpha_2 + \alpha_4)\mu + \alpha_2 r)}{(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_3\theta)(r + \mu)}}{\sum_{j=1}^{N} \frac{\alpha_4 (wh_t^t + rk_t^j)}{(r(r + \mu))(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_3\theta)}} . \tag{16}$$

Simplifying this expression, we get:

$$\lambda_t = \lambda = \mu r \frac{\mu(\alpha_2 + \alpha_4) + \alpha_2 r}{\alpha_4} . \tag{17}$$

3. THEORETICAL ANALYSIS OF THE MODEL

In this section, we analyze the most important properties of our model. Most importantly, as laid down in propositions 1, 2 and 3, there is convergence in income and factor holdings. Hence, all families will eventually end up with the same stocks of physical and human capital.

Proposition 1: If $0 < \theta < 1$*, the human capital stock of every individual will converge to the same level, irrespective of the initial distribution of human capital.*

Proof:

See Appendix.

Intuitively, an individual with a low stock of human capital will optimally choose a higher investment share than an individual with a high capital stock (see Equation (10)).

Eventually, this leads to convergence. In addition, accumulation of human capital in Equation (2) is associated with decreasing returns to scale if, as it is assumed in this proposition, $0 < \theta < 1$. These specifications ensure that there is eventually convergence in human capital.

Proposition 2: Irrespective of the initial distribution of physical capital, the stock of every individual's physical capital will converge to the same level.

Proof:

See Appendix.

Intuitively, an individual with a low capital stock will show a higher investment share than an individual with a high capital stock (see Equation (11)). Eventually, this leads to convergence. If the individuals have equal access to finance, the financial system demands the same loan-to-value ratio from every individual and thus, does not prevent convergence to happen.

Proposition 3: The income of every individual will converge to the same level for given institutions.

Proof:

See Appendix.

Hence, under the assumption of equal access to financial and educational institutions, heterogeneity in capital holdings disappears over time for any kind of initial distribution. Thus, income inequality disappears and, for given institutions, the economy reaches a stationary state. The reason for this feature of the model is that the growth rates of income and the capital stocks are higher for poor invdividuals than for rich individuals. Convergence of individual incomes is a feature that our model shares with the related literature (see e.g., Viaene and Zilcha, 2001 and Glomm and Kaganovich, 2008).

One aspect we will illustrate in Section 5 is how institutional development affects economic growth and the level of the stationary state. In addition, we ask how institutional quality affects income inequality on the path to convergence. Propositions 4, 5 and 6 show how institutional development impacts on the economy.

Proposition 4: The development of educational and financial institutions increases the growth rates of income and the two capital stocks. Eventually, the economy reaches a higher stationary level of these variables with institutional development.

Proof:

See Appendix.

Proposition 5: An improvement of financial institutions increases income inequality at any date t.

Proof :

See Appendix.

As shown in the appendix, investment by an individual decreases with a one time improvement of financial institutions. This decrease in investment is stronger for rich individuals, leaving them with higher current income. Income inequality will thus temporarily increase. However, investment of poor individuals compared with rich individuals will rise, the more the income gap widens. As a result, there will eventually be income convergence.

Proposition 6: An improvement of educational institutions leads to less income inequality at any date t.

Proof:

See Appendix.

For the development of educational institutions, there is no effect on income through a change in investment decisions. The reason for this is that schooling is a public good and more schooling is not associated with direct costs for the individuals. Hence, if better educational institutions increase the growth rate of human capital relatively more for the poor, then this will lead to less inequality for any level of capital at any date t.

It is important to note that different combinations of parameter values of μ and G can lead to the same level of income. This means that one of two equally prosperous countries may be relatively abundant in physical capital, while the other may be abundant in human capital. In addition, as stated in Proposition 7, there are spillover effects, i.e., one type of institutions affects income and inequality of both types of capital holdings.

Proposition 7: Both types of institutions affect the accumulation of both physical and human capital. Hence, the development of educational or financial institutions does not only impact on the type of capital it directly affects, but also on the accumulation of the other type of capital.

Proof:

See Appendix.

The intuition for this result is straightforward. As can be seen in (10) and (11), investment decisions depend on total potential factor income of the individuals. An increase in h_t^j or k_t^j due to institutional development will therefore increase investment in both types of capital.

4. CHOICE OF PARAMETERS

This section describes the choice of the parameter values, which are used to illustrate the characteristics of the model. Note that while we try to capture some observable features in the parametrization, this section mainly intends to illustrate the qualitative properties of the model doing simulations. We do not try to calibrate our parameters to match the characteristics of a specific economy. The chosen parameter values for our model are depicted in Table 1. For the preference parameters, we first normalize $\alpha_1 = 1$. Then, assuming an annual discount rate of 0.96 and a period of 25 years, we get $\alpha_2 \approx 0.37$. By choosing this period length, we follow Azariadis and de la Croix (2002) and assume that the first period of the model corresponds to ages 12-37 and the second one corresponds to ages 37-62. We assume an annual interest rate of 4%, which implies that the interest over 25 years is $r \approx 1.66$. We assume that the return to one unit of human capital equals the return to one unit of physical capital implying $w = r$. Next, we assume that $\alpha_3 = \alpha_4$. Given *r*, *w* and α_2, the two parameters α_3 and α_4 have to be chosen. We choose them in a way such that the saving rate of an individual is approximately 0.2. This implies $\alpha_3 = \alpha_4 = 0.19$. These two parameter values determine the degree of altruism, however they should not be directly compared to the values of α_1 and α_2 because the consumption is a flow variable and capital is a stock variable. According to Glomm and Ravikumar (2003), the value for θ is difficult to choose and lies somewhere between 0.2 and 0.6. We choose a value in between, namely 0.4. For the baseline choice of parameter values, we assume $\mu = 0.5$ and $G = 3$. For simplicity, we assume $\varsigma = 1$. Initial endowments of physical and human capital are depicted in Table 2. The inital Gini coefficient is approximately 0.4 and computed by using the following standard formula:

