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EDUCATIONAL IMPLICATIONS OF SCHOOL SYSTEMS AT DIFFERENT STAGES OF SCHOOLING

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In educating students national public school systems use different methods of grouping students by ability across schools. We consider four different school systems of student allocation at different stages of schooling and their educational implications. Our two-period model suggests that both frequency and sequence of ability grouping play an important role in producing educational implications. As different households prefer different combinations of school systems, the overall performance of a school system is determined by how households are distributed over income and a child's ability and the voting of households.

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1. INTRODUCTION

In educating students national public school systems around the globe use different methods of grouping (or ungrouping) students by ability across schools. Some countries such as Germany, pre-1960s U.K., China and Singapore adopt selective systems based on school entrance exams to place students to schools; other countries such as Scandinavian nations, contemporary U.K., U.S. and South Korea employ comprehensive systems based on residential school districts to allocate students.

Within the group of countries that rely on selective systems, the starting point of ability grouping across schools varies in the procession of schooling, although most countries adopt mixed-ability schooling in an early part of primary education. For example, Germany first divides different-ability children into different school tracks (*Gymnasium*, *Realschule* and *Hauptschule*) at age 10; prior to school reforms in 1960s U.K. placed children to different types of schools according to ability (grammar and

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secondary modern schools) at age 11; China and Singapore send primary-school graduates to ability-stratified secondary schools at age 13; Japan employs similar grouping at age 16. In contrast, such ability grouping across schools does not exist in countries that in principle adopt comprehensive systems of public primary and secondary education, although ability-tracking within mixed-ability schools is exercised in some cases (e.g., U.S.; rare in South Korea).

Despite the heated debates for last decades as to whether to group students by ability and from when to group them, educational experts and parents have yet to reach consensus on these issues. Proponents of comprehensive systems argue that the major objective of a school system is to provide equal educational opportunities to all students irrespective of family and social backgrounds. They contend that by giving students rights to attend any school in the neighborhood, education can serve as a method for narrowing educational inequality (Jenkins *et al.* (forthcoming), Leschinsky and Mayer (1990), Oakes (1985)). In contrast, advocates of selective systems argue that selective schooling maximizes educational outcomes because it is easier for teachers to instruct groups of students with a low variance of ability and for students to learn with similar-ability students in school (Gamoran (1986), Lazear (2001)). They also believe that a selective system is more fair since student allocation is based upon ability alone, not upon family background.[1]

In contrast to grown interests in school systems of student allocation, economic studies of education are rare that directly deal with educational implications of comprehensive and selective school systems. Even more scarce are studies that examine an optimal timing and sequence of ability grouping across schools in terms of efficient and equitable educational production.[2]

Nevertheless, there exist at least four papers that are directly related to school systems by exploring the effects of ability grouping and mixing in education. A selective school system concerns ability grouping across schools; a comprehensive system concerns ability mixing across schools. Benabou (1996) examines implications of ability grouping and mixing of students, arguing that ability mixing tends to slow down the short-run growth but raise the long-run growth. Fernandez and Gali (1999) compare the relative performance of markets and tournaments—ability grouping by a test—in an economy with borrowing constraints. They show that tournaments dominate markets in

[1] Further details on comprehensive and selective schooling and their effects on education are well surveyed by Gamoran and Mare (1989), Ireson and Hallam (1999), Kerckhoff (1986) and Slavin (1987, 1990) among others.

[2] This situation is a little surprising given that such issues are under continuing discussions in many countries, and often more emphasized outside North America than school resource effectiveness (e.g., Hanushek and Woessmann (2006)). Recent research in economics of education that shows the presence of positive and nonlinear peer effects in primary and secondary education also underscores the importance of different methods of student grouping in educational production (e.g., Ding and Lehrer (2007), Kang (2007)).

terms of matching efficiency. Lazear (2001) investigates how educational outputs are affected by grouping methods of different-quality students to show that the total educational output is maximized when students are grouped according to quality. Finally, Hur and Kang (2007) rely on a simple economic model for households' choices on consumption and educational investments to explore varying educational implications of comprehensive and selective school systems for efficiency and equity of education. All these studies are, however, completely silent about when it is optimal to start ability grouping across schools to achieve educational goals of a society.[3]

For want of relevant studies, the current paper examines two issues that are of primary concern in the construction of school systems: whether and when to group students by ability across schools. Given that there are in general two stages of public schooling--primary and secondary levels--that produce the ultimate educational outcome, we investigate whether it is desirable to adopt ability grouping and when it is optimal to start (or not start) such grouping in order to achieve often-conflicting educational goals of efficiency and equity.[4] To this end, we extend Hur and Kang's (2007) one-period model to the case where the decisions are made in two periods reflecting two stages of public education before college. In the analysis each of comprehensive and selective systems is characterized by the role of a student's own quality prior to school entrance and the parent's choice of residential district in the determination of quality of school peers. We then compare efficiency and equity of educational outcomes across different school systems by examining educational expenditures, the role of family income and student ability in the determination of the educational outcome, and the variance and level of such an outcome.

Our two-period model of school systems gives rise to several points that have failed to emerge in previous discussions on school systems. Above all, it suggests that not only the frequency of ability grouping in two stages of schooling but its *sequence* also matter in drawing educational implications of school systems. Given that there are two methods of grouping students (comprehensive and selective) and two stages of schooling (primary and secondary), we have four different school systems in educating students: (1) the comprehensive-comprehensive (CC) system in which the comprehensive system is employed in stage 1 (e.g., primary level) and stage 2 (e.g., secondary level) alike; (2) the selective-selective (SS) system in which the selective system is adopted in both

[3] See Hur and Kang (2007) for further theoretical discussions on these four papers and empirical findings about the effects of comprehensive and selective systems on efficiency and equity of education.

[4] In this paper our primary concern is about how to allocate students across schools, not across classes within a school given that the students are already placed to the school. For issues and impacts of within-school ability grouping and mixing on education, see, e.g., Argys *et al*. (1996), Betts and Shkolnik (2000), Figlio and Page (2002), Gamoran (1987), and Slavin (1990). In addition, we focus exclusively on public school systems, ignoring the presence of private schools. For issues concerning the role of ability grouping in the relationship between public and private schools, see, e.g., Epple *et al*. (2002).

stages; (3) the comprehensive-selective (CS) system in which the comprehensive (selective) system is used in stage 1 (stage 2); (4) the selective-comprehensive (SC) system in which the selective (comprehensive) system is used in stage 1 (stage 2). The first three systems (i.e., CC, SS and CS) are often observed in existing educational institutions. However, the SC system is hard to find. Below we show that the SC system at times reveals more desirable educational implications than the CC and CS systems. Thus the sequence of ability grouping can make differences in achieving educational goals.

Second, we find that different households that are characterized by a pair of family income and the child's innate ability prefer different combinations of school systems. For example, households in which the income is high but the child's ability is low tend to like CC but dislike SS; households in which the income is high and the child's ability is also high tend to like CS but dislike SC; and so on. Thus the overall performance of a school system is determined by how households are distributed over income and child ability and how they prefer each school system. Which sequence of school systems is adopted in a nation's education system will be ultimately determined by voting of households.

Third, if the entrance exam in the selective system tests a student's endowed ability alone but never her nurtured ability, then the systems that adopt the selective method in the later stage of education show more favorable educational implications than if the exam tests a student's endowed ability as well as her nurtured ability. Whether or not the entrance exam should signal the endowed ability alone in the selective system will also be determined by voting of households, because different households prefer different settings of the entrance exam in the selective system.

The rest of the paper proceeds as follows: section 2 extends Hur and Kang's (2007) one-period model to a two-period case in which comprehensive and selective systems are combined with different sequences. In section 3 we discuss analytical results of the four school systems in terms of efficiency and equity of education. Section 4 introduces a new setting of the entrance exam in the selective system and examines its educational implications under the four systems. Section 5 concludes the paper.

2. THE MODEL

Let us first set up a utility function of a household that consists of a parent and one child:

$$U = \ln U_1 + \rho \ln U_2, \tag{1}$$

where $U_t = x_t^{\alpha} A_t = x_t^{\alpha}(A_{t-1}\theta_t^{\beta} e_t^{\gamma})$, $\alpha + \beta + \gamma = 1$, $0 < \alpha, \beta, \gamma, \rho < 1$, and $t = 1, 2$.[5]

Here the household's total utility (U) is expressed by a sum (in natural log) of period-1 utility (U_1) and period-2 utility (U_2) that is discounted by a factor ρ. A period-t utility is determined by the amount of private goods consumption (x_t) and the child's educational outcome (A_t) at period t. The educational outcome at t is produced by the child's educational outcome in the previous period (A_{t-1}), the average quality of peers in school (θ_t), and the amount of private educational services (e_t) that can be purchased in a form of private tutoring in the market.[6] The child's time-invariant ability endowed from birth is denoted by A_0.

Now we consider two major school systems that exist in public primary and secondary education: comprehensive and selective school systems. At each stage of schooling, the comprehensive system is characterized by school districts based on residential location. In this system parents choose a school district and their children are placed into one school within the school district. In contrast, the selective system is characterized by entrance tests. In this system students take an entrance exam and are admitted to a school according to their ranking in the exam.

In our analysis of comprehensive systems, we suppose, for simplicity, that student placement into a school is random within a school district, while parents can choose the school district by moving across school districts.[7] We also assume that there is a fixed number of school districts to serve the entire population of students, and that each district is identical in terms of the number of schools and school resources such as class

[5] The current Cobb-Douglas form of utility and educational production functions plays an important role in drawing differences in educational implications across different school systems in this paper. However, the adoption of such a functional form is not unique to the current study. They are often employed by the economics of education literature (e.g., Epple *et al*. (2002), Epple and Romano (1998), Ferreyra (2007), Lazear (2001), Nechyba (2000)). See Hur and Kang (2007) for further discussions on alternative functional forms.

[6] To focus exclusively on the impacts of student allocation across schools, we assume that every public school spends an equal amount of resources for a student, setting monetary resource allocation across schools and districts aside. For issues related to school financing and the distribution of the resources, see, e.g., Fernandez and Rogerson (2003), and Hoxby (2001).

