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AN ANALYSIS OF THE CONDITIONAL VOLATILITY DYNAMICS OF THE AUSTRALIAN BUSINESS CYCLE

KIN YIP HO, ALBERT K TSUI AND ZHAOYONG ZHANG*

Colgate University, Cornell University
National University of Singapore and Edith Cowan University

In this paper, we analyse the conditional variance of the Australian real gross domestic product (GDP) and the expenditure components by a variety of generalised autoregressive conditional heteroskedasticity (GARCH) models. First, we test the plausibility of the constant-correlation assumption by employing Tse's (2000) Lagrange Multiplier (LM) test and the Bera and Kim's (2002) Information Matrix (IM) test. Our results indicate that the correlations among the shocks to real GDP and its various expenditure components are invariant over time. In addition, these shocks are not highly correlated with one another. Second, we examine if volatility asymmetry exists in the Australian business cycle by proposing four bivariate asymmetric GARCH specifications. Except for the case of gross fixed capital formation, the evidence of asymmetric conditional volatility in the growth rates of the Australian real GDP and the other components is weak. Despite the weak evidence of asymmetric volatility, higher volatility is generally associated with the contractionary phase of the Australian business cycle. This finding has important implications for macroeconomic policy and forecasting for business cycle.

Keywords: Business Cycle, Real GDP, Information Matrix Test, Lagrange Multiplier Test, Multivariate Asymmetric GARCH, Asymmetric Conditional Volatility
JEL classification: E32, E37

1. INTRODUCTION

Business cycle analysis has mainly focused on the asymmetric nature of macroeconomic variables and the co-movements among such variables. In a seminal article, Neftci (1984) advocates a nonparametric test for "steepness" in economic time series and concludes that contractions are steeper than expansions for postwar unemployment data. On the other hand, Sichel (1993) proposes a test for "deepness" and finds evidence in unemployment variables that contractions are deeper than expansions.

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More recently, researchers have introduced specific non-linear parametric models. One category of these models posits that the non-linear behaviour occurs in the conditional mean function. For instance, Terasvirta and Anderson (1992) and Beaudry and Koop (1993) have used the threshold autoregressive model to study cyclical asymmetries. Their results generally show that contractions are less persistent than expansions.

An alternative category of models concentrates on the property of conditional heteroskedasticity. A noteworthy example is Engle (1982), who models the time-varying variance of the UK inflation by proposing the Autoregressive Conditional Heteroskedasticity (ARCH) model. Weiss (1984) also finds evidence of ARCH in the US industrial production. However, not many researchers have attempted to model asymmetries in the conditional variance of business-cycle variables.[1] Brunner (1992) has used the semi-nonparametric (SNP) approach to construct the changing conditional density of the US real GNP and found strong evidence of cyclical asymmetry in real GNP growth rates. More recently, Hamori (2000) applies a family of univariate GARCH models to the real GDP of Japan, the UK and the US, and concludes that these series do not exhibit significant asymmetric conditional volatility. This conclusion is reversed by Ho and Tsui (2001, 2003 and 2004), who find significant volatility asymmetry in the real GDP of the developed economies, such as Canada, Japan, the UK and the US, and the developing economies of Greater China.

One major drawback of univariate GARCH models is that they fail to capture the co-movement of business-cycle variables, an important feature stressed by researchers such as Lucas (1977). Diebold and Rudebusch (1996) have further proposed that a successful model for business cycles should not only take into account the co-movement of macroeconomic variables but also their possible asymmetries or non-linearities. As such, it is more apposite to consider various asymmetric GARCH models in multivariate contexts.

Several recent papers are on the issues of comovements and/or asymmetries of the Australian business cycle (see Fisher *et al.* (1996), Olekalns (1998), Bodman (1998), Henry and Summers (2000), Cashin and Ouliaris (2001), and Bodman and Crosby (2002)). Except Fisher *et al.* (1996), who have documented the volatilities of the cyclical (detrended) series and their cross correlations with cyclical real output of Australia, all of the other papers address the issue of asymmetry with emphasis on the conditional mean function, thereby ignoring the conditional volatility of real output.

A deeper understanding of the conditional heteroskedasticity and volatility asymmetry of business cycles has important implications for macroeconomic and business cycle theory and forecasting. If business cycles are conditionally heteroskedastic and exhibit volatility asymmetry, then any theory assuming the absence of either of these properties is probably inadequate. This is related to Valderrama's (2001) main

[1] To the best of our knowledge, the few exceptions are Brunner (1992, 1997), French and Sichel (1993), Hamori (2000), and Ho and Tsui (2001, 2003, and 2004).

criticism of the real business cycle model. In addition, neglecting ARCH effects may lead to a loss in asymptotic efficiency of parameter estimation, over-rejection of conventional tests for serial correlation (Milhoj (1985), and Diebold (1987)), and the identification of overparameterised ARMA models (Weiss (1984)). For practical purposes, a proper understanding of conditional variance helps develop a more realistic confidence interval for forecasting purposes (see Engle (1982)).