$$g_t = \frac{1}{2n(n-1)\bar{y}_t} \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{n} \left| y_i - y_j \right|,$$

where *n* stands for the number of families or individuals, $\bar{y}_t$ is average income and y_i and y_j are individual incomes. For the purpose of illustrating the basic features of the model, we consider a discrete number of 10 individuals or families as it is often done in the related literature. In this basic setting, we assume that each of these 10 individuals has the same initial holdings of physical and human capital. The median is below the mean, which corresponds to the actual pattern of the income distribution.

Table 1. Parameter Values

Parameter	Value
α_1	1
α_2	0.37
α_3	0.19
α_4	0.19
θ	0.4
μ	0.5
G	3
r	1.66
w	1.66
ς	1

Table 2. Initial Distribution of Capital

Family	Physical Capital	Human Capital
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	11	11
9	15	15
10	18	18

5. SIMULATION RESULTS

5.1. Financial Development

This section illustrates the effects of varying the degree of aggregate financial efficiency, which is represented by the parameter μ. For the purpose of illustrating the impact of μ on the other variables, we look at the following values of μ: 0.2 (solid lines); 0.5 (dashed lines); 0.8 (dotted lines). Figure 1 depicts the effects on aggregate income of the young (inc_young) and the old (inc_old), as well as the evolution of income inequality within these two groups (gini_incyoung and gini_incold). In addition, the impacts on aggregate physical capital (phys_cap) and aggregate human capital (hum_cap) are shown.

As discussed in Section 3, a higher μ means that more loans are available, which increases aggregate income and consumption of the young and old. One can also see that inequality is higher when the financial system is more efficient due to Proposition 5. Financial efficiency does not only impact on physical, but also on human capital accumulation. Thus, there is a positive spillover effect as it was shown in Proposition 7. If better financial institutions increase income of the individuals, they invest more in both physical and human capital.

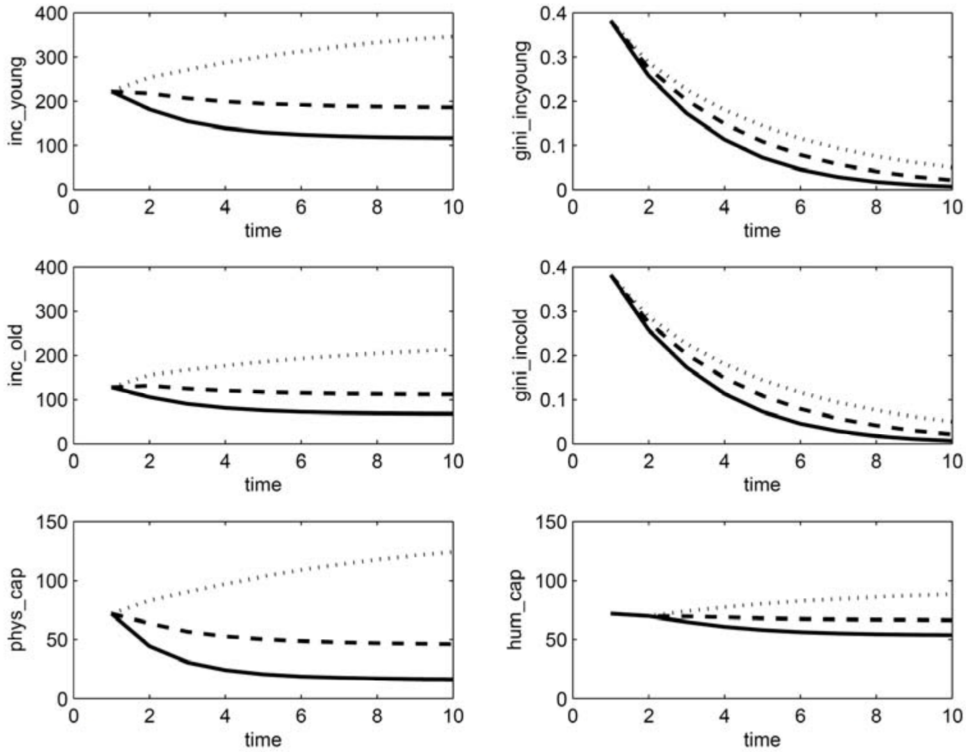

Figure 1. Financial Development
($\mu = 0.2$: solid lines; $\mu = 0.5$: dashed lines; $\mu = 0.8$: dotted lines)

5.2. The Development of Educational Institutions

The impact of better educational institutions on the other variables is illustrated in this section. This is done by varying *G*. The following parameter values for *G* are chosen: 2 (solid lines); 3 (dashed lines); 4 (dotted lines). Better educational institutions increase the income of the young and the old (see Figure 2). If educational institutions improve, inequality is reduced faster, which is in contrast to what we have seen for financial institutions and illustrates Proposition 5. In accordance with Proposition 7, there is a positive spillover effect from human capital institutions on physical capital accumulation.