[7] Under comprehensive systems, there are largely two methods of student placement within a school district. The first is to assign each student randomly to one school within the district. This method has been employed in South Korea since 1969 to allocate students to middle schools and general (or non-vocational) high schools within a school district. See OECD (1998, Chapters 1 and 2) for details. The second is to assign each student to one school within a district in consideration of the preferences of the student and parent. Such a method is more common in comprehensive systems than random assignment. In our analysis, however, we assume random assignment within a district for conceptual sharpness.

size, teacher quality, etc. Under the comprehensive system, the quality of school peers (θ_t) is exogenously given to individual students once a school district is decided: within a district, students face the same average quality of peers in any school due to randomization. Nonetheless, parents may choose the quality of school peers indirectly by moving across school districts. The children's quality of a school district is reflected in house prices (Black (1999), Gibbons and Machin (2003)). Let us denote the children's average quality of a school district by d_t and its unit price by p_{dt}. Parents spend a total of $p_{dt}d_t$ to purchase a house in a certain school district; a high house price in a good school district is expressed by a high value of d_t, hence $p_{dt}d_t$. In the model, for simplicity again, we suppose that there are no commuting costs between the house and school, and no changes in living environment over time.

In contrast to comprehensive systems, the choice of a school district does not matter in the determination of school peer quality under selective systems, because schools give admissions to students based on the entrance exam ranking alone, not on the school district. Note that the total number of schools and school resources remain unchanged compared with those of the comprehensive system; only school districts disappear in the selective system. In this system parents choose the quality of school peers solely by raising the entrance exam ranking of the child; the quality of school peers (θ_t) is determined by the child's own quality at the end of the previous period (A_{t-1}). That is, the level of θ_1 is decided by that of A_0 —the endowed quality—and the level of θ_2 by that of A_1 —the endowed plus nurtured quality just prior to period 2.[8]

To characterize different systems of student allocation to schools at t, we introduce the following model for θ_t:

$$\theta_t = A_{t-1}^{\tau_t} d_t^{\delta_t} \quad \text{for} \quad t = 1, 2. \tag{2}$$

The comprehensive system is characterized by $\tau_t = 0$ and $\delta_t = 1$, and the selective system by $\tau_t = 1$ and $\delta_t = 0$. δ_t is equal to one in the comprehensive system that employs within-district randomization, because θ_t, the average peer quality in a school, is ultimately determined by d_t, the average quality of children in a school district.[9] For

[8] In section 4 we analyze educational implications of selective school systems where the entrance exam tests a student's endowed ability A_0 alone (not A_{t-1}) at each stage of schooling, and compare them with those obtained in the current setting of the entrance exam. An interesting result is that inequality of the educational outcome under selective systems is reduced in the system relying on A_0 relative to the system relying on A_{t-1}. Details are presented in section 4.

[9] If a within-district allocation of students is not random, then δ_t may deviate from one.

the selective system we assume that $\tau_t = 1$ for simplicity of analysis. Namely, in the selective system the average quality of school peers is equal to a student's own quality. If either $\tau_t > 1$ or $0 < \tau_t < 1$, then student allocation is infeasible given a pool of students to be assigned to schools.[10]

In sum, a household's utility function at t is denoted by

$$U_t = x_t^{\alpha} A_t = x_t^{\alpha} \{A_{t-1}(A_{t-1}^{\tau_t} d_t^{\delta_t})^{\beta} e_t^{\gamma}\} = x_t^{\alpha} (A_{t-1}^{1+\tau_t \beta} d_t^{\delta_t \beta} e_t^{\gamma}) . \tag{4}$$

And the total utility is given by:

$$\begin{aligned} U = {} & \alpha[\ln x_1 + \rho \ln x_2] + (1 + \rho + \rho\beta\tau_2)[(1 + \beta\tau_1) \ln A_0 + \beta\delta_1 \ln d_1 + \gamma \ln e_1] \\ & + \rho[\beta\delta_2 \ln d_2 + \gamma \ln e_2]. \end{aligned} \tag{5}$$

Now consider the household's budget constraint:

$$x_1 + p_{d1} d_1 + p_{e1} e_1 + \frac{1}{1+g}(p_{x2} x_2 + p_{d2} d_2 + p_{e2} e_2) \le y_1 + \frac{y_2}{1+g} \equiv y_T , \tag{6}$$

where y_t is the post-tax disposable income of the household at t, y_T is its total present-value disposable income, and g is the interest rate of the private credit market. The price of x_1 is assumed to be one for normalization. All the prices are determined in their own perfectly competitive markets. Subject to the budget constraint, the household,

[10] The assumption that $\tau_t = 1$ in the selective system holds strictly only in a very restrictive condition that all students in a given school have the same value of A_{t-1}. For illustration, suppose that there are a total of 100 students to be placed to a total of 10 schools in the selective system. Suppose also that the best 10 students score 10 in the exam that determines A_{t-1}; the second best 10 students score 9; the third best 10 students score 8; and so on. In such a case the best school (say, school 1) is attended by the best 10 students with an identical score 10; the second best school (say, school 2) by the second best 10 students with score 9; and so on. Each school is attended by students with the same value of A_{t-1}. In this case alone θ_t is exactly equal to A_{t-1}, hence $\tau_t = 1$; the entire functional relationship between θ_t and A_{t-1} can be described by a discrete step function whereby the value of θ_t jumps up by 1 as A_{t-1} rises by 1. In more realistic cases where individual students have different (continuous) scores of A_{t-1}, the exact relationship between θ_t and A_{t-1} will also be a similar step function; the difference, however, is that within a given school θ_t falls as A_{t-1} rises, because θ_t is determined by the average value of A_{t-1} among the schoolmates excluding the self. There is a jump in θ_t across different schools as A_{t-1} rises. Such a step function can be approximated by a linear function with slope 1, i.e., $\theta_t = A_{t-1}$. We use such an approximation to characterize the selective system.

which is uniquely characterized by a pre-determined set of A_0, y_1 and y_2, chooses the total utility maximizing levels of x_1, x_2, e_1 and e_2 together with those of d_1 and d_2 the choices of which depend on the school system adopted.

The optimal quantities of the choice variables, the educational outcome and indirect utilities are obtained as following:

$$x_1^* = \left(\frac{\alpha}{(1+\rho)\alpha + (1+\rho+\rho\beta\tau_2)(\beta\delta_1+\gamma) + \rho(\beta\delta_2+\gamma)} \right) \cdot y_T , \tag{7}$$

$$d_1^* = \left(\frac{(1+\rho+\rho\beta\tau_2)\cdot\beta\delta_1}{(1+\rho)\alpha + (1+\rho+\rho\beta\tau_2)(\beta\delta_1+\gamma) + \rho(\beta\delta_2+\gamma)} \right) \cdot \frac{y_T}{p_{d1}} , \tag{8}$$

$$e_1^* = \left(\frac{(1+\rho+\rho\beta\tau_2)\cdot\gamma}{(1+\rho)\alpha + (1+\rho+\rho\beta\tau_2)(\beta\delta_1+\gamma) + \rho(\beta\delta_2+\gamma)} \right) \cdot \frac{y_T}{p_{e1}} , \tag{9}$$

$$x_2^* = \left(\frac{\rho\alpha}{(1+\rho)\alpha + (1+\rho+\rho\beta\tau_2)(\beta\delta_1+\gamma) + \rho(\beta\delta_2+\gamma)} \right) \cdot \frac{(1+g)y_T}{p_{x2}} , \tag{10}$$

$$d_2^* = \left(\frac{\rho\beta\delta_2}{(1+\rho)\alpha + (1+\rho+\rho\beta\tau_2)(\beta\delta_1+\gamma) + \rho(\beta\delta_2+\gamma)} \right) \cdot \frac{(1+g)y_T}{p_{d2}} , \tag{11}$$

$$e_2^* = \left(\frac{\rho\gamma}{(1+\rho)\alpha + (1+\rho+\rho\beta\tau_2)(\beta\delta_1+\gamma) + \rho(\beta\delta_2+\gamma)} \right) \cdot \frac{(1+g)y_T}{p_{e2}} , \tag{12}$$

$$A_t^* = (A_{t-1}^*)^{1+\tau_t\beta} (d_t^*)^{\delta_t\beta} (e_t^*)^{\gamma}, \quad (t=1,2) , \tag{13}$$

$$U_t^* = (x_t^*)^{\alpha} A_t^*, \quad (t=1,2) , \tag{14}$$

$$U^* = \ln U_1^* + \rho \ln U_2^* . \tag{15}$$

Hereafter, we will drop the asterisks of the variables, unless the optimized value may be confused with the variable itself.

3. SCHOOL SYSTEMS AND EDUCATIONAL IMPLICATIONS

Now we consider four different school systems of student allocation at different stages of schooling (e.g., primary and secondary education): the comprehensive-comprehensive (CC) system, the selective-selective (SS) system, the comprehensive-selective (CS) system and the selective-comprehensive (SC) system, as defined previously. From the discussions in section 2, the CC system is characterized by $\tau_1 = \tau_2 = 0$ and $\delta_1 = \delta_2 = 1$; the SS system by $\tau_1 = \tau_2 = 1$ and $\delta_1 = \delta_2 = 0$; the CS system by $\tau_1 = 0$, $\tau_2 = 1$, $\delta_1 = 1$ and $\delta_2 = 0$; the SC system by $\tau_1 = 1$, $\tau_2 = 0$, $\delta_1 = 0$ and $\delta_2 = 1$.

Given such characterizations of different school systems, we employ four criteria to

compare educational implications across different systems. These criteria include the consumption levels (x_t), educational expenditures ($E_t \equiv p_{dt}d_t + p_{et}e_t$), and the average level and variance of the final educational outcome (A_2), because they are often examined in theoretical and empirical discussions on school systems (see Hur and Kang (2007) for details). Comparisons based on utilities are theoretically feasible but empirically difficult. Nonetheless, unreported comparisons based on the total utility (U^*) produce qualitatively similar results to those of the current comparisons based on A_2^*. The size of educational expenditure is included as a criterion for a comparison between different school systems, because excessive private tutoring that reduces household consumptions is believed to exist for primary and secondary school students in some Asian countries (Baker *et al.* (2001), Bray and Kwok (2003), Time Asia (2006)), and Hur and Kang (2007) show that school systems are responsible for the presence of such excess tutoring. Table 1 summarizes the values of the comparison measures for each school system.

First, the level of a household's optimal consumption varies by school systems. From Table 1 it is easy to show that $x_1^{SS} > x_1^{SC} > x_1^{CS} > x_1^{CC}$ and $x_2^{SS} > x_2^{SC} > x_2^{CS} > x_2^{CC}$. In both periods 1 and 2 consumption levels are largest in SS and smallest in CC. Consumption levels in SC are larger than those in CS, while both being intermediate between SS and CC. Such patterns of household consumptions are closely related to patterns of educational expenditures in different school systems.

Proposition 1. (Educational Expenditures) Let E_1^k be the optimal educational expenditure of a household with y_T $(\equiv y_1 + \frac{y_2}{1+g})$ at period 1 in school system k (i.e., $E_1 \equiv p_{d1}d_1 + p_{e1}e_1$). Also let E_2^k be the optimal (present-value) educational expenditure of the household at period 2 in school system k (i.e., $E_2 \equiv \frac{1}{1+g}(p_{d2}d_2 + p_{e2}e_2)$). Let E_T^k be the total educational expenditure in school system k as a sum of E_1^k and E_2^k. Then we have the following rankings for the expenditures.