In this paper, we examine the conditional volatility of the Australian real GDP and expenditure components by proposing a variety of GARCH models in multivariate contexts. The major problem with multivariate GARCH (MGARCH) models is that they inevitably increase the number of parameters to be estimated and complicate the specifications of the conditional variance-covariance matrix.[2] Among others, Bollerslev (1990) has proposed the constant conditional correlations (CC-MGARCH) model to tackle this problem. However, the constant-correlation assumption is strong, and its validity in many contexts has been rejected in recent studies (Tsui and Yu (1999), Tse (2000), and Bera and Kim (2002)). This over-rejection may adversely affect the robustness of the parameter estimates in the conditional variance equation. As such, one main focus of this paper is to test the plausibility of the constant-correlation assumption. Specifically, we employ the recently developed Tse's (2000) Lagrange Multiplier (LM) test and the Bera and Kim's (2002) Information Matrix (IM) test to verify this assumption. Our results indicate that the correlations among the shocks to real GDP and the various expenditure components of Australia are probably invariant over time, and that these shocks are not highly correlated with one another.

By extending Bollerslev's (1990) work, we propose 4 different bivariate asymmetric GARCH specifications to test for volatility asymmetry. These include the Quadratic GARCH (QGARCH), Leveraged GARCH (LGARCH), Threshold GARCH (TGARCH), and Asymmetric Power ARCH (APARCH) models, respectively. Except for the case of gross fixed capital formation, our estimation results do not find evidence of asymmetric conditional volatility in growth rates of the Australian National Accounts. This is in contrast to the findings of volatility asymmetry in GDP growth rates reported by French and Sichel (1993) and Ho and Tsui (2001, 2003 and 2004) for the U.S., Canada, and Greater China. Our results are robust to a variety of alternative model specifications. Despite the weak evidence for asymmetry, we find that higher volatility is generally associated with the contractionary phase of the Australian business cycle.

The rest of the paper is organised as follows. In Sections 2 and 3, we discuss the theoretical framework and methodology for this study along with Tse's (2000) and Bera and Kim's (2002) tests, and propose the new bivariate asymmetric GARCH models. We then proceed to analyse the estimation results in Section 4, before providing the concluding remarks in Section 5.

[2] Details are discussed in Bera and Higgins (1993).

2. TESTING FOR CONSTANT CONDITIONAL CORRELATIONS

To examine the conditional volatility of the Australian real GDP and expenditure components it is useful to test the plausibility of the constant-correlation assumption before estimating the bivariate asymmetric GARCH specifications. Bera and Kim (2002) developed the following IM test to verify the assumption for the bivariate CC-GARCH model. Let $y_t = (y_{1t}, y_{2t})'$ be the bivariate vector of interest with time-varying covariance matrix H_t, and let $\mu_t(\xi)$ be the arbitrary mean functions which depend on ξ, a column vector of parameters. A typical bivariate GARCH model may be specified as follows:

$$y_{1t} = \mu_{1t}(\xi) + \varepsilon_{1t}, \tag{1}$$

$$y_{2t} = \mu_{2t}(\xi) + \varepsilon_{2t}, \tag{2}$$

and

$$(\varepsilon_{1t}, \varepsilon_{2t})' \,|\, \psi_{t-1} \sim BN(O, H_t). \tag{3}$$

Note that ψ_{t-1} is the σ-algebra generated by all the available information up to time $(t-1)$. Denoting the $ij-th$ element ($i, j = 1,2$) in H_t by h_{ijt}, the conditional correlation coefficient is given by $\rho_t = h_{12t} / \sqrt{h_{11t} h_{22t}}$. Bollerslev (1990) assumes that the conditional correlation is constant over time ($\rho_t = \rho$ for all *t*), so that all the variations in the conditional covariances are due to changes in the conditional variances only. The conditional covariance is therefore given by

$$h_{12t} = \rho \sqrt{h_{11t}} \sqrt{h_{22t}}. \tag{4}$$

Consequently, the conditional covariance matrix H_t can be partitioned as

$$H_t = \begin{pmatrix} h_{11t} & h_{12t} \\ h_{21t} & h_{22t} \end{pmatrix} = D_t \Gamma_t D_t, \tag{5}$$

where

$$D_t = \begin{pmatrix} \sqrt{h_{11t}} & 0 \\ 0 & \sqrt{h_{22t}} \end{pmatrix} \text{ and } \Gamma_t = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}. \tag{6}$$

Here the conditional variance equations are defined as

$$h_{iit} = \omega_i + \alpha_i \varepsilon_{it}^2 + \beta_i h_{iit-1}, i = 1,2 . \tag{7}$$

Note that H_t is almost surely positive definite for all *t*. Under the assumption of constant conditional correlations, the maximum likelihood estimate (MLE) of the correlation matrix is equal to the sample correlation matrix. As the sample correlation matrix is always positive definite, the optimisation will not fail as long as the conditional variances h_{11t} and h_{22t} themselves are positive.