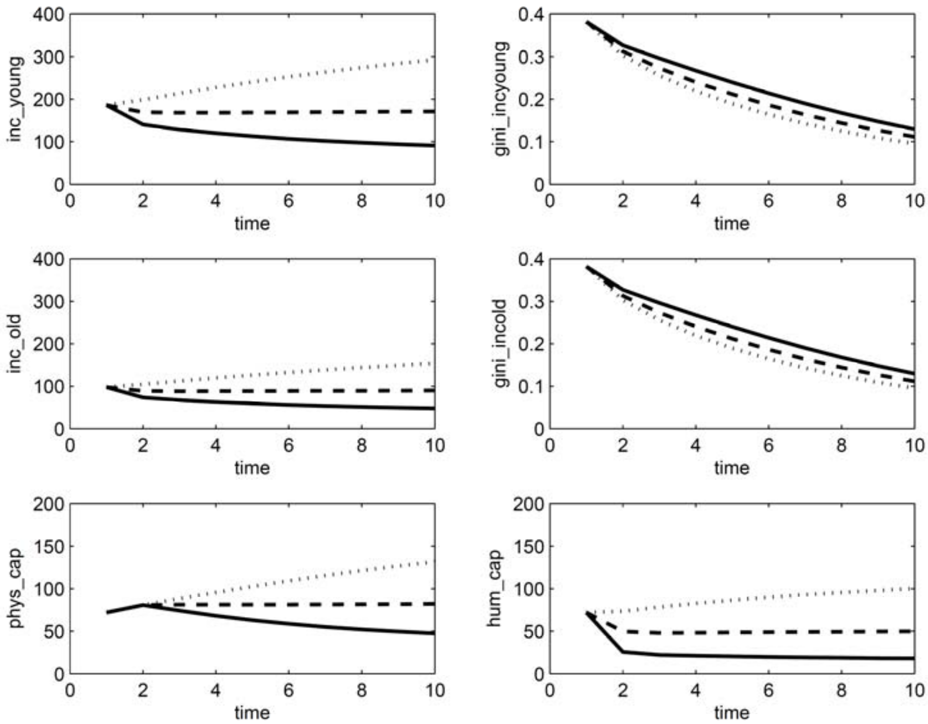

Figure 2. The Development of Educational Institutions
($G = 2$: solid lines; $G = 3$: dashed lines; $G = 4$: dotted lines)

6. UNEQUAL ACCESS TO INSTITUTIONS

So far, we have assumed that every individual or family has equal access to institutions that promote physical and human capital accumulation. In this section, we analyze what happens when individuals do not have equal access to these institutions. In particular, we assume that the access of an individual to educational institutions depends on his stock of human capital and access to financial institutions depends on the stock of physical capital. One can motivate this assumption from various perspectives. Following de Soto (2003), one can argue that poor people have a lot of "dead" capital because property rights are not well defined. Another possible view is that it is not profitable for a bank to consider requests for low loans. A small capital stock is therefore "useless" in getting access to the financial system, but can still be used in production. This is related to Bertola, Foellmi and Zweimueller (2005) and Matsuyama (2008). Following this literature, the model is modified by assuming a threshold level for private investment below which an individual does not get access to a particular institution. For educational institutions, we denote this threshold level as k^e. Formally, human capital accumulation is then given by: $h_{t+1}^j = G^{1-\theta}(h_t^j n_t^j)^\theta$ if $n_t^j h_t^j \geq k^e$ and $h_{t+1}^j = (h_t^j n_t^j)^\theta$ if $n_t^j h_t^j < k^e$, where k^e is the threshold level that determines whether individuals have access to public education or not. Similarly, for physical capital, we define the threshold level as k^f. We then have $k_{t+1}^j = x_t^j k_t^j (1+\lambda_t)$ if $x_t^j k_t^j \geq k^f$ and $k_{t+1}^j = x_t^j k_t^j$ if $x_t^j k_t^j < k^f$. Unequal access to financial institutions adds one further complication to the model. Unequal access to the financial system lessens borrowing constraints for those individuals who have access to financial institutions. Since constrained individuals do not get loans, only the *N-M* unconstrained individuals will get loans. Hence, we need to modify (16) to get:

$$\lambda_t = \mu \frac{\sum_{j=1}^{N} \frac{(wh_t^j + rk_t^j)((\alpha_2+\alpha_4)E_t^j(\lambda_t^j) + \alpha_2 r)}{(\alpha_1+\alpha_2+\alpha_4+\alpha_3\theta)(r+E_t^j(\lambda_t^j))}}{\sum_{j=1}^{N-M} \frac{\alpha_4(wh_t^t + rk_t^j)}{r(r+E_t^j(\lambda_t^j))(\alpha_1+\alpha_2+\alpha_4+\alpha_3\theta)}},$$

which can be simplified to get:

$$\lambda_t = \mu r \frac{\sum_{j=1}^{N}(wh_t^j + rk_t^j)((\alpha_2+\alpha_4)E_t^j\{\lambda_t^j\} + \alpha_2 r)}{\alpha_4 \sum_{j=1}^{N-M}(wh_t^j + rk_t^j)}. \tag{18}$$