(i) If $\gamma \geq \frac{(1+\rho)^2\alpha}{\rho(\alpha+\rho)}$, then $E_1^{CS} > E_1^{SS} \geq E_1^{CC} > E_1^{SC}$;

if $\gamma < \frac{(1+\rho)^2\alpha}{\rho(\alpha+\rho)}$, then $E_1^{CS} > E_1^{CC} > E_1^{SS} > E_1^{SC}$.

(ii) $E_2^{SC} > E_2^{CC} > E_2^{SS} > E_2^{CS}$.

(iii) $E_T^{CC} > E_T^{CS} > E_T^{SC} > E_T^{SS}$.

Table 1. Optimal Levels of Consumption and Educational Expenditures and the Final Educational Outcome under Different School Systems

School system	Consumption goods, x_1	Consumption goods, x_2
CC	$x_1^{CC} = \left(\frac{\alpha}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)} \right) \cdot y_T$	$x_2^{CC} = \left(\frac{\rho\alpha}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)} \right) \cdot \frac{(1+g)y_T}{p_{x2}}$
SS	$x_1^{SS} = \left(\frac{\alpha}{(1+\rho)\alpha + (1+2\rho+\rho\beta)\gamma} \right) \cdot y_T$	$x_2^{SS} = \left(\frac{\rho\alpha}{(1+\rho)\alpha + (1+2\rho+\rho\beta)\gamma} \right) \cdot \frac{(1+g)y_T}{p_{x2}}$
CS	$x_1^{CS} = \left(\frac{\alpha}{(1+\rho)\alpha + (1+\rho+\rho\beta)(\beta+\gamma) + \rho\gamma} \right) \cdot y_T$	$x_2^{CS} = \left(\frac{\rho\alpha}{(1+\rho)\alpha + (1+\rho+\rho\beta)(\beta+\gamma) + \rho\gamma} \right) \times \frac{(1+g)y_T}{p_{x2}}$
SC	$x_1^{SC} = \left(\frac{\alpha}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma} \right) \cdot y_T$	$x_2^{SC} = \left(\frac{\rho\alpha}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma} \right) \cdot \frac{(1+g)y_T}{p_{x2}}$
	Educational expenditure, $E_1 \equiv p_{d1}d_1 + p_{e1}e_1$	Educational expenditure, $E_2 \equiv \frac{1}{1+g}(p_{d2}d_2 + p_{e2}e_2)$
CC	$E_1^{CC} = \left(\frac{(1+\rho)(\beta+\gamma)}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)} \right) \cdot y_T$	$E_2^{CC} = \left(\frac{\rho(\beta+\gamma)}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)} \right) \cdot y_T$
SS	$E_1^{SS} = \left(\frac{(1+\rho+\rho\beta)\gamma}{(1+\rho)\alpha + (1+2\rho+\rho\beta)\gamma} \right) \cdot y_T$	$E_2^{SS} = \left(\frac{\rho\gamma}{(1+\rho)\alpha + (1+2\rho+\rho\beta)\gamma} \right) \cdot y_T$
CS	$E_1^{CS} = \left(\frac{(1+\rho+\rho\beta)(\beta+\gamma)}{(1+\rho)\alpha + (1+\rho+\rho\beta)(\beta+\gamma) + \rho\gamma} \right) \cdot y_T$	$E_2^{CS} = \left(\frac{\rho\gamma}{(1+\rho)\alpha + (1+\rho+\rho\beta)(\beta+\gamma) + \rho\gamma} \right) \cdot y_T$
SC	$E_1^{SC} = \left(\frac{(1+\rho)\gamma}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma} \right) \cdot y_T$	$E_2^{SC} = \left(\frac{\rho(\beta+\gamma)}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma} \right) \cdot y_T$
	Total educational expenditure, $E_T \equiv E_1 + E_2$	

CC	$E_T^{CC} = \left(\dfrac{(1+2\rho)(\beta+\gamma)}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)} \right) \cdot y_T$
SS	$E_T^{SS} = \left(\dfrac{(1+2\rho+\rho\beta)\gamma}{(1+\rho)\alpha + (1+2\rho+\rho\beta)\gamma} \right) \cdot y_T$
CS	$E_T^{CS} = \left(\dfrac{(1+\rho+\rho\beta)(\beta+\gamma)+\rho\gamma}{(1+\rho)\alpha + (1+\rho+\rho\beta)(\beta+\gamma)+\rho\gamma} \right) \cdot y_T$
SC	$E_T^{SC} = \left(\dfrac{\rho\beta + (1+2\rho)\gamma}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma} \right) \cdot y_T$
	Final educational outcome, A_2
CC	$A_2^{CC} = \Gamma_{CC} \cdot A_0 y_T^{2(\beta+\gamma)}$, $\Gamma_{CC} = [\rho(1+\rho)(1+g)]^{\beta+\gamma} [p_{d1}p_{d2}]^{-\beta} [p_{e1}p_{e2}]^{-\gamma} \beta^{2\beta}\gamma^{2\gamma} \times [(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)]^{-2(\beta+\gamma)}$
SS	$A_2^{SS} = \Gamma_{SS} \cdot A_0^{(1+\beta)^2} y_T^{\gamma(2+\beta)}$, $\Gamma_{SS} = [\rho(1+g)]^{\gamma} [p_{e1}]^{-\gamma(1+\beta)} [p_{e2}]^{-\gamma} [(1+\rho+\rho\beta)]^{\gamma(1+\beta)} \gamma^{\gamma(2+\beta)} \times [(1+\rho)\alpha + (1+2\rho+\rho\beta)\gamma]^{-\gamma(2+\beta)}$
CS	$A_2^{CS} = \Gamma_{CS} \cdot A_0^{(1+\beta)} y_T^{(\beta+\gamma)(1+\beta)+\gamma}$, $\Gamma_{CS} = [\rho(1+g)]^{\gamma} [p_{d1}^{\beta} p_{e1}^{\gamma}]^{-(1+\beta)} [p_{e2}]^{-\gamma} (1+\rho+\rho\beta)^{(\beta+\gamma)(1+\beta)} \times \beta^{\beta(1+\beta)} \gamma^{\gamma(2+\beta)} \times [(1+\rho)\alpha + (1+\rho+\rho\beta)(\beta+\gamma) + \rho\gamma]^{-(\beta+\gamma)(1+\beta)-\gamma}$
SC	$A_2^{SC} = \Gamma_{SC} \cdot A_0^{(1+\beta)} y_T^{\beta+2\gamma}$, $\Gamma_{SC} = (1+\rho)^{\gamma} [\rho(1+g)]^{\beta+\gamma} [p_{d2}]^{-\beta} [p_{e1}p_{e2}]^{-\gamma} \beta^{\beta}\gamma^{2\gamma} \times [(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma]^{-(\beta+2\gamma)}$

Note: CC denotes the comprehensive system in both periods 1 and 2; SS the selective system in both periods; CS the comprehensive system in period 1 and the selective system in period 2; SC the selective system in period 1 and the comprehensive system in period 2.

Proof.

(i) Comparisons between two different E_1^k's are given as following:

$E_1^{CC} - E_1^{SS} = \beta[(1+\rho)^2\alpha - \gamma\rho(\alpha+\rho)] \times const. \leq (>) 0$ if $\gamma \geq (<) \frac{(1+\rho)^2\alpha}{\rho(\alpha+\rho)}$;

$E_1^{CC} - E_1^{CS} = -\rho[\beta(1+2\rho)(1-\alpha) + 2\gamma(1+\rho)] \times const. < 0$;

$E_1^{CC} - E_1^{SC} = \beta(1+\rho) \times const. > 0$;

$E_1^{SS} - E_1^{CS} = -\beta[(1+\rho)\alpha + \rho\gamma] \times const. < 0$;

$E_1^{SS} - E_1^{SC} = \rho\beta[1+2\rho+\alpha] \times const. > 0$;

$E_1^{CS} - E_1^{SC} = (1+\rho)\beta[1+\rho+\rho(\beta+2\gamma)] \times const. > 0$.

Proposition (i) is trivial.

(ii) Comparisons between two different E_2^k's are given as following:

$E_2^{CC} - E_2^{SS} = \beta[(1+\rho)\alpha + \rho(\beta+\gamma)\gamma] \times const. > 0$;

$E_2^{CC} - E_2^{CS} = \beta[(1+\rho)\alpha + \{1+\rho+\rho(\beta+\gamma)\}(\beta+\gamma)] \times const. > 0$;

$E_2^{CC} - E_2^{SC} = -(1+\rho)\beta \times const. < 0$;

$E_2^{SS} - E_2^{CS} = (1+\rho+\rho\beta)\beta \times const. > 0$;

$E_2^{SS} - E_2^{SC} = -\beta[(1+\rho)\alpha + \{1+\rho(2-\alpha)\}\gamma] \times const. < 0$;

$E_2^{CS} - E_2^{SC} = -[\beta\{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma)\} + \beta(\beta+\gamma)\{1+\rho(\beta+\gamma)\}] \times const. < 0$.

Proposition (ii) is trivial.

(iii) Comparisons between two different E_T^k's are given as following:

$E_T^{CC} - E_T^{SS} = (1+\rho)\alpha\beta[1+\rho(2-\gamma)] \times const. > 0$;

$E_T^{CC} - E_T^{CS} = \rho(1+\rho)\alpha^2\beta \times const. > 0$;

$E_T^{CC} - E_T^{SC} = (1+\rho)^2\alpha\beta \times const. > 0$;

$E_T^{SS} - E_T^{CS} = -(1+\rho)(1+\rho+\rho\beta)\alpha\beta \times const. < 0$;

$E_T^{SS} - E_T^{SC} = -\rho(1+\rho)\alpha\beta(1-\gamma) \times const. < 0$;

$E_T^{CS} - E_T^{SC} = (1+\rho)(1+\rho\beta+\rho\gamma)\alpha\beta \times const. > 0$.

Proposition (iii) is trivial. **QED**

Irrespective of the values of α and γ, a household's educational expenditure at period 1 is the largest in CS and the smallest in SC. It is primarily because in CS parents overspend on education at period 1 to equip the children for the entrance exam at period 2. Such a need, however, does not exist at period 1 in SC. As a consequence of this pattern of educational spending at period 1, the household's goods consumption level at period 1 is lower in CS than in SC. In contrast, the relative size of the household's educational expenditures in CC and SS depends on the values of α and γ. If γ is

greater than a threshold $\left(\frac{1+\rho)^2\alpha}{\rho(\alpha+\rho)}\right)$, that is, if private tutoring at period 1 is relatively effective in preparing children for the entrance exam at period 2, then parents spend more on education at period 1 in SS than in CC. If γ is less than the threshold—private tutoring at period 1 is less effective for the entrance exam at period 2, however, parents spend less on education at period 1 in SS than in CC. In contrast to varying rankings of the two systems in educational spending, the household's consumption level at period 1 is always greater in SS than in CC.