Assuming conditional normality and ignoring the constant, the likelihood function of all unknown parameters θ is

$$l(\theta) = -\frac{1}{2}\log|H_t| - \frac{1}{2}(\varepsilon_{1t}, \varepsilon_{2t}) H_t^{-1} (\varepsilon_{1t}, \varepsilon_{2t})' , \tag{8}$$

or,

$$l(\theta) = -\frac{1}{2}\log|D_t \Gamma_t D_t| - \frac{1}{2}(\varepsilon_{1t}, \varepsilon_{2t}) D_t^{-1} \Gamma_t^{-1} D_t^{-1} (\varepsilon_{1t}, \varepsilon_{2t})' .$$

ε_{it} is assumed to have the following structure:

$$\varepsilon_{it} = \sqrt{h_{iit}} e_{it} \text{, where } e_{it} \sim N(0,1), i = 1,2 . \tag{9}$$

In a bivariate CC-GARCH model, Equation (8) becomes

$$l = -\frac{1}{2}\log(1-\rho^2) - \frac{e_{1t}^2 + e_{2t}^2 - 2\rho e_{1t} e_{2t}}{2(1-\rho^2)} . \tag{8'}$$

Bera and Kim (2002) derive the “efficient score” version of the IM test for the bivariate CC-GARCH model:

$$IM_\rho = \frac{\left[\sum_{t=1}^{T} (v_{1t}^{*2} v_{2t}^{*2} - 1 - 2\rho^{*2})\right]^2}{4T(1 + 4\rho^{*2} + \rho^{*4})} , \tag{10}$$

where

$$v_{1t}^{*} = \frac{e_{1t}^{*} - \rho^{*} e_{2t}^{*}}{\sqrt{1-\rho^{*2}}} , \tag{11}$$

$$v_{1t}^{*} = \frac{e_{2t}^{*} - \rho^{*} e_{1t}^{*}}{\sqrt{1-\rho^{*2}}} , \tag{12}$$

$$\rho^{*} = \frac{1}{T}\sum_{t=1}^{T} e_{1t}^{*} e_{2t}^{*} , \tag{13}$$

and

$$e_{it}^{*} = \varepsilon_{it}^{*} \Big/ \sqrt{h_{it}} , i = 1,2 , \tag{14}$$

where $e_{it}^{*}, \varepsilon_{it}^{*}$ and ρ^{*} are the maximum likelihood estimates (MLE) of e_{it}, ε_{it} and ρ, respectively. Under the null hypothesis of constant conditional correlations, the IM_{ρ} asymptotically follows a chi-squared distribution with one degree of freedom.

Tse (2000) proposes the following specification to capture the time-varying conditional correlations:

$$\rho_t = \rho + \delta \varepsilon_{1,t-1} \varepsilon_{2,t-1} . \tag{15}$$

From (15), the conditional covariances are given by

$$h_{12t} = \rho_t \sqrt{h_{11t}} \sqrt{h_{22t}} . \tag{16}$$

This extended bivariate model with time-varying correlations has 8 parameters. The constant-correlation hypothesis can subsequently be tested by examining the null hypothesis $H_0 : \delta = 0$.

Define the following derivatives of h_{iit} with respect to ω_i, α_i, and β_i for $i = 1,2$:

$$d_{it} = \frac{\partial h_{iit}}{\partial \omega_i}, f_{it} = \frac{\partial h_{iit}}{\partial \alpha_i}, g_{it} = \frac{\partial h_{iit}}{\partial \beta_i} . \tag{17}$$

These derivatives are computed iteratively by the following recursions:

$$
\begin{aligned}
d_{it} &= 1 + \alpha_i d_{it-1}, \\
f_{it} &= h_{iit-1} + \alpha_i f_{it-1}, \\
g_{it} &= \alpha_i g_{it-1} + \varepsilon_{it-1}^2,
\end{aligned}
\tag{18}
$$

where the starting values are given by $d_{i1} = 1, f_{i1} = h_{ii0}$, and $g_{i1} = \varepsilon_{i0}^2$. These partial derivatives can be used to calculate the following first partial derivatives of l with respect to the model parameters:

$$
\begin{aligned}
\frac{\partial l}{\partial \omega_i} &= \frac{(e_{it}^{**} e_{it} - 1) d_{it}}{2h_{iit}}, \\
\frac{\partial l}{\partial \alpha_i} &= \frac{(e_{it}^{**} e_{it} - 1) f_{it}}{2h_{iit}}, \\
\frac{\partial l}{\partial \beta_i} &= \frac{(e_{it}^{**} e_{it} - 1) g_{it}}{2h_{iit}}, \\
\frac{\partial l}{\partial \rho_t} &= e_{1t}^{**} e_{2t}^{**} + \frac{\rho_t}{1 - \rho_t^2}, \\
\frac{\partial l}{\partial \delta} &= (e_{1t}^{**} e_{2t}^{**} + \frac{\rho_t}{1 - \rho_t^2}) \varepsilon_{1t} \varepsilon_{2t},
\end{aligned}
\tag{19}
$$

where $e_t^{**} = \Gamma_t^{-1} e_t$. Denoting the 8 parameters of the model as $\theta = (\omega_1, \omega_2, \alpha_1, \alpha_2, \beta_1, \beta_2, \rho, \delta)'$, the derivative $\partial 1 / \partial \theta$ can be calculated from (19). Let θ^* be the MLE of θ under H_0, s the 8-element score vector given by $s = \partial 1 / \partial \theta$, and V be the (8×8) information matrix given by $V = E(-\partial^2 1 / \partial \theta \partial \theta')$, where $E(.)$ denotes the expectation operator. The LM statistic for H_0 is given by $s^{*'} V^{*-1} s^*$, where the asterisks denote evaluation at θ^*. For practical purposes, Tse (2000) recommends the following LM statistic denoted by