As in Section 3, we assume that $E_t^j\{\lambda_t^j\} = \mu$ for those individuals whose parents had access to loans. For those individuals whose parents had no access to borrowing, we take $E_t^j\{\lambda_t^j\} = 0$. This leads to:

$$\lambda_t = \mu r \frac{(\alpha_2+\alpha_4)\mu + \alpha_2 r}{\alpha_4} + \mu r \frac{\sum_{j=N-M}^{N}(wh_t^j + rk_t^j)\alpha_2 r}{\alpha_4 \sum_{j=1}^{N-M}(wh_t^j + rk_t^j)}. \tag{19}$$

By comparing (17) with (19), one can clearly see that those individuals with access to financial institutions will get more bigger loans than under a system of equal access.

6.1. Theoretical Analysis of the Model's Modifications

Proposition 8: Under unequal access to financial institutions, there is no persistent inequality in physical capital holdings if

1. λ_t is sufficiently low

2. inequality in physical capital is sufficiently higher than in human capital

Proof. Consider two individuals i and j who differ in physical capital holdings with $k_t^i > k_t^j$. Assume that $x_t^i k_t^i > k^f$ and $x_t^j k_t^j < k^f$. The growth rate of physical capital of individual i is given by $(g_t^k)^i = \frac{k_{t+1}^i}{k_t^i} = x_t^i(1+\lambda_t)$ and for individual j, it is $(g_t^k)^j = \frac{k_{t+1}^j}{k_t^j} = x_t^j(1+\lambda_t)$. Next, we use (11) to substitute for x_t^i and x_t^j and check whether $g_k^i < g_k^j$, which would show us that there is convergence in physical capital holdings. We then get:

$$\frac{k_t^i}{(1+\lambda_t^i)k_t^j} > \frac{wh_t^i + rk_t^i}{wh_t^j + rk_t^j}. \tag{20}$$

This inequality is more likely to hold if the conditions in Proposition 8 hold.

Proposition 9: Under unequal access to educational institutions, there is no persistent inequality in human capital holdings if

1. G is sufficiently low

2. inequality in human capital is sufficiently higher than in physical capital

3. θ is sufficiently low

Proof. Consider two individuals i and j who differ in human capital holdings with $h_t^i > h_t^j$. Assume that $n_t^i h_t^i > k^e$ and $n_t^j h_t^j > k^e$. Thus, the higher human capital of individual i enables him to get access to education G, whereas individual j has no access to education. The growth rates of the human capital stocks of these two individuals are given by: $g_h^i = \frac{G(B(wh_t^i + rk_t^i))^\theta}{h_t^i}$ and $g_h^j = \frac{(B(wh_t^j + rk_t^j))^\theta}{h_t^j}$.

If there is convergence in human capital holdings across individuals, we must have $g_h^i > g_h^j$ or

$$\frac{G(B(wh_t^i + rk_t^i))^\theta}{h_t^i} < \frac{(B(wh_t^j + rk_t^j))^\theta}{h_t^j} . \tag{21}$$

Rearranging yields:

$$\frac{h_t^i}{Gh_t^j} > \left(\frac{wh_t^i + rk_t^i}{wh_t^j + rk_t^j} \right)^\theta . \tag{22}$$

This inequality is more likely to hold under the conditions given in proposition 9.

Propositions 8 and 9 mainly show two things. First, institutional development makes persistent inequality more likely if there is unequal access. Second, if there is convergence in one type of capital, convergence for the other type of capital becomes more likely.

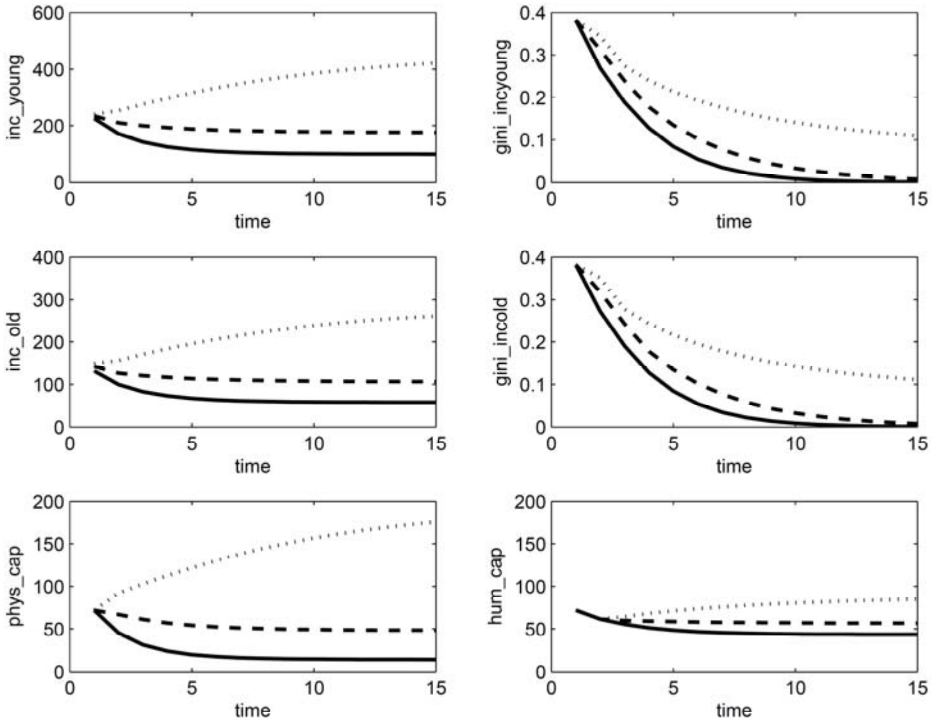

Figure 3. Unequal Access to the Financial System
($\mu = 0.2$: solid lines; $\mu = 0.5$: dashed lines; $\mu = 0.8$: dotted lines)