As for a household's educational expenditure at period 2, different rankings emerge. The educational expenditure at period 2 is the largest in SC and the smallest in CS. Such a ranking is the exact reverse of that for the period-1 educational expenditure. In addition, the period-2 educational expenditure is greater in CC than in SS. One notable pattern of the period-2 educational expenditure is that educational spending is in general smaller under the selective system of period 2 (SS and CS) than under the comprehensive system (SC and CC). Since parents are not able to purchase the quality of children's school peers in the selective system at period 2 given the children's period-1 performance (A_1), they spend less for education and consume more in the selective system than in the comprehensive system.

Given the current predictions on educational spending, it would be illuminating to see if our theoretical rankings of educational expenditures in general agree with empirical regularities. For this, we rely on differences in patterns of private tutoring that is prominently observed in East Asian countries—South Korea and Japan in particular. Parents in both countries share a similar cultural background and degree of zeal for education, while letting their children subject to school systems that are slightly different at the secondary level of education. At the primary (grades 1 to 6) and lower secondary (grades 7 to 9; middle school) levels of schooling, both countries adopt a comprehensive system based on school districts. In contrast, at the upper secondary level (grades 10 to 12; high school), South Korea continues to employ a comprehensive system but Japan switches to a selective system based on school entrance exams. In South Korea that adopts CC in our terms, private tutoring is widespread over all stages of education up to the upper secondary level; in Japan that adopts a system relatively close to CS, however, private tutoring is pronounced only at the primary and lower secondary levels, while relatively less widespread at the upper secondary level (Bray (1999)). Direct comparisons of the values of tutoring expenditures at each level of education between the two countries may not offer much insight, however, because each country may have unique educational markets for school districts and private educational services. Nonetheless, the overall patterns of private tutoring at different stages of schooling remain informative to highlight the effects of school systems on educational expenditures.

To the extent that the ranking and absolute size of educational spending may reverse across two periods, a comparison of total (present-value) expenditures between different systems may be of greater interest to households and policy-makers. Proposition 1.(iii)

shows that the total expenditure is the largest in CC and the smallest in SS; in the middle, the expenditure is larger in CS than in SC. Given that parents are not permitted to purchase the quality of children's school peers in selective systems, it is little surprising that total educational spending is much smaller in SS than in CC. Notable, however, is that such spending is larger in CS than in SC. Not only does the frequency of ability grouping across schools matter to the patterns of educational spending, but the *sequence* also makes differences in them. Since parents spend more on education at period 1 in CS to prepare the upcoming entrance exam than they economize at period 2 in SC, the total expenditure is larger in CS than in SC. Such differences in spending patterns across school systems in turn give rise to differences in the role of family income (and student endowed ability) in determining the final educational outcome (A_2).

Proposition 2. (Gradients of A_2) Let η_{A0}^k be the gradient of A_2 with respect to A_0 , a child's endowed ability, in school system k (i.e., $\eta_{A0}^k \equiv \frac{\partial \ln A_2}{\partial \ln A_0}$). And let η_y^k be the gradient of A_2 with respect to y_T , a parent's total income, in school system k (i.e., $\eta_y^k \equiv \frac{\partial \ln A_2}{\partial \ln y_T}$). Then we have the following rankings.

(i) $\eta_{A0}^{SS} > \eta_{A0}^{CS} = \eta_{A0}^{SC} > \eta_{A0}^{CC}$

(ii) $\eta_y^{CC} = 2(\beta + \gamma)$

Proof.

From Table 1, $\eta_{A0}^{CC} = 1$, $\eta_{A0}^{SS} = (1+\beta)^2$, $\eta_{A0}^{CS} = 1+\beta$ and $\eta_{A0}^{SC} = 1+\beta$; $\eta_y^{CC} = 2(\beta+\gamma)$, $\eta_y^{SS} = \gamma(2+\beta)$, $\eta_y^{CS} = (\beta+\gamma)(1+\beta)+\gamma$ and $\eta_y^{SC} = \beta + 2\gamma$.

Propositions (i) and (ii) are trivial. **QED**

To the extent that a child's endowed ability matters in school peer selection under the selective system alone, it is easy to see that the child-ability gradient η_{A0} is the largest in SS, which groups similar-quality students in one school twice at periods 1 and 2, and the smallest in CC, which never exercises such grouping at both periods. In CC, the initial difference in endowed ability among children is reflected proportionally without inflation (or deflation) into that in the final outcome A_2 . In SS, however, the initial difference in ability widens the difference in A_2 via peer selection in school. The values of η_{A0} are equal between CS and SC, and are at an intermediate level, because CS and SC group students across schools only once and a child's endowed ability does not vary over time.

Echoing the patterns of total educational spending in different school systems, the parental-income gradient η_y is the largest in CC but the smallest in SS. To the extent

that a child's endowed ability does not function in peer selection in comprehensive systems, the role of the parent's background in the determination of the educational output is the strongest in CC but the weakest in SS. Similar to the comparison of E_T between CS and SC, the parental-income gradient η_y is larger in CS than in SC, while it is at an intermediate level in both systems. Again, both frequency and sequence of ability mixing across schools matter to the role of the parent's background in the determination of the final output, because in CS (but not in SC) educational investments at period 1 may change the quality of school peers at period 2, which in turn improves a student's educational outcome.

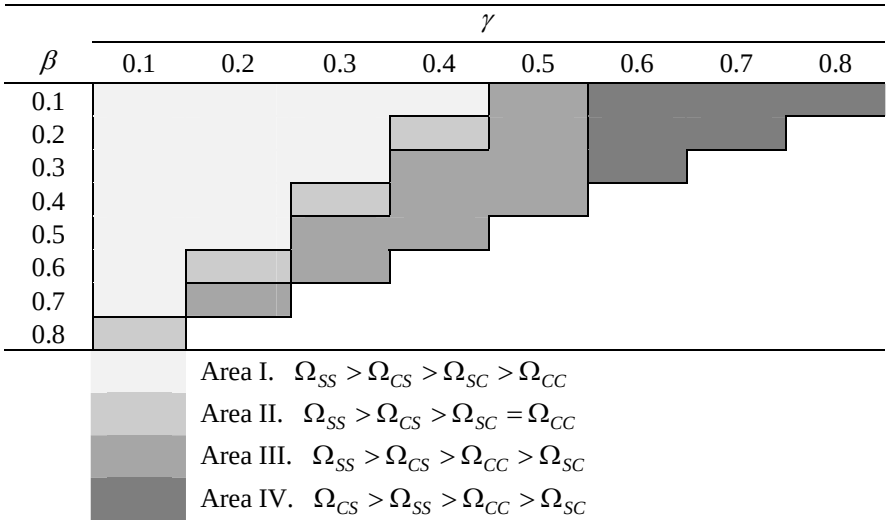
Our theoretical prediction that the parental-income gradient is largest in CC but smallest in SS is in fact in sharp contrast to the argument that comprehensive schooling functions to minimize the effects of family backgrounds on children. To the extent there exist private tutoring markets outside the p school system, rich parents may exp ca ir c en fun th of family b chi y led to emerg na sc a e deb we examin predictions, a rare empirical study by Galindo-Rueda and Vignoles (2005) investigating the impacts of student ability on ational attainments across different secondary school systems in the U.K. agrees wit r theory; abilit a stronger influence on the outcome in a selective system (CS in our model) than in a comprehensive system (CC). Empirical evidence on the cha the child-ability gradient due to the changes between other school systems, how ound.

In contrast to the evidence on the changes in the che che chl-c(um)16(enpc(um)16 5Gg)-501 Tc77hs4(e)-5

In accordance with the theoretical predictions, rare empirical studies that examine the impacts of different school systems on educational inequality show inconclusive findings. Using international data sets, Gorard and Smith (2004) and Hanushek and Woessmann (2006) present that a selective system in secondary education (CS in our model) exacerbates the inequality in educational outcomes relative to a comprehensive system (CC); in a study on ability tracking in the U.S. secondary schools, however, Figlio and Page (2002) show tracking helps low-ability students and reduces the inequality of academic performance, suggesting $V(\ln A_2)$ decreases as the school system switches from CC to CS.

Although we do not know definitely which system yields what level of variance in the final outcome, we can at least attempt to draw a rough picture of inequality in different school systems by substituting appropriate values for $V(\ln A_0)$, $V(\ln y_T)$ and $Cov(\ln A_0, \ln y_T)$. Below we assign values to constituents of $V(\ln A_2)$ that are plausible from previous empirical research. A value of one is assigned to each of $V(\ln A_0)$ and $V(\ln y_T)$ for normalization, and 0.4 is given to $Cov(\ln A_0, \ln y_T)$, since studies on intergenerational income and educational mobility suggest that $Cov(\ln A_0, \ln y_T)$ is around 0.4 (Solon (1992), Zimmerman (1992), Epple and Romano (1998)). Given such values, the rankings of $V(\ln A_2)$ between different school systems are illustrated in Table 2 for varying pairs of β and γ. There are four unique areas in the table. Area I is pairs of β and γ for which $V(\ln A_2)$ is ranked by $\Omega_{SS} > \Omega_{CS} > \Omega_{SC} > \Omega_{CC}$; area II is those for which $\Omega_{SS} > \Omega_{CS} > \Omega_{SC} = \Omega_{CC}$; area III is those for which $\Omega_{SS} > \Omega_{CS} > \Omega_{CC} > \Omega_{SC}$; area IV is those for which $\Omega_{CS} > \Omega_{SS} > \Omega_{CC} > \Omega_{SC}$.

Table 2. Rankings of the Variance in $\ln A_2$ for Different Pairs of β and γ



Two patterns are noteworthy in Table 2 for the given values of $V(\ln A_0)$, $V(\ln y_T)$ and $Cov(\ln A_0, \ln y_T)$. First, for all feasible pairs of β and γ, $V(\ln A_2)$ is larger in the systems that employ the selective method in period 2 (SS and CS) than in those that adopt the comprehensive method in period 2 (CC and SC). This suggests that the comprehensive method in the later stages of schooling is effective in mitigating the inequality of the educational outcome. Second, for a given value of β below 0.4, the ranking of Ω_{SS} and Ω_{CS} reverses as γ rises above 0.5; for a given value of β, the ranking of Ω_{SC} and Ω_{CC} reverses as γ rises. They suggest that as the role of monetary expenditures (γ) increases in educational production, the comprehensive method in the early stage of schooling yields a higher variance in the final outcome than the selective method; the variance-reducing function of the comprehensive system weakens as γ rises, because family income widens the difference in the outcome under comprehensive systems due to the increasing effectiveness of private tutoring. Yet the first pattern that the variance is smaller under the comprehensive system at period 2 than under the selective system holds for all feasible pairs of β and γ.