$$
LMC = s^{*'} (S^{*'} S^*)^{-1} s^* = 1' S^* (S^{*'} S^*)^{-1} S^{*'} 1, \tag{20}
$$

where 1 is the ($T \times 1$) column vector of ones, S^* is S evaluated at θ^*, and S is the ($T \times 8$) matrix, the rows of which are the partial derivatives $\partial 1 / \partial \theta'$, for $t = 1,...,T$. Note that LMC is distributed asymptotically as chi-squared with one degree of freedom.

3. MODELING ASYMMETRIC CONDITIONAL VOLATILITY

It is understood that the CC-GARCH model does not capture asymmetric volatility, whereby the effect of a negative shock on future volatilities is different from that of a positive shock of the same magnitude. In order to incorporate asymmetric conditional volatility we propose the following 4 different bivariate asymmetric specifications, i.e., the QGARCH, LGARCH, TGARCH, and APARCH models. These specifications are less restrictive since they nest several versions of popular GARCH models, thereby reducing the possibility of conditional variance misspecification.

Denote Y_{it} as the i_{th} variable of interest and y_{it} as the growth rate (in percentage) calculated on a continuously compounding basis. We have

$$y_{it} = \log(\frac{Y_{it}}{Y_{it-1}}) \times 100, \; i = 1,2 . \tag{21}$$

Assume that the conditional mean equation for each variable *i* is effectively captured by an autoregressive AR(*k*) filter:

$$y_{it} = \pi_0 + \sum_{j=1}^{k} \pi_j y_{it-j} + \varepsilon_{it}, \; i = 1,2 , \tag{22}$$

where ε_{it} is the serially uncorrelated random disturbance term. ε_{it} is assumed to have the structure as stated in Equation (9), with h_{iit} modified to capture the possible existence of asymmetric volatility.

QGARCH model

Sentana's (1995) QGARCH(1,1) model is specified as

$$h_{iit} = \eta_i + \gamma_i \varepsilon_{it-1} + \alpha_{it-1} \varepsilon_{it-1}^2 + \beta_i h_{iit-1} , \tag{23}$$

where γ is the asymmetric coefficient. It represents the most general quadratic version possible within the ARCH class and encompasses many existing quadratic variance functions. It may be nested into non-parametric approaches to dynamic conditional heteroskedasticity.

LGARCH, TGARCH and APARCH models

The setups are as follows:

$$\varepsilon_{it} = h_{iit} e_{it},\ e_{it} \sim N(0,1),\ i = 1,2,$$
$$h_{iit}^{\delta} = \eta_i + \alpha_i (|\,\varepsilon_{it-1}\,| - \gamma \varepsilon_{it-1})^{\delta} + \beta_i h_{iit-1}^{\delta}. \quad (24)$$

When $\delta = 2$, this is the LGARCH(1,1) model. Alternatively, when $\delta = 1$, this is the TGARCH(1,1) model. When δ is not restricted to any positive value, this becomes the APARCH(1,1) model (See Ding *et al.* (1993)).

4. DATA AND RESULTS

All data are quarterly, seasonally adjusted, and expressed at 1999-2000 prices. Our data set was obtained from the Quarterly National Accounts of the OECD Statistical Directorate, spanning from 1960Q1 to 2000Q4 with 164 observations. It comprises Australia's gross domestic product (GDP), private final consumption expenditure (C), gross fixed capital formation (I), government final consumption expenditure (G), exports (X) and imports (M) of goods and services.

Table 1. Summary Statistics of Australian Real GDP and the Expenditure Components

Country	GDP	C	I	G	X	M
Panel A: Moments, Maximum, Minimum						
Mean	0.0092	0.0088	0.0079	0.0100	0.0150	0.0139
Median	0.0086	0.0097	0.0073	0.0125	0.0168	0.0162
Maximum	0.0469	0.0333	0.0782	0.0794	0.1641	0.1188
Minimum	-0.0359	-0.0173	-0.0891	-0.0558	-0.1055	-0.1574
Standard Deviation	0.0139	0.0083	0.0288	0.0232	0.0457	0.0454
Skewness	0.1204	-0.2250	-0.3186	0.0202	0.3849	-0.5457
Kurtosis	3.7633	3.4300	3.2575	3.2817	4.1102	4.3865
Observations	164	164	164	164	164	164
Panel B: Jarque-Bera Test						
Jarque-Bera	4.3511	2.6310	3.2071	0.5501	12.3963	21.1463
Panel C: Ljung-Box Q-statistic						
12 lags	47.0942	5.3560	19.8458	33.0850	23.1763	36.5518
Panel D: BDS Test						
$e = 3, 1 = 1.5$	4.3895	0.8492	0.9347	4.3515	2.2742	3.3595
$e = 5, 1 = 1.5$	5.6276	2.1390	1.2489	3.8647	3.4647	3.3290
$e = 3, 1 = 1.0$	5.6680	0.5255	1.3181	3.3700	2.3816	3.4417
$e = 5, 1 = 1.0$	8.1576	1.5119	1.4470	3.8293	4.0583	3.4913

Notes: [1] The Jarque-Bera test statistics and the Ljung-Box Q-statistics follow the chi-squared distribution with 2 and 12 degrees of freedom respectively. [2] For the BDS test, *e* represents the embedding dimension whereas *l* represents the distance between pairs of consecutive observations, measured as a multiple of the standard deviation of the series.