6.2. Simulated Effects of Unequal Access to Financial Institutions

This section illustrates Propositions 8 and 9. Thus, we show the effects of unequal access to institutions for several degrees of aggregate financial efficiency. We assume that the threshold level k^f is given by $k^f = 0.5\overline{x_t k_t}$ where $\overline{x_t k_t}$ is the average investment level in the population. Figures 3 illustrates what happens when financial institutions develop and there is only inequality in the access to financial institutions. One can see that despite unequal access to financial institutions, there is income convergence due to equal access to educational institutions. This confirms Proposition 8 that sufficiently equal access to one type of institutions is enough to generate income convergence across individuals.

Figure 4 illustrates the effects when there is also unequal access to educational institutions; where we assume $k^e = 0.5\overline{n_t h_t}$. In this case, there is only income convergence under the lowest considered financial development as shown in Proposition 8.

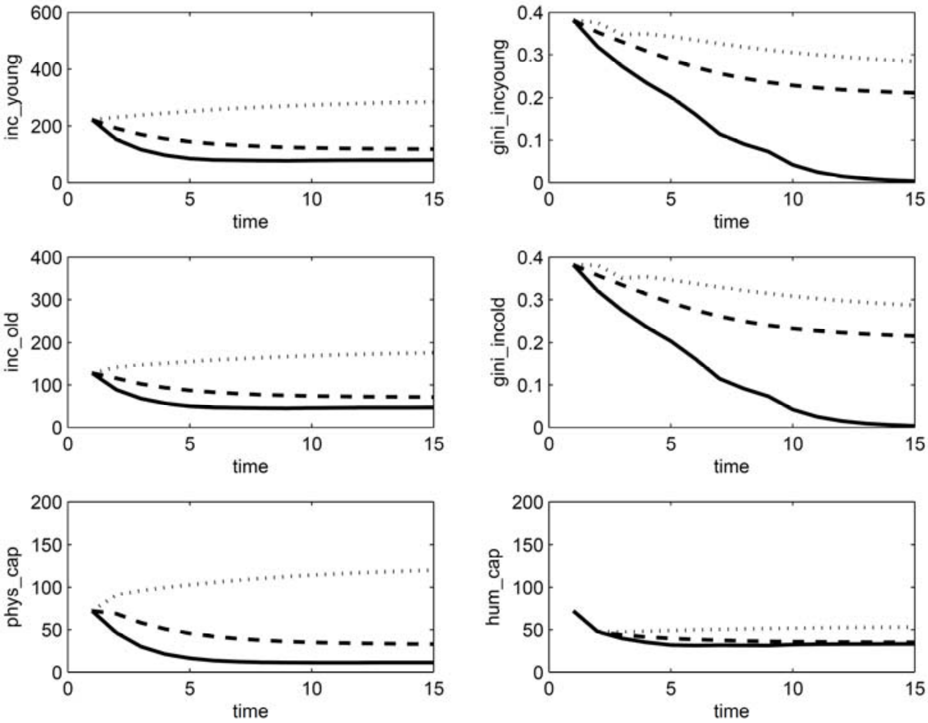

Figure 4. Unequal Access to the Financial and Educational Systems ($\mu = 0.2$: solid lines; $\mu = 0.5$: dashed lines; $\mu = 0.8$: dotted lines)

Finally, Figure 5 shows what happens when the threshold levels are increased to $k^f = 0.8\overline{x_t k_t}$ and $k^e = 0.8\overline{x_t k_t}$. In this case, there is no income converegence any more even for the lowest depicted level of financial development. Comparing Figures 3-5, one can observe that aggregate income and the two aggregate capital stocks decrease with more unequal access to institutions.