Previous research suggests that β is approximately between 0.2 and 0.4 for elementary and secondary school students (Hanushek *et al.* (2003), Hoxby (2000), Kang (2007)) and that γ is between 0.1 and 0.2 (Card and Krueger (1996, p.37)). Therefore, for the current given values of $V(\ln A_0)$, $V(\ln y_T)$ and $Cov(\ln A_0, \ln y_T)$, the most plausible rankings would be shown by $\Omega_{SS} > \Omega_{CS} > \Omega_{SC} > \Omega_{CC}$.

Proposition 4. (Size of A_2)

(i) The ranking of A_2 between any two systems is not uniquely determined. Either $\frac{A_2^k}{A_2^l} \geq 1$ or $\frac{A_2^k}{A_2^l} < 1$ for any pairs of k and l $(k \neq l; k, l = CC, SS, CS, SC)$. Each ranking depends on a household's A_0 and y_T, values of parameters and prices given in each school system.

(ii) Each ratio of $\frac{A_2^{CC}}{A_2^{SS}}$, $\frac{A_2^{CC}}{A_2^{CS}}$, $\frac{A_2^{CC}}{A_2^{SC}}$, $\frac{A_2^{CS}}{A_2^{SS}}$, and $\frac{A_2^{SC}}{A_2^{SS}}$ for a given household is negatively related to A_0 but positively related to y_T. In contrast, the value of $\frac{A_2^{CS}}{A_2^{SC}}$ is not related to A_0 but it is positively related to y_T.

Proof. From Table 1,

$$\frac{A_2^{CC}}{A_2^{SS}} = \frac{\Gamma_{CC}}{\Gamma_{SS}} \cdot A_0^{-\beta(2+\beta)} y_T^{\beta(2-\gamma)}; \quad \frac{A_2^{CC}}{A_2^{CS}} = \frac{\Gamma_{CC}}{\Gamma_{CS}} \cdot A_0^{-\beta} y_T^{\beta(1-\beta-\gamma)}; \quad \frac{A_2^{CC}}{A_2^{SC}} = \frac{\Gamma_{CC}}{\Gamma_{SC}} \cdot A_0^{-\beta} y_T^{\beta};$$

$$\frac{A_2^{CS}}{A_2^{SS}} = \frac{\Gamma_{CS}}{\Gamma_{SS}} \cdot A_0^{-\beta(1+\beta)} y_T^{\beta(1+\beta)}\ ;\ \frac{A_2^{SC}}{A_2^{SS}} = \frac{\Gamma_{SC}}{\Gamma_{SS}} \cdot A_0^{-\beta(1+\beta)} y_T^{\beta(1-\gamma)}\ ;\ \frac{A_2^{CS}}{A_2^{SC}} = \frac{\Gamma_{CS}}{\Gamma_{SC}} \cdot y_T^{\beta(\beta+\gamma)}\ .$$

Since each of the constants Γ_{CC}, Γ_{SS}, Γ_{CS} and Γ_{SC} contains prices (p_{d1}, p_{d2}, p_{e1} and p_{e2}), its value is indeterminate without further information on the markets in different school systems. Hence comes proposition (i). Proposition (ii) are trivial given the preceding formulas. **QED**

Proposition 4.(i) suggests that it is difficult a priori to decide which sequence of school systems yields greater educational outputs. This is in contrast to previous studies that usually support selective systems as an efficient method of student allocation to schools (e.g., Fernandez and Gali (1999), Lazear (2001)). Focusing on technological aspects of educational production in consideration of peer effects, these studies in general ignore the possibility that parents may adjust their optimal choices of consumption and educational investments in response to different grouping methods in schooling. If parents optimally respond to ways in which students are assigned across schools, a pool of students to be grouped in a school may be subject to change when different systems of student allocation are adopted in different stages of schooling (e.g., Epple *et al* (2002)). Our model addresses such a possibility via prices (p_{d1}, p_{d2}, p_{e1} and p_{e2}) at stages of schooling. Because sorting of students into schools varies by school systems, the ranking of the ultimate educational outcome mediated through peer effects is indeterminate between any two systems. We can only infer that high prices of d and e lead to poor educational outcomes.

In line with such theoretical predictions, empirical studies report inconclusive evidence on the impact of school systems on overall educational outcomes. While Kang *et al.* (2007) show that a transition from a selective (CS) to a comprehensive (CC) system in secondary education significantly increased the average educational outcome (i.e., adulthood earnings) in South Korea, Hanushek and Woessmann (2006) find that school systems fail to yield significant differences in the average outcome (i.e., test score in secondary school), using international data sets.[11]

[11] Our theoretical model casts doubt on the capability of transnational comparisons based on cross-sectional data to reveal the true strength of ability grouping (as opposed to ability mixing) in raising the level of educational outputs. Because each country possesses unique educational markets for school districts and private educational services, the prices are likely to differ across countries. Even if parents' preferences are identical, the educational output may vary across countries solely by the differences in the prices, not by the method of ability grouping in school. Hanushek and Woessmann (2006) attempt to overcome such a problem by applying difference-in-difference methods that use educational outputs of both primary and secondary schooling in a country. An implicit assumption of that paper is that there are no substantial differences in markets for school districts and private educational services between primary and secondary education in each country. A before-after comparison for a single country such as Kang *et al.* (2007) may also be a valid empirical strategy; a corresponding assumption is that there are no changes in the educational markets in the vicinity of the exogenous policy change.

Proposition 4.(ii) implies that households characterized by different combinations of (A_0, y_1, y_2) prefer different pairs of school systems. Namely, the rankings depend on the values of A_0 and y_T. In the absence of detail information on the parameter values and prices, however, it is difficult to know a priori which pairs of school systems are liked or disliked by what types of households characterized by (A_0, y_T). In order to draw a rough picture of the pairs of school systems preferred by different households, as in the case of variance comparisons, we employ a simulation method by substituting plausible values for parameters and prices.

The set-up of the simulation is as follows: A value of 0.4 is assigned to β, because peer effects literature suggests values between 0.2 and 0.4 for the effect of average peer quality on a student's achievement (Hanushek *et al.* (2003), Hoxby (2000), Kang (2007)); a value of 0.1 is allocated to γ, because Card and Krueger (1996, p.37) summarize that a 10 percent increase in public school spending leads to about a 1-2 percent increase in subsequent earnings; thus α is equal to 0.5; we set all of the prices p_{d1}, p_{d2}, p_{e1} and p_{e2} equal to one, as we have no useful information on them and wish to avoid arbitrary price differences; finally, we set $g = 0.05$ and $\rho = 0.95$.

Figure 1 shows six lines each of which represents the households that are indifferent between two school systems in terms of A_2.[12] Line I represents the households that are indifferent between CC and SS; Line II those indifferent between CC and CS; Line III those indifferent between CC and SC; Line IV those indifferent between SS and CS; Line V those indifferent between SS and SC; Line VI those indifferent between CS and SC. The six lines divide the entire region by a total of 17 unique areas that are individually numbered from (1) to (17). Table 3 reports the orderings of A_2 among four different school systems in a given area. For example, $A_2^{CC} > A_2^{CS} > A_2^{SC} > A_2^{SS}$ for households whose (A_0, y_T) is located in area (1), etc.

From the orderings of A_2 in the 17 areas, CC is most preferred by the households in areas (1) and (9), but least preferred by those in areas (6), (7), (8), (12), (15) and (17); SS is most preferred by those in areas (8), (16) and (17), but least preferred by those in areas (1), (2), (5), (9), (10) and (11); CS is most preferred by those in areas (2), (3), (4), (5), (6) and (7), but least preferred by those in areas (13), (14) and (16); SC is most preferred by those in areas (10), (11), (12), (13), (14) and (15), but least preferred by those in areas (3) and (4).

[12] In addition to comparisons based on A_2, we can employ the total indirect utility (i.e., $U^* = \ln U_1^* + \rho \ln U_2^*$) as a criterion of such comparisons. The results based on the utility, however, produce qualitatively similar pictures given here. Such results are available upon request.

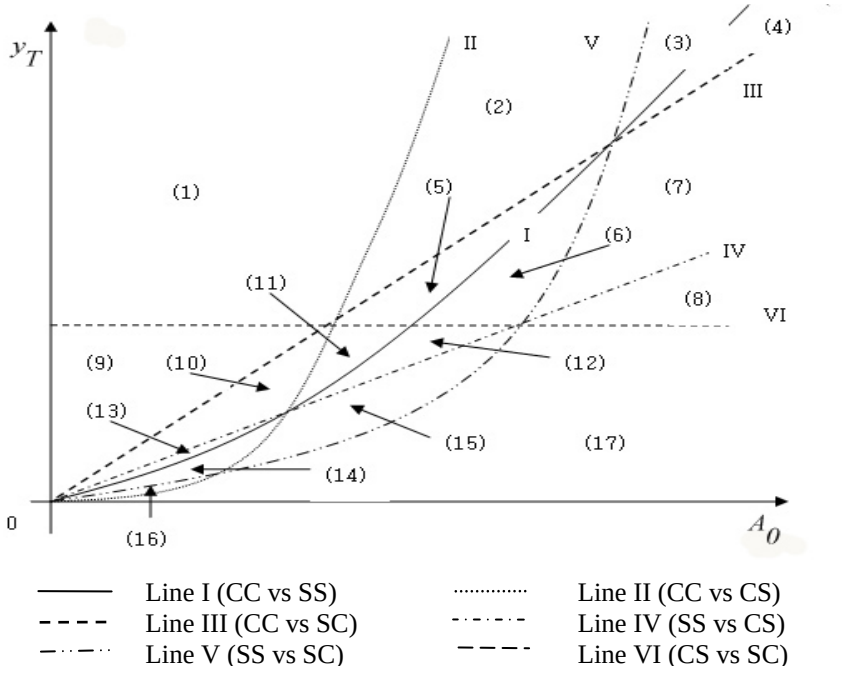


Figure 1. Combinations of A_0 and y_T that Yield Highest A_2

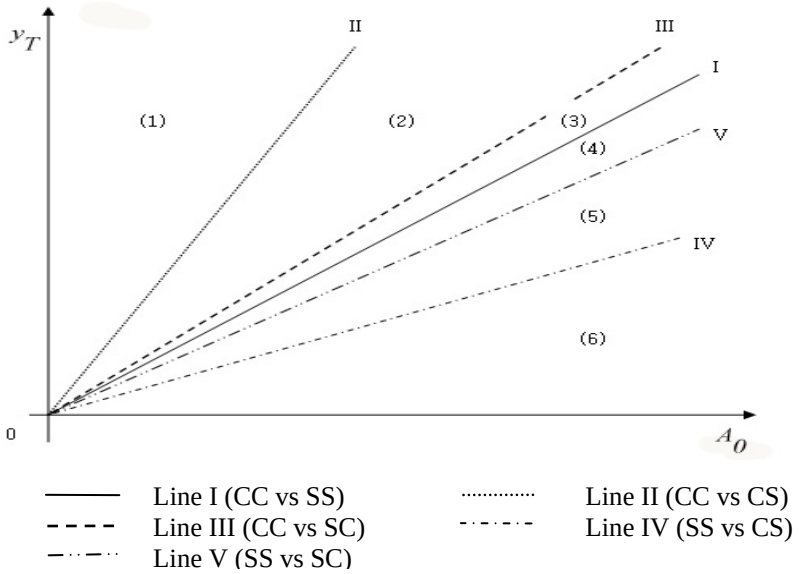


Figure 2. Combinations of A_0 and y_T that Yield Highest $\overline{A}_2$

Table 3. Rankings of A_2 and $\overline{A}_2$ in Different Areas of Figures 1 and 2