Table 1 reports the summary statistics of the Australian real GDP and the expenditure components. As can be seen from Table 1, the series of variables I, G, X and M exhibit higher standard deviation relative to real GDP. In particular, X and M have the highest standard deviation and exhibit significant leptokurtosis. The Ljung-Box Q-statistics in Panel C suggests that significant serial correlation is detected for GDP, G, X, and M. Also, the BDS test statistics in Panel D indicate that most of the series, such as GDP, C, G, X, and M, are not independently and identically distributed (IID). As argued by Hsieh (1993), such departures from IID may be ascribed to the presence of conditional heteroskedasticity in the data sets.

Several estimation issues deserve mention. First, we use both the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests to check for stationarity in order to ensure reliable statistical inference (results are available on request). Both tests suggest that all the series are stationary. In addition, diagnostic checking of the ADF model indicates the absence of serial correlation in the residuals. Second, we have tried different lag lengths and found that an AR(1) model is a more suitable filter for the mean equation. Third, the conditional mean and the variance/covariance matrix are simultaneously estimated assuming normality using the quasi-maximum likelihood estimation (QMLE) method *a la* Bollerslev and Wooldridge (1992). All programmes are coded in Gauss, with the convergence criterion (tolerance level) set to 10^{-5}. To ensure the stability of the parameter estimates, we have also used stricter criteria (up to 10^{-8}) and all results are found to be similar. Our results are generally invariant to the choice of the initial values. Fourth, we adopt an incremental approach by first estimating the most restrictive model (CC-GARCH) and then proceeding gradually to the least restrictive one (CC-APARCH). Conceivably, such procedures can be tedious but they help to secure convergence more expeditiously during estimation.

Table 2. Estimation Results of the AR(1)-CC-GARCH(1,1) Model: $y_t = \pi_0 + \pi_1 y_{t-1} + \varepsilon_t ; h_t = \eta + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$

Variable	π_0	π_1	η	α	β	Correlations	LMC	IMC	Likelihood Value
GDP	0.8663	0.0261	0.0146	0.1333	0.8551	0.1670	1.1935	0.0521	-306.2672
	(0.1404)	(0.0914)	(0.0287)	(0.0940)	(0.0947)	(0.0751)			
G	1.3153	-0.3527	0.1132	0.0521	0.9181				
	(0.1941)	(0.0835)	(0.2214)	(0.0546)	(0.0681)				
GDP	0.8141	0.0638	0.0088	0.0908	0.8936	0.2314	1.5376	6.7244**	-418.2727
	(0.1327)	(0.0831)	(0.0326)	(0.1213)	(0.1302)	(0.0779)			
X	1.7693	-0.0955	0.0560	0.0217	0.9732				
	(0.3676)	(0.0875)	(0.3606)	(0.0483)	(0.0682)				
GDP	0.8904	-0.0053	0.0173	0.1292	0.8576	0.2974	5.3040*	1.1651	-416.6329
	(0.1048)	(0.0532)	(0.0274)	(0.0789)	(0.0808)	(0.0657)			
M	1.1762	0.2226	8.6014	0.2367	0.3303				
	(0.3549)	(0.0923)	(3.4806)	(0.1559)	(0.1813)				
G	1.3314	-0.3601	0.1886	0.0474	0.9062	0.0425	5.3044*	1.9643	-509.3897
	(0.1901)	(0.0827)	(0.3301)	(0.0523)	(0.0925)	(0.0778)			
X	1.7982	-0.1014	0.0062	0.0309	0.9617				
	(0.3479)	(0.0882)	(0.0392)	(0.0313)	(0.0306)				
G	1.3195	-0.3334	0.2424	0.0468	0.8948	0.1427	0.2263	0.0097	-508.736
	(0.1907)	(0.0856)	(0.4328)	(0.0512)	(0.1100)	(0.0773)			
M	1.1218	0.2731	7.4597	0.2715	0.3660				
	(0.3597)	(0.0931)	(3.7901)	(0.1742)	(0.2297)				
X	1.7573	-0.0736	0.0097	0.0314	0.9611	0.1235	1.0843	23.1602**	-623.6999
	(0.3496)	(0.0890)	(0.0408)	(0.0338)	(0.0334)	(0.0885)			
M	1.0921	0.2867	8.1404	0.2466	0.3454				
	(0.3549)	(0.0942)	(3.9945)	(0.1760)	(0.2364)				
GDP	0.8974	-0.0231	0.0060	0.0886	0.9003	0.3567	0.4087	1.0193	-145.8156
	(0.1489)	(0.0979)	(0.0231)	(0.0767)	(0.0807)	(0.0600)			