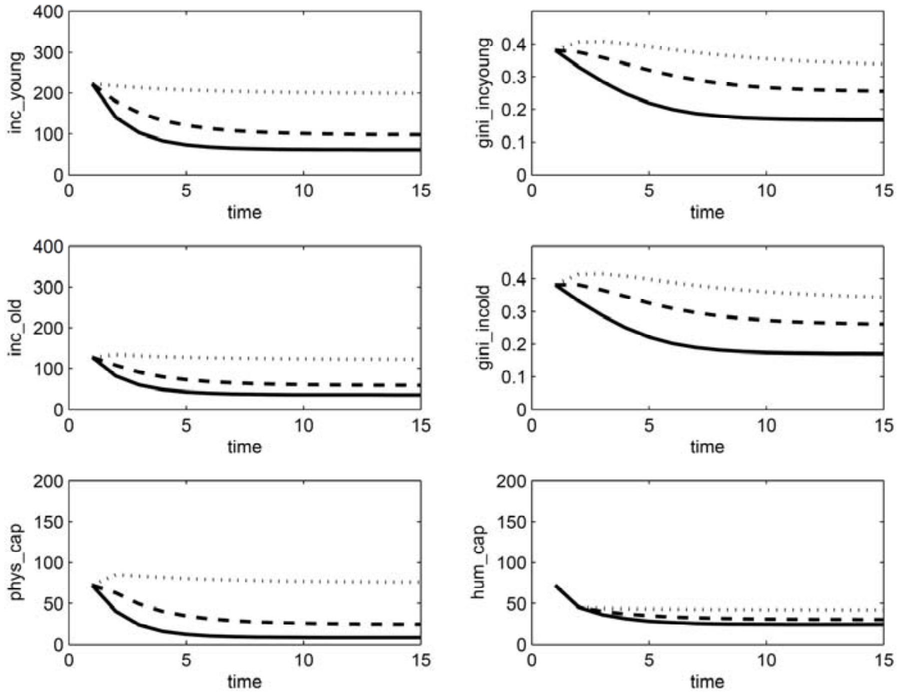

Figure 5. Increased Unequal Access to the Financial and Educational Systems ($\mu = 0.2$: solid lines; $\mu = 0.5$: dashed lines; $\mu = 0.8$: dotted lines)

6.3. Simulated Effects of Unequal Access to the Educational System

Figure 6 shows the impact of an improvement in human capital institutions when there is only unequal access to educational institutions but equal access to financial institutions. Again, we see that equal access to one type of institution, which is here the financial system, is sufficient for income convergence if educational development is low.

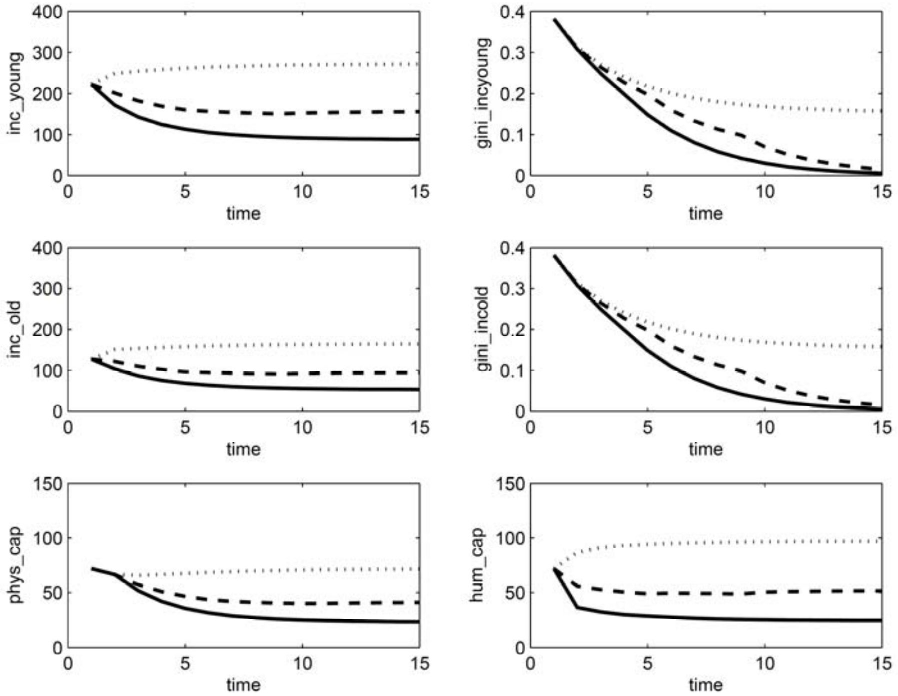

Figure 6. Unequal Access to the Educational System
($G = 2$: solid lines; $G = 3$: dashed lines; $G = 4$: dotted lines)

The case where individuals have unequal access to both types of institutions is illustrated in Figure 7. One can see that there is now only income convergence for low educational development. Thus, if there is unequal access to both types of institutions, income convergence becomes less likely.