Area	Ordering in Figure 1	Area	Ordering in Figure 2
(1)	$A_2^{CC} > A_2^{CS} > A_2^{SC} > A_2^{SS}$	(1)	$\overline{A}_2^{CC} > \overline{A}_2^{CS} > \overline{A}_2^{SC} > \overline{A}_2^{SS}$
(2)	$A_2^{CS} > A_2^{CC} > A_2^{SC} > A_2^{SS}$	(2)	$\overline{A}_2^{CS} > \overline{A}_2^{CC} > \overline{A}_2^{SC} > \overline{A}_2^{SS}$
(3)	$A_2^{CS} > A_2^{CC} > A_2^{SS} > A_2^{SC}$	(3)	$\overline{A}_2^{CS} > \overline{A}_2^{SC} > \overline{A}_2^{CC} > \overline{A}_2^{SS}$
(4)	$A_2^{CS} > A_2^{SS} > A_2^{CC} > A_2^{SC}$	(4)	$\overline{A}_2^{CS} > \overline{A}_2^{SC} > \overline{A}_2^{SS} > \overline{A}_2^{CC}$
(5)	$A_2^{CS} > A_2^{SC} > A_2^{CC} > A_2^{SS}$	(5)	$\overline{A}_2^{CS} > \overline{A}_2^{SS} > \overline{A}_2^{SC} > \overline{A}_2^{CC}$
(6)	$A_2^{CS} > A_2^{SC} > A_2^{SS} > A_2^{CC}$	(6)	$\overline{A}_2^{SS} > \overline{A}_2^{CS} > \overline{A}_2^{SC} > \overline{A}_2^{CC}$
(7)	$A_2^{CS} > A_2^{SS} > A_2^{SC} > A_2^{CC}$		
(8)	$A_2^{SS} > A_2^{CS} > A_2^{SC} > A_2^{CC}$		
(9)	$A_2^{CC} > A_2^{SC} > A_2^{CS} > A_2^{SS}$		
(10)	$A_2^{SC} > A_2^{CC} > A_2^{CS} > A_2^{SS}$		
(11)	$A_2^{SC} > A_2^{CS} > A_2^{CC} > A_2^{SS}$		
(12)	$A_2^{SC} > A_2^{CS} > A_2^{SS} > A_2^{CC}$		
(13)	$A_2^{SC} > A_2^{CC} > A_2^{SS} > A_2^{CS}$		
(14)	$A_2^{SC} > A_2^{SS} > A_2^{CC} > A_2^{CS}$		
(15)	$A_2^{SC} > A_2^{SS} > A_2^{CS} > A_2^{CC}$		
(16)	$A_2^{SS} > A_2^{SC} > A_2^{CC} > A_2^{CS}$		
(17)	$A_2^{SS} > A_2^{SC} > A_2^{CS} > A_2^{CC}$		

Households with low A_0 and high y_T tend to like CC but dislike SS; households with high A_0 and low y_T tend to like SS but dislike CC; households with high A_0 and high y_T tend to like CS but dislike SC; households with low A_0 and low y_T tend to like SC but dislike CS. In sum, the overall performance of a school system does not only depend on the frequency and sequence of ability grouping across schools. It is also determined by how households are distributed over A_0 and y_T and how they prefer each school system. Thus which sequence of school systems is adopted in a nation's education system will be ultimately determined by voting of households. Our analysis sheds light on decisions of households and policy-makers by offering comparisons about educational implications that can be drawn from four different sequences of school systems.

Our analysis also suggests that there always exist conflicts in interests surrounding the choice of school systems, unless policy-makers offer different combinations of school systems in different regions so that households self-select into the most preferred

school system. To the extent that policy-makers have to decide over a limited set of school systems in a nation, a nation's school system will ultimately be determined by how they distribute weights to each of educational criteria (e.g., consumption level, the inequality and overall level of the educational outcome, etc) in order to achieve educational goals. The presence of different school systems, as observed in different nations around the globe, will reflect transnational differences in such weights given to each of a variety of educational targets.

4. EDUCATIONAL IMPLICATIONS WHEN THE ENTRANCE EXAMS SIGNAL A STUDENT'S ENDOWED ABILITY ALONE

In the preceding sections we suppose that the entrance exams in selective systems test academic quality of students just prior to the school placement. Namely, the entrance exam for period-1 schooling tests the level of A_0, while the exam for period-2 schooling tests the level of A_1. Such exams are, however, one particular type of entrance exams; one can imagine a different setting of entrance exams. For example, an exam can test a student's endowed ability (A_0) alone, but not her nurtured ability. An IQ test seems relatively close to such a test, although to what extent an IQ test measures a person's endowed ability remains controversial (Neisser *et al*. (1996)). Ferdinandez and Gali (1999) and Hur and Kang (2007) suggest that how strongly the entrance exam in the selective system signals a student's endowed ability relative to her nurtured ability matters for educational implications of comprehensive and selective systems. In this section, we extend the ideas of Ferdinandez and Gali (1999) and Hur and Kang (2007) to school systems of different stages of schooling by employing an (hypothetical) entrance exam that signals a student's endowed ability alone (but not her nurtured ability) for school admission. We modify the model for θ_t by replacing A_{t-1} with A_0, as follows:

$$\theta_t = A_0^{\tau_t} d_t^{\delta_t}, \quad (t = 1, 2). \tag{16}$$

As discussed in section 3, given within-district randomization in comprehensive systems, CC is characterized by $\tau_1 = \tau_2 = 0$ and $\delta_1 = \delta_2 = 1$; SS by $\tau_1 = \tau_2 = 1$ and $\delta_1 = \delta_2 = 0$; CS by $\tau_1 = 0$, $\tau_2 = 1$, $\delta_1 = 1$ and $\delta_2 = 0$; SC by $\tau_1 = 1$, $\tau_2 = 0$, $\delta_1 = 0$ and $\delta_2 = 1$. Note that the entrance exam of the period-1 selective system remains the same as earlier, and that only the entrance exam of the period-2 selective system has changed. Thus a household's choices under CS and SS alone are expected to be affected, while those under CC and SC remain unaffected.

A household's new utility function at t is expressed by:

$$U_t = x_t^{\alpha} A_t = x_t^{\alpha} \{A_{t-1} (A_0^{\tau_t} d_t^{\delta_t})^{\beta} e_t^{\gamma}\} = x_t^{\alpha} (A_0^{\tau_t \beta} A_{t-1} d_t^{\delta_t \beta} e_t^{\gamma}). \tag{17}$$

Table 4. Optimal Levels of Consumption and Educational Expenditures and the Final Educational Outcome When the Entrance Exam Signals Endowed Ability Alone

School system	Consumption goods, $\bar{x}_1$	Consumption goods, $\bar{x}_2$
CC	$\bar{x}_1^{CC} = \left(\frac{\alpha}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)}\right) \cdot y_T$	$\bar{x}_2^{CC} = \left(\frac{\rho\alpha}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)}\right) \cdot \frac{(1+g)y_T}{p_{x2}}$
SS	$\bar{x}_1^{SS} = \left(\frac{\alpha}{(1+\rho)\alpha + (1+2\rho)\gamma}\right) \cdot y_T$	$\bar{x}_2^{SS} = \left(\frac{\rho\alpha}{(1+\rho)\alpha + (1+2\rho)\gamma}\right) \cdot \frac{(1+g)y_T}{p_{x2}}$
CS	$\bar{x}_1^{CS} = \left(\frac{\alpha}{(1+\rho)(\alpha+\beta) + (1+2\rho)\gamma}\right) \cdot y_T$	$\bar{x}_2^{CS} = \left(\frac{\rho\alpha}{(1+\rho)(\alpha+\beta) + (1+2\rho)\gamma}\right) \frac{(1+g)y_T}{p_{x2}}$
SC	$\bar{x}_1^{SC} = \left(\frac{\alpha}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma}\right) \cdot y_T$	$\bar{x}_2^{SC} = \left(\frac{\rho\alpha}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma}\right) \cdot \frac{(1+g)y_T}{p_{x2}}$
	Educational expenditure, $\bar{E}_1 \equiv p_{d1}\bar{d}_1 + p_{e1}\bar{e}_1$	Educational expenditure, $\bar{E}_2 \equiv \frac{1}{1+g}(p_{d2}\bar{d}_2 + p_{e2}\bar{e}_2)$
CC	$\bar{E}_1^{CC} = \left(\frac{(1+\rho)(\beta+\gamma)}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)}\right) \cdot y_T$	$\bar{E}_2^{CC} = \left(\frac{\rho(\beta+\gamma)}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)}\right) \cdot y_T$
SS	$\bar{E}_1^{SS} = \left(\frac{(1+\rho)\gamma}{(1+\rho)\alpha + (1+2\rho)\gamma}\right) \cdot y_T$	$\bar{E}_2^{SS} = \left(\frac{\rho\gamma}{(1+\rho)\alpha + (1+2\rho)\gamma}\right) \cdot y_T$
CS	$\bar{E}_1^{CS} = \left(\frac{(1+\rho)(\beta+\gamma)}{(1+\rho)(\alpha+\beta) + (1+2\rho)\gamma}\right) \cdot y_T$	$\bar{E}_2^{CS} = \left(\frac{\rho\gamma}{(1+\rho)(\alpha+\beta) + (1+2\rho)\gamma}\right) \cdot y_T$
SC	$\bar{E}_1^{SC} = \left(\frac{(1+\rho)\gamma}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma}\right) \cdot y_T$	$\bar{E}_2^{SC} = \left(\frac{\rho(\beta+\gamma)}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma}\right) \cdot y_T$