Variable	π_0	π_1	η	α	β	Correlations	LMC	IMC	Likelihood Value
C	0.8444	0.0548	0.0336	0.0726	0.8733				
	(0.1329)	(0.0796)	(0.2114)	(0.1787)	(0.4888)				
C	0.8271	0.0823	0.0432	0.0954	0.8369	0.1821	0.6418	0.7670	-240.6874
	(0.1006)	(0.0809)	(0.0636)	(0.0935)	(0.1625)	(0.0776)			
G	1.2889	-0.3141	0.1497	0.0432	0.9188				
	(0.1986)	(0.0900)	(0.2491)	(0.0530)	(0.0698)				
C	0.8227	0.0863	0.0597	0.0823	0.8238	-0.0265	0.1543	5.0524*	-357.8241
	(0.1014)	(0.0856)	(0.0965)	(0.1028)	(0.2247)	(0.0787)			
X	1.7777	-0.0943	-0.0158	0.0292	0.9640				
	(0.3583)	(0.0886)	(0.1137)	(0.0346)	(0.0374)				
C	0.8467	0.0826	0.0705	0.1256	0.7696	0.3002	2.2488	1.3017	-351.2096
	(0.1035)	(0.0879)	(0.0593)	(0.1097)	(0.1530)	(0.0779)			
M	1.2301	0.2414	8.2578	0.2678	0.3273				
	(0.3613)	(0.0963)	(3.3091)	(0.1723)	(0.1767)				

Notes: [1] The LMC is the Lagrange Multiplier statistics for constant correlations and is calculated using Tse's (2000) test. It is asymptotically distributed as chi-squared with 1 degree of freedom. [2] The IMC is the Information Matrix statistics for constant correlations and is calculated using Bera and Kim's (2002) test. It is asymptotically distributed as chi-squared with 1 degree of freedom. [3] All standard errors are the heteroskedastic-consistent Bollerslev-Wooldridge standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique. [4] ** and * indicate statistical significance at the 1% and 5% levels respectively.

Table 2 reports the test statistics for constant conditional correlations. With the exception of the correlation coefficients involving <GDP, M> and, <G, X>, respectively, Tse's LM test statistics on the whole suggest that the conditional correlations among the components of real GDP are time-invariant. As for Bera and Kim's (2002) IM test, the null hypothesis of constant conditional correlations cannot be rejected at the 5% level except for the cases of <GDP, X>, <X, M> and <C, X>.[3] Combining the results of Tse's (2000) LM and Bera and Kim's (2002) IM test, there are five cases (<GDP, G>, <GDP, C>, <G, M>, <C, G> and <C, M>) in which both tests consistently indicate that the constant-correlation hypothesis cannot be rejected at the 5% level. Nevertheless, there are several cases of inconsistency between the two tests. For instance, albeit Tse's (2000) LM test suggests that the null hypothesis of constant correlations should be rejected for <GDP, X> and <G, X>, the IM test indicates otherwise. In contrast, for <GDP, X>, <X, M> and <C, X>, the converse is true. Given the inconclusive nature of the results in these cases, we refer to the other residual diagnostic tests to examine if the model of constant conditional correlations is misspecified. In particular, we will focus on the Ljung-Box Q-statistics and the runs test statistics calculated based on the cross-product of the standardised residuals from the CC-GARCH(1,1) model. Tables 3-4 present these tests results.

[3] Bera and Kim (2002) have alternatively suggested a studentised version of the IM test, which is purported to have better finite-sample behaviour compared to the original one. We have calculated these statistics for those cases in which the IM test rejects the null hypothesis and found that it cannot be rejected, consistent with the LM test. The complete results are available from the authors upon request.

Table 3. Summary Statistics of Standardised Residuals of AR(1)-CC-GARCH(1,1) Model:

$$y_t = \pi_0 + \pi_1 y_{t-1} + \varepsilon_t ; h_t = \eta + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