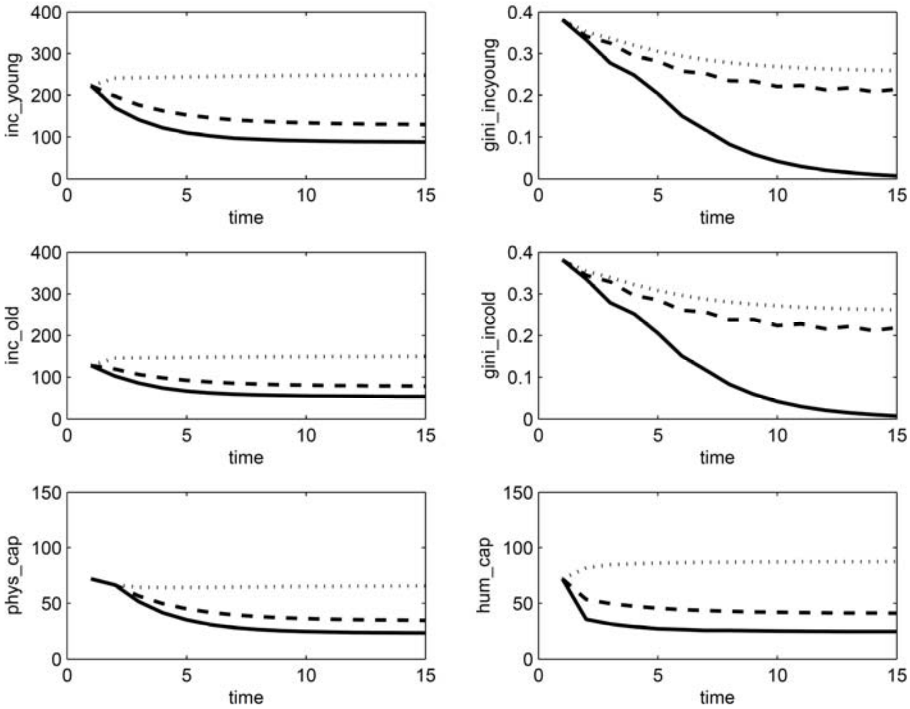

Figure 7. Unequal Access to the Educational and Financial Systems
($G = 2$: solid lines; $G = 3$: dashed lines; $G = 4$: dotted lines)

As in the previous section, we increase the degree of inequality in the access to those institutions and assume now that $k^f = 0.8\overline{x_t k_t}$ and $k^e = 0.8\overline{x_t k_t}$. As can be seen in Figure 8, there is no income convergence any longer for any level of the development of the educational system. Comparing Figures 6 to 8, we see that aggregate income and capital holdings decrease with higher inequality in the access to institutions, because some resources are not used by poor individuals under unequal access.

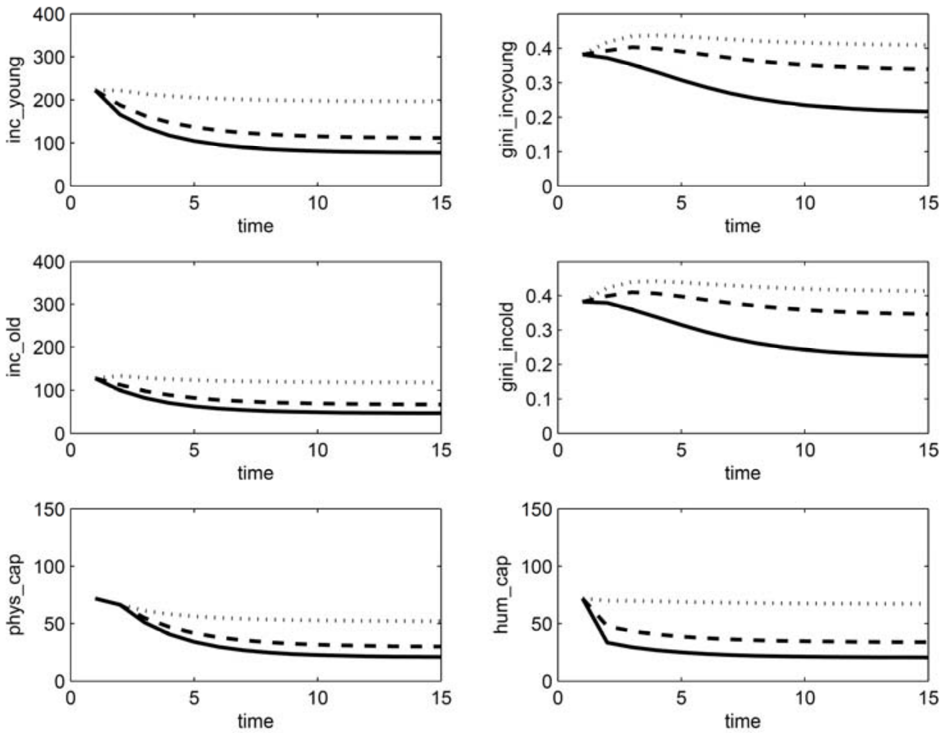

Figure 8. Increased unequal Access to the Educational and Financial Systems ($G = 2$: solid lines; $G = 3$: dashed lines; $G = 4$: dotted lines)

7. CONCLUSION

This paper examines the relationship between institutions, income and inequality within a simple model. The analysis focuses on financial and educational institutions. While both types of institutions foster income growth, the impacts on income inequality are different. Since access to private loans is a function of the underlying collateral, which is higher for rich individuals, better financial institutions make holdings of physical capital more pronounced. In contrast, making the public component of human capital accumulation more important reduces inequality. Besides aggregate measures of institutional quality, this paper also considers heterogenous access to these institutions. Unequal access to financial and educational institutions leads to lower income and higher inequality. For certain parameter values, however, inequality is not persistent, if there is equal access to one of these two institutions. This may be one possible explanation why we observe persistent inequality. It also shows that economic analysis should take into account both aggregate measures of institutional quality and individual access to these institutions.

APPENDIX

Proofs of Propositions in Section 3

Proposition 1: If $0 < \theta < 1$ *, the human capital stock of every individual will converge to the same level, irrespective of the initial distribution of human capital.*

Proof. The growth rate of human capital $(g_t^h)^j$ is given by

$$(g_t^h)^j = \frac{h_{t+1}^j}{h_t^j} = \frac{G^{1-\theta}(h_t^j n_t^j)^\theta}{h_t^j}.$$

Use (10) to substitute for n_t^j to get:

$$(g_t^h)^j = \frac{h_{t+1}^j}{h_t^j} = \frac{G^{1-\theta}}{h_t^j}\left(\frac{\alpha_3\theta(wh_t^j + rk_t^j)}{w(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5\theta)}\right)^\theta.$$

Take the derivative with respect to h_t^j :

$$\frac{\partial(g_t^h)^j}{\partial h_t^j} = \frac{G^{1-\theta}B}{(h_t^j)^2}(wh_t + rk_t)^{\theta-1}(wh_t^j(\theta-1) - rk_t^j) < 0\,, \tag{23}$$

where $B = \left(\frac{\alpha_3\theta}{w(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5\theta)}\right)^\theta$.