	Total educational expenditure, $\overline{E}_T \equiv \overline{E}_1 + \overline{E}_2$
CC	$\overline{E}_T^{CC} = \left(\dfrac{(1+2\rho)(\beta+\gamma)}{(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)} \right) \cdot y_T$
SS	$\overline{E}_T^{SS} = \left(\dfrac{(1+2\rho)\gamma}{(1+\rho)\alpha + (1+2\rho)\gamma} \right) \cdot y_T$
CS	$\overline{E}_T^{CS} = \left(\dfrac{(1+\rho)\beta + (1+2\rho)\gamma}{(1+\rho)(\alpha+\beta) + (1+2\rho)\gamma} \right) \cdot y_T$
SC	$\overline{E}_T^{SC} = \left(\dfrac{\rho\beta + (1+2\rho)\gamma}{(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma} \right) \cdot y_T$
	Final educational outcome, $\overline{A}_2$
CC	$\overline{A}_2^{CC} = \Phi_{CC} \cdot A_0 y_T^{2(\beta+\gamma)}$, $\Phi_{CC} = [\rho(1+\rho)(1+g)]^{\beta+\gamma} [p_{d1} p_{d2}]^{-\beta} [p_{e1} p_{e2}]^{-\gamma} \beta^{2\beta} \gamma^{2\gamma} \times [(1+\rho)\alpha + (1+2\rho)(\beta+\gamma)]^{-2(\beta+\gamma)}$
SS	$\overline{A}_2^{SS} = \Phi_{SS} \cdot A_0^{1+2\beta} y_T^{2\gamma}$, $\Phi_{SS} = [\rho(1+\rho)(1+g)]^{\gamma} [p_{e1} p_{e2}]^{-\gamma} \gamma^{2\gamma} [(1+\rho)\alpha + (1+2\rho)\gamma]^{-2\gamma}$
CS	$\overline{A}_2^{CS} = \Phi_{CS} \cdot A_0^{1+\beta} y_T^{\beta+2\gamma}$, $\Phi_{CS} = (1+\rho)^{\beta+\gamma} [\rho(1+g)]^{\gamma} [p_{d1}]^{-\beta} [p_{e1} p_{e2}]^{-\gamma} \beta^{\beta} \gamma^{2\gamma} \times [(1+\rho)(\alpha+\beta) + (1+2\rho)\gamma]^{-(\beta+2\gamma)}$
SC	$\overline{A}_2^{SC} = \Phi_{SC} \cdot A_0^{1+\beta} y_T^{\beta+2\gamma}$, $\Phi_{SC} = (1+\rho)^{\gamma} [\rho(1+g)]^{\beta+\gamma} [p_{d2}]^{-\beta} [p_{e1} p_{e2}]^{-\gamma} \beta^{\beta} \gamma^{2\gamma} \times [(1+\rho)\alpha + \rho\beta + (1+2\rho)\gamma]^{-(\beta+2\gamma)}$

Note: CC denotes the comprehensive system in both periods 1 and 2; SS the selective system in both periods; CS the comprehensive system in period 1 and the selective system in period 2; SC the selective system in period 1 and the comprehensive system in period 2.

Table 4 summarizes the optimal consumption level ($\bar{x}_t$), educational expenditure ($\bar{E}_t$), and the final educational outcome ($\bar{A}_2$) for each school system given the new utility function. To distinguish from those in section 3, all the optimal values of the variables in this section are over-lined.

From Table 4 it is easy to show that $\bar{x}_1^{SS} > \bar{x}_1^{SC} > \bar{x}_1^{CS} > \bar{x}_1^{CC}$ and $\bar{x}_2^{SS} > \bar{x}_2^{SC} > \bar{x}_2^{CS} > \bar{x}_2^{CC}$, which mirror the rankings in consumption levels in section 3. In both periods 1 and 2 the consumption levels are largest in SS and smallest in CC. The levels are larger in SC than those in CS, while both levels are intermediate between SS and CC.

It is also noteworthy that while $\bar{x}_t^{CC} = x_t^{CC}$ and $\bar{x}_t^{SC} = x_t^{SC}$, $\bar{x}_t^{SS} > x_t^{SS}$ and $\bar{x}_t^{CS} > x_t^{CS}$ for each $t\,(=1,2)$. If the entrance exam at period 2 tests the level of A_0 alone, but not A_1, the consumption levels at both periods rise in SS and CS, which employ the selective method at period 2, while they do not change in CC and SC, which employ the comprehensive method at period 2. Since over-investments for A_1 in period 1 do not lead to the change in school peer quality in period 2 even under SS and CS, parents spend less on education and consume more in the new institutional setting than in the old setting where the entrance exam tests the level of A_1. This can be also seen in patterns of educational expenditures that follow.

Proposition 5. (Educational Expenditures)[13]

(i) $\bar{E}_1^{CS} > \bar{E}_1^{SS} > \bar{E}_1^{CC} > \bar{E}_1^{SC}$.

(ii) $\bar{E}_2^{SC} > \bar{E}_2^{CC} > \bar{E}_2^{SS} > \bar{E}_2^{CS}$.

(iii) $\bar{E}_T^{CC} > \bar{E}_T^{CS} > \bar{E}_T^{SC} > \bar{E}_T^{SS}$.

(iv) $\bar{E}_1^{CC} = E_1^{CC}$; $\bar{E}_1^{SC} = E_1^{SC}$; $\bar{E}_1^{SS} < E_1^{SS}$; $\bar{E}_1^{CS} < E_1^{CS}$

(v) $\bar{E}_2^{CC} = E_2^{CC}$; $\bar{E}_2^{SC} = E_2^{SC}$; $\bar{E}_2^{SS} > E_2^{SS}$; $\bar{E}_2^{CS} > E_2^{CS}$

(vi) $\bar{E}_T^{CC} = E_T^{CC}$; $\bar{E}_T^{SC} = E_T^{SC}$; $\bar{E}_T^{SS} < E_T^{SS}$; $\bar{E}_T^{CS} < E_T^{CS}$

If the entrance exam in the period-2 selective system tests A_0 alone, patterns of educational spending across school systems remain similar to those in proposition 1. A household's educational expenditure at period 1 is the largest in CS and the smallest in SC. In contrast to proposition 1.(i), the expenditure at period 1 is always lower in CC than in SS. The educational expenditure at period 2 is the largest in SC and the smallest in CS; the period-2 expenditure in CC are greater than that in SS, while both being at an intermediate level. The total expenditure is the largest in CC and the smallest in SS; in

[13] Proofs of propositions 5 to 8 are suppressed but available upon request.

the middle, it is larger in CS than in SC.

Propositions 5.(iv) to 5.(vi) suggest that once the entrance exam in the period-2 selective system tests A_0 alone, the sizes of educational expenditure in SS and CS change from those in the old exam setting, but the direction varies by the period. While the expenditures in CC and SC remain constant in both periods, the educational expenditures in SS and CS fall in period 1 but rise in period 2; overall, the total educational expenditures in SS and CS alike fall if the A_0-biased entrance exam is offered at period 2. Such changes take place primarily because over-investments for A_1 at period 1 do not lead to the improvement in school peer quality at period 2 in SS and CS under the new exam setting; parents would rather choose to consume more in the new institutional setting. Such patterns in educational spending and consumption in turn give rise to differences in the role of family income (and student endowed ability) in determining the final educational outcome (A_2) under the new exam setting.

Proposition 6. (Gradients of $\overline{A}_2$)

(i) $\overline{\eta}_{A0}^{SS} > \overline{\eta}_{A0}^{CS} = \overline{\eta}_{A0}^{SC} > \overline{\eta}_{A0}^{CC}$

(ii) $\overline{\eta}_{y}^{CC} > \overline{\eta}_{y}^{CS} = \overline{\eta}_{y}^{SC} > \overline{\eta}_{y}^{SS}$

(iii) $\overline{\eta}_{A0}^{CC} = \eta_{A0}^{CC}$; $\overline{\eta}_{A0}^{SS} < \eta_{A0}^{SS}$; $\overline{\eta}_{A0}^{CS} = \eta_{A0}^{CS}$; $\overline{\eta}_{A0}^{SC} = \eta_{A0}^{SC}$.

(iv) $\overline{\eta}_{y}^{CC} = \eta_{y}^{CC}$; $\overline{\eta}_{y}^{SS} < \eta_{y}^{SS}$; $\overline{\eta}_{y}^{CS} < \eta_{y}^{CS}$; $\overline{\eta}_{y}^{SC} = \eta_{y}^{SC}$.

As in proposition 2, the child-ability gradient η_{A0} is the largest in SS and the smallest in CC, while it is at an intermediate level in CS and SC; the family-income gradient η_y is the largest in CC and the smallest in SS, while it is at an intermediate level in CS and SC. A difference from proposition 2.(ii) is that η_y is equal between CS and SC, since educational spending at period 1 does not lead to the change in school peer quality at period 2.

Proposition 6.(iii) suggests that since peer quality in period-2 schooling is less affected by the initial difference in A_0 under the new setting, η_{A0} falls in SS, while it remains constant in CC, CS and SC. Proposition 6.(iv) suggests that η_y falls in SS and CS under the new setting of the entrance exam, because over-investments for A_1 fail to change the peer quality in period-2 schooling. Such patterns in gradients are reflected in the variance in $\ln \overline{A}_2$ for each school system.

Proposition 7. (Variance of $\ln \overline{A}_2$) Let us suppose that $Cov(\ln A_0, \ln y_T) > 0$.

(i) $\overline{\Omega}_{CS} = \overline{\Omega}_{SC}$

(ii) Other rankings of variances among $\overline{\Omega}_{CC}$, $\overline{\Omega}_{SS}$, $\overline{\Omega}_{CS}$ and $\overline{\Omega}_{SC}$ are not

uniquely determined.

(iii) $\overline{\Omega}_{CC} = \Omega_{CC}$; $\overline{\Omega}_{SS} < \Omega_{SS}$; $\overline{\Omega}_{CS} < \Omega_{CS}$; $\overline{\Omega}_{SC} = \Omega_{SC}$.