Variable	<GDP, G>		<GDP, X>		<GDP, M>		<G, X>		<G, M>	
	GDP	G	GDP	X	GDP	M	G	X	G	M
Panel A: Moments, Maximum, Minimum										
Mean	-0.0101	0.0276	0.0076	-0.0067	-0.0055	-0.0234	0.0252	-0.0096	0.0195	-0.0295
Median	-0.0231	-0.0195	-0.0152	-0.0181	-0.0176	0.0454	-0.0247	-0.0274	-0.0189	0.0696
Maximum	2.2684	2.9140	2.3830	3.0584	2.2574	2.6180	2.9242	2.9672	2.9574	2.6706
Minimum	-2.6616	-2.6566	-2.8429	-2.7889	-2.6181	-3.4301	-2.6533	-2.8033	-2.5565	-3.4068
Std. Dev.	1.0153	1.0134	1.0290	1.0580	1.0102	0.9999	1.0109	1.0425	1.0100	0.9996
Skewness	-0.1927	0.0763	-0.1433	0.3200	-0.2036	-0.3123	0.0929	0.3277	0.1131	-0.2954
Kurtosis	2.8538	2.9057	2.8414	3.5564	2.8360	3.7528	2.9172	3.5702	2.9250	3.6858
Panel B: Jarque-Bera Test										
Jarque-Bera	1.1464	0.2173	0.7243	4.8542	1.3009	6.4592	0.2792	5.0936	0.3835	5.5302
Panel C: Ljung-Box Q-statistics										
Q-statistic (4)	3.3694	1.7997	3.9969	9.4548	3.5694	5.5095	1.7762	9.3632	1.8627	5.0431
Q-statistic (8)	9.2719	10.4699	9.9893	11.8101	9.7631	13.0426	10.6126	11.7809	10.5375	11.9015
Q-statistic (12)	14.7331	19.8086	16.1174	17.4509	15.6190	17.0784	20.0817	17.2115	19.8660	16.2978
Panel D: McLeod-Li Test										
McLeod-Li (4)	1.6747	1.9968	2.6332	1.3990	1.8370	0.7517	1.9692	1.5917	1.6936	0.9448
McLeod-Li (8)	2.3902	4.9820	3.7762	3.7660	2.6390	2.0286	4.9333	3.9969	4.5087	2.0718
McLeod-Li (12)	4.7923	30.2000	6.2108	6.7175	4.8641	4.1608	31.7500	6.6860	31.3658	3.8054
Panel E: BDS Test										
$e = 3$, $1 = 1.5$	-0.4798	0.1300	-0.0149	0.6752	0.1172	-0.0058	0.1646	0.6609	-0.0697	-0.1212
$e = 5$, $1 = 1.5$	0.2809	0.0013	0.8904	1.6266	0.6929	0.4199	0.0705	1.5364	-0.0340	0.2000
$e = 3$, $1 = 1.0$	-0.0588	0.1942	0.2406	0.2526	0.0796	-0.4522	0.5134	0.3013	0.5392	-0.9135
$e = 5$, $1 = 1.0$	0.6342	1.0667	1.0467	1.0985	0.5132	-0.2240	1.3724	1.0918	1.5692	-0.7432

Variable	<GDP, G>		<GDP, X>		<GDP, M>		<G, X>		<G, M>	
	GDP	G	GDP	X	GDP	M	G	X	G	M
Panel F: Runs Test										
R_1	0.0019	1.4207	0.0019	0.3153	0.0019	-0.1926	1.4208	0.3172	1.4208	0.1976
R_2	-0.8526	0.5611	-1.0609	-0.0713	-1.4503	1.1065	0.5611	0.1904	0.5178	0.8823
R_3	0.3008	-0.1775	-0.2927	-0.9465	-0.3982	0.3935	-0.7646	-0.7869	-0.0575	0.3935
Panel G: Ljung-Box Q-statistics (Cross Product)										
4 lags	5.1545		3.9904		1.7969		9.0141		1.6605	
8 lags	15.1665		7.0383		8.0105		13.2256		6.2727	
12 lags	21.5595		23.8878		10.2833		18.4233		7.9739	
Panel H: Runs Test (Cross Product)										
R_1	0.0961		-0.1745		0.7108		0.6818		1.0809	
R_2	0.5379		-0.0350		0.3699		-0.1537		0.1921	
R_3	1.1432		-1.3686		-0.1924		-0.1662		-0.5883	

Notes: [1] The Jarque-Bera statistics follows the chi-square distribution with 1 degree of freedom. [2] For the Ljung-Box Q-statistics and the McLeod-Li test statistics, the number in brackets refers to the number of lags. [3] For the BDS Test, e represents the embedding dimension whereas l represents the distance between pairs of consecutive observations, measured as a multiple of the standard deviation of the series. Under the null hypothesis of independence, the test statistic is asymptotically distributed as standard normal. [4] For the Runs Test, R_i for i = 1, 2, 3 denote the runs tests of the series R_t, $|R_t|$, and R_t^2 respectively. Under the null hypothesis that successive observations in the series are independent, the test statistic is asymptotically standard normal. [5] The Ljung-Box Q-statistics (Cross Product) refers to the Q-statistic applied to the cross-product of the standardised residuals. 4, 8, and 12 lags are used. [6] The Runs Test (Cross Product) refers to the runs test applied to the cross-product of the standardised residuals. R_i for i = 1, 2, 3 denote the runs tests of the series R_t, $|R_t|$, and R_t^2 respectively.

Table 4. Summary Statistics of Standardised Residuals of AR(1)-CC-GARCH(1,1) Model:

$$y_t = \pi_0 + \pi_1 y_{t-1} + \varepsilon_t ; h_t = \eta + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