This expression is negative if $0 < \theta < 1$. Since we also have $\frac{\partial(g_t^h)^j}{\partial(h_t^j)^2} > 0$, the human capital holdings of the individuals converge to the same level.

Proposition 2: Irrespective of the initial distribution of physical capital, the stock of every individual's physical capital will converge to the same level.

Proof. Take the growth rate of physical capital, $(g_t^k)^j = \frac{k_{t+1}^j}{k_t^j} = x_t^j(1+\lambda_t)$, use (11) to substitute for x_t^j and take the first derivative with respect to k_t^j.

$$\frac{\partial (g_t^k)^j}{\partial k_t^j} = \frac{-Awh_t^j(1+\lambda_t)}{(k_t^j)^2} < 0\,, \tag{24}$$

where $A = \dfrac{\alpha_4}{r(r+\mu)(\alpha_1+\alpha_4+\alpha_2+\alpha_3\theta)}$.

Since $A > 0$ and $\dfrac{\partial^2 (g_t^k)^j}{\partial (k_t^j)^2} > 0$, the accumulation of capital is bounded, which implies that the physical capital stock of the individuals will converge to the same level of physical capital holdings.

Proposition 3: The income of every individual will converge to the same level.

Proof. Since physical and human capital are the only factors of production in the economy, this follows directly from Propositions 1 and 2.

Proposition 4: The development of educational and financial institutions increases the growth rates of the individuals' incomes and the two capital stocks. Eventually, the economy reaches a higher stationary level for these variables with institutional development.

Proof. Take the derivatives $\dfrac{\partial (g_t^h)^j}{\partial G}$ and $\dfrac{\partial (g_t^k)^j}{\partial \mu}$ and show that the growth rates are higher under institutional development. For human capital, we get:

$$\frac{\partial (g_t^h)^j}{\partial G} = (1-\theta)\frac{G^{-\theta}}{h_t^j}\left(\frac{\alpha_3\theta(wh_t^j + rk_t^j)}{w(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5\theta)}\right)^{\theta} > 0\,. \tag{25}$$

For physical capital, we obtain:

$$\begin{aligned}\frac{\partial (g_t^k)^j}{\partial \mu} &= \frac{A(wh_t^j + rk_t^j)}{k_t^j}\left(\frac{\partial \lambda_t}{\partial \mu} - \frac{(1+\lambda_t)}{r+\mu}\right) \\ &= \frac{A(wh_t^j + rk_t^j)}{k_t^j}\left(\frac{\mu^2 r(\alpha_2+\alpha_4) + \alpha_2 r^2\mu + r(2\mu r(\alpha_2+\alpha_4) + \alpha_2 r^2) - \alpha_4}{\alpha_4(r+\mu)}\right) > 0.\end{aligned} \tag{26}$$

The last condition holds if $r > 1$ and $\alpha_2 > \alpha_4$.

Proposition 5: An improvement of financial institutions increases income inequality *at any date t.*

Proof. Investment by an individual depends negatively on aggregate financial development. For any given levels of k_t^j and h_t^j, this can be seen by taking the derivative of $x_t^j k_t^j$ with respect to μ:

$$\frac{\partial x_t^j k_t^j}{\partial \mu} = \frac{-A_t(rk_t^j + wh_t^j)}{(r + E_t\{\lambda_t\})} \frac{\partial E_t\{\lambda_t\}}{\partial \mu} < 0 . \tag{27}$$

Thus, when aggregate financial efficiency improves, individuals will decrease private investments because they get more loans. As can be seen from (27), this effect is stronger for individuals with higher capital holdings. Noting that $\frac{\partial n_t^j h_t^j}{\partial \mu} = 0$, it follows that income given by $(1 - n_t^j)h_t^j + (1 - x_t^j)k_t^j$ increases more for individuals with high capital holdings, which increases income inequality.

Proposition 6: An improvement of educational institutions leads to less income inequality at any date t.

Proof. This can be seen by taking the derivative of $n_t^j h_t^j$ with respect to *G* holding h_t^j and k_t^j constant:

$$\frac{\partial n_t^j h_t^j}{\partial G} = 0 . \tag{28}$$

Proposition 7: Both types of institutions affect the accumulation of both physical and human capital. Hence, the development of educational or financial institutions does not only impact on the type of capital it directly affects, but also on the accumulation of the other type of capital.

Proof. The amount an individual invests on human and physical capital are given by:

$$n_t^j h_t^j = \frac{\alpha_3 \theta (wh_t^j + rk_t^j)}{w(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_3\theta)} ,$$

$$x_t^j k_t^j = \frac{\alpha_4 (wh_t^j + rk_t^j)}{r(r + E_t\{\lambda_t\})(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_3\theta)} .$$

These investments depend on total potential factor income of the individuals. An increase in h_t^j or k_t^j due to institutional development will therefore increase investment in both types of capital.

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