As in proposition 3, the ranking is uniquely determined only between $\overline{\Omega}_{CS}$ and $\overline{\Omega}_{SC}$. $V(\ln \overline{A}_2)$ is equal between CS and SC. Since the entrance exams test A_0 alone in both periods 1 and 2, educational spending at period 1 does not lead to the change in school peer quality at period 2; the initial difference in A_0 also change school peer quality equally in CS and SC. Other rankings fail to be uniquely determined, because the variance depends on many factors that countervail across different school systems. Although the ranking of the component of $V(\ln \overline{A}_2)$ that is related to each of $V(\ln A_0)$ and $V(\ln y_T)$ is determined as in propositions 6.(i) and 6.(ii), respectively, the role of $Cov(\ln A_0, \ln y_T)$ in the determination of $V(\ln \overline{A}_2)$ is not uniform across school systems.

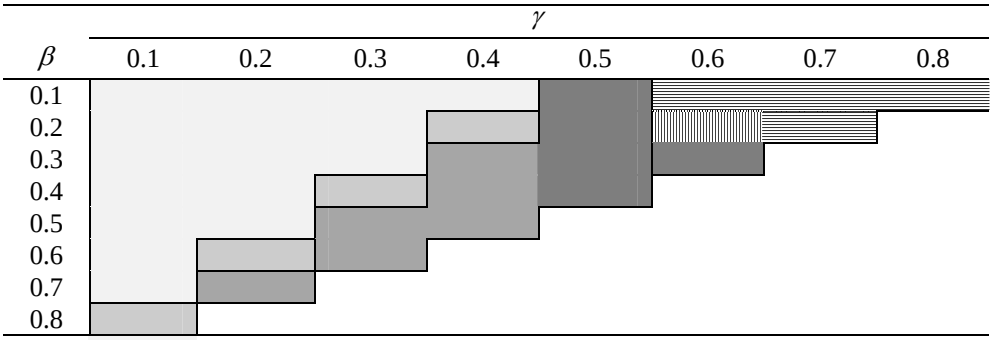
Proposition 7.(iii) suggests that while it remains constant in CC and SC, $V(\ln \overline{A}_2)$ falls in SS and CS under the new setting of the entrance exam. Again, it is primarily because peer quality in period-2 schooling is less affected by the initial difference in A_0 under the new setting, and over-investments for A_1 fail to increase the peer quality in period-2 schooling.

Since we do not know a priori which system yields what level of inequality in the final outcome, we attempt to draw a rough picture of $V(\ln \overline{A}_2)$ in different school systems by substituting $V(\ln A_0)$ for one, $V(\ln y_T)$ for one and $Cov(\ln A_0, \ln y_T)$ for 0.4, as in section 3. Given such values, the rankings of $V(\ln \overline{A}_2)$ are illustrated in Table 5 for varying pairs of β and γ. There are six unique areas in the table.

A notable pattern is that for a given value of β below 0.3, the ranking of $\overline{\Omega}_{SS}$ and $\overline{\Omega}_{CC}$ dramatically reverses as γ rises above 0.5. For β below 0.3, $\overline{\Omega}_{SS}$ is the largest and $\overline{\Omega}_{CC}$ is the smallest if γ is below 0.5. In stark contrast, $\overline{\Omega}_{SS}$ is the smallest and $\overline{\Omega}_{CC}$ is the largest if γ is above 0.5. This suggests that, if the role of peer quality (β) is relatively weak in educational production but that of educational expenditure (γ) is relatively strong, comprehensive systems amplify initial inequality of family income but selective systems narrow it in the determination of the final educational output. In such a case, selective systems are more effective to reduce inequality in educational attainments to the extent that the entrance exams test A_0 alone in selective systems. See Hur and Kang (2007) for the similar finding.

Previous research, however, suggests that β is approximately between 0.2 and 0.4 and that γ is between 0.1 and 0.2. Therefore, for the current given values of $V(\ln A_0)$, $V(\ln y_T)$ and $Cov(\ln A_0, \ln y_T)$, the most plausible rankings would be shown by $\overline{\Omega}_{SS} > \overline{\Omega}_{CS} = \overline{\Omega}_{SC} > \overline{\Omega}_{CC}$.

Table 5. Rankings of the Variance in $\ln \overline{A}_2$ for Different Pairs of β and γ



Area I. $\overline{\Omega}_{SS} > \overline{\Omega}_{CS} = \overline{\Omega}_{SC} > \overline{\Omega}_{CC}$

Area II. $\overline{\Omega}_{SS} > \overline{\Omega}_{CS} = \overline{\Omega}_{SC} = \overline{\Omega}_{CC}$

Area III. $\overline{\Omega}_{SS} > \overline{\Omega}_{CC} > \overline{\Omega}_{CS} = \overline{\Omega}_{SC}$

Area IV. $\overline{\Omega}_{CC} = \overline{\Omega}_{SS} > \overline{\Omega}_{CS} = \overline{\Omega}_{SC}$

Area V. $\overline{\Omega}_{CC} > \overline{\Omega}_{CS} = \overline{\Omega}_{SC} > \overline{\Omega}_{SS}$

Area VI. $\overline{\Omega}_{CC} > \overline{\Omega}_{CS} = \overline{\Omega}_{SC} = \overline{\Omega}_{SS}$

Proposition 8. (Size of $\overline{A}_2$)

(i) The ranking of $\overline{A}_2$ between any two systems is not uniquely determined. Either $\frac{\overline{A}_2^k}{\overline{A}_2^l} \geq 1$ or $\frac{\overline{A}_2^k}{\overline{A}_2^l} < 1$ for any pairs of k and l $(k \neq l; k,l = CC, SS, CS, SC)$. Each ranking depends on a household's A_0 and y_T, values of parameters and prices given in each school system.

(ii) Each ratio of $\frac{\overline{A}_2^{CC}}{\overline{A}_2^{SS}}$, $\frac{\overline{A}_2^{CC}}{\overline{A}_2^{CS}}$, $\frac{\overline{A}_2^{CC}}{\overline{A}_2^{SC}}$, $\frac{\overline{A}_2^{CS}}{\overline{A}_2^{SS}}$, and $\frac{\overline{A}_2^{SC}}{\overline{A}_2^{SS}}$ for a given household is negatively related to A_0 but positively related to y_T. In contrast, the value of $\frac{\overline{A}_2^{CS}}{\overline{A}_2^{SC}}$ is not related to A_0 nor y_T.

(iii) $\frac{\overline{A}_2^{CC}}{A_2^{CC}} = 1$; $\frac{\overline{A}_2^{SS}}{A_2^{SS}} = \frac{\Phi_{SS}}{\Gamma_{SS}} \cdot A_0^{-\beta^2} y_T^{-\beta\gamma}$; $\frac{\overline{A}_2^{CS}}{A_2^{CS}} = \frac{\Phi_{CS}}{\Gamma_{CS}} \cdot y_T^{-\beta(\beta+\gamma)}$; $\frac{\overline{A}_2^{SC}}{A_2^{SC}} = 1$.

Proposition 8.(i) suggests that it is difficult a priori to decide which sequence of school systems yields greater educational outputs, as in proposition 4.(i). However, proposition 8.(ii) implies that households characterized by different combinations of (A_0, y_1, y_2) prefer different pairs of school systems. The rankings depend on the values

of A_0 and y_T. Here we also employ a simulation method by substituting the same values for parameters and prices as in section 3.

Figure 2 shows five lines each of which represents the households that are indifferent between two school systems in terms of $\overline{A}_2$. (For the current given values, CS is preferred to SC by every household.) Each line represents the households that are indifferent between the same pairs of the school systems as in Figure 1. The five lines divide the entire region by a total of 6 unique areas. Table 3 (right panel) reports the orderings of $\overline{A}_2$ among four school systems in a given area.

From the orderings of $\overline{A}_2$, CC is most preferred by the households in area (1), but least preferred by those in areas (4), (5) and (6); SS is most preferred by those in area (6), but least preferred by those in areas (1), (2) and (3); CS is most preferred by those in areas (2), (3), (4) and (5). Namely, households with low A_0 and high y_T tend to like CC but dislike SS; households with high A_0 and low y_T tend to like SS but dislike CC; households with medium A_0 and medium y_T tend to like CS but dislike SS or CC, depending on their characteristics. The conclusion is similar to that in section 3. In the new setting of the entrance exam also, the overall performance of a school system does not only depend on the frequency and sequence of ability grouping across schools, but on how households are distributed over A_0 and y_T and how they prefer each school system.

Proposition 8.(iii) suggests that the rankings between $\overline{A}_2^{SS}$ and A_2^{SS}, on the one hand, and between $\overline{A}_2^{CS}$ and A_2^{CS}, on the other, also vary by household characteristics, while the final outcomes are equal between the old and new settings in CC and SC. $\overline{A}_2^{SS}$ is greater (smaller) than A_2^{SS} for households with small (large) values of both A_0 and y_T. In contrast, $\overline{A}_2^{CS}$ is greater (smaller) than A_2^{CS} for households with small (large) y_T, while A_0 does not affect the preference ranking. Thus whether or not the entrance exam should be biased toward A_0 in the selective system will be ultimately determined by voting of households, to the extent that each type of the exam chosen has the educational implications that are outlined in our preceding discussions. This reveals the conflicts in interests surrounding the choice of school systems and exam settings as shown in section 3. Similarly, the ultimate decision will be made based on a society's weights given to a variety of educational targets.

5. CONCLUDING REMARKS

In this paper, using a simple two-period model of household choices, we consider four different school systems of student allocation at different stages of schooling and their educational implications. Our model first suggests that both frequency and sequence of ability grouping play an important role in producing educational implications of school systems. We next find that different households that are characterized by a pair of family income and the child's innate ability prefer different combinations of school systems. As a result, the overall performance of a school system is determined by how households are distributed over income and a child's ability. Which sequence of school systems is adopted in a nation's education system will be ultimately determined by voting of households. Finally, if the entrance exam in the selective system tests a student's endowed ability alone but never her nurtured ability, then the systems that adopt the selective method in the later stage of education show more favorable educational implications than if the exam tests a student's endowed ability as well as her nurtured ability. Whether or not the entrance exam should signal the endowed ability alone in the selective system will also be determined by voting of households, because different households prefer different settings of the entrance exam in the selective system.

Although useful policy implications can be drawn from our approach, it is not, of course, free of drawbacks. First, the model is based on partial equilibrium, where prices are fixed across different school systems. In the context of general equilibrium, however, the prices vary by the structure of markets. We do not model the changes in markets in response to those in demand for each in different school systems. Second, our education production is based on a Cobb-Douglas form, where peer effects increase with a student's ability (i.e., $A = b\theta^{\beta}e^{\gamma}$). Such a form of peer effects are frequently used in economic theories of education (e.g., Fernandez and Gali (1999), Lazear (2001)). By assuming a Cobb-Douglas form, we avoid complications that may be introduced by different functional forms of educational production. Nevertheless, there is a need to employ different specifications of peer effects over ability in educational production for future research.

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