Variable	<X, M>		<GDP, C>		<C, G>		<C, X>		<C, M>	
	X	M	GDP	C	C	G	C	X	C	M
Panel A: Moments, Maximum, Minimum										
Mean	-0.0101	-0.0264	0.0034	-0.0118	-0.0189	0.0242	-0.0188	-0.0076	-0.0412	-0.0425
Median	-0.0115	0.0822	0.0013	0.0754	0.0467	-0.0008	0.0458	-0.0189	0.0263	0.0269
Maximum	2.9863	2.6757	2.1935	2.7045	2.6962	2.9555	2.7575	2.9828	2.6876	2.6182
Minimum	-2.8311	-3.3797	-2.6945	-2.6742	-2.6457	-2.5538	-2.6420	-2.7933	-2.7064	-3.4185
Std. Dev.	1.0465	1.0003	1.0171	1.0190	1.0199	1.0128	1.0186	1.0432	1.0147	0.9986
Skewness	0.3231	-0.2818	-0.2061	-0.1693	-0.1415	0.1048	-0.1411	0.3272	-0.1159	-0.2985
Kurtosis	3.5750	3.6756	2.8077	3.0713	3.0508	2.9185	3.0921	3.5715	3.0822	3.7040
Panel B: Jarque-Bera Test										
Jarque-Bera	5.0507	5.2252	1.3967	0.8083	0.5580	0.3412	0.5946	5.0959	0.4080	5.7510
Panel C: Ljung-Box Q-statistics										
Q-statistic (4)	9.1974	5.0491	4.0750	3.1261	3.1230	2.0758	2.9226	9.3286	3.1754	5.2832
Q-statistic (8)	11.6013	11.6573	10.0353	5.6736	5.7623	10.6129	5.5363	11.7249	5.9477	12.6011
Q-statistic (12)	17.1501	16.2199	16.6665	6.4538	6.6000	19.5699	6.3567	17.2362	6.8786	16.7429
Panel D: McLeod-Li Test										
McLeod-Li (4)	1.6233	0.9117	3.3253	4.3323	3.5354	1.8486	3.9185	1.5664	3.0499	0.8585
McLeod-Li (8)	3.9860	2.0245	4.5204	6.2180	5.8289	4.5679	5.8453	3.9467	5.1419	2.0351
McLeod-Li (12)	6.4702	3.7551	6.3711	12.7166	12.5383	29.8139	12.4473	6.6996	12.1675	3.8336
Panel E: BDS Test										
$e = 3,\ 1 = 1.5$	0.6568	0.0267	0.5482	-0.2860	-0.4631	-0.0438	-0.1932	0.6686	-0.5660	-0.0953
$e = 5,\ 1 = 1.5$	1.5496	0.3826	1.3368	0.5544	0.3600	-0.0082	0.6952	1.5131	0.1586	0.3358
$e = 3,\ 1 = 1.0$	0.2285	-0.6659	0.4656	-0.2198	-0.6906	0.2223	-0.4416	0.2593	-0.6770	-0.7508
$e = 5,\ 1 = 1.0$	0.7396	-0.5540	1.3627	0.4018	-0.0549	1.4406	0.0259	0.9691	-0.4785	-0.4719

Variable	<X, M>		<GDP, C>		<C, G>		<C, X>		<C, M>	
	X	M	GDP	C	C	G	C	X	C	M
Panel F: Runs Test										
R_1	0.3153	0.1976	0.0019	-1.0962	-1.0962	1.4208	-0.7717	0.3153	-1.0963	-0.1596
R_2	0.9183	0.2399	-1.5056	-0.5261	-0.5261	0.5178	-0.5261	-0.0713	0.0682	1.3085
R_3	-0.6183	0.3935	-0.6426	-0.1051	0.0520	0.3446	-0.3311	-1.3347	-0.1701	0.3935
Panel G: Ljung-Box Q-statistics (Cross Product)										
4 lags	2.7808		2.2470		2.3148		3.5035		5.9051	
8 lags	5.1641		6.5143		5.1583		8.5934		9.5045	
12 lags	6.3327		8.6399		8.3915		14.0112		16.4491	
Panel H: Runs Test (Cross Product)										
R_1	-0.5631		-1.4464		-0.6333		-0.9006		1.0175	
R_2	2.1656		-2.1023		0.1775		0.9944		0.6062	
R_3	0.5703		-3.1177		-0.1736		0.3724		0.5536	

Notes: [1] The Jarque-Bera statistics follows the chi-square distribution with 1 degree of freedom. [2] For the Ljung-Box Q-statistics and the McLeod-Li test statistics, the number in brackets refers to the number of lags. [3] For the BDS Test, *e* represents the embedding dimension whereas *l* represents the distance between pairs of consecutive observations, measured as a multiple of the standard deviation of the series. Under the null hypothesis of independence, the test statistic is asymptotically distributed as standard normal. [4] For the Runs Test, R_i for i = 1, 2, 3 denote the runs tests of the series R_t, $|R_t|$, and R_t^2 respectively. Under the null hypothesis that successive observations in the series are independent, the test statistic is asymptotically standard normal. [5] The Ljung-Box Q-statistics (Cross Product) refers to the Q-statistic applied to the cross-product of the standardised residuals. 4, 8, and 12 lags are used. [6] The Runs Test (Cross Product) refers to the runs test applied to the cross-product of the standardised residuals. R_i for i = 1, 2, 3 denote the runs tests of the series R_t, $|R_t|$, and R_t^2 respectively.

As shown in Tables 3 and 4 , the Ljung-Box Q-statistics for the cross-product of the standardised residuals in the models involving <GDP, M> and <G, X> indicate that the residuals are serially uncorrelated. This is corroborated by the nonparametric runs test. Apparently, the constant correlation model is adequately specified in these cases. On the other hand, for the models involving <GDP, X>, <X, M> and <C, X>, most Ljung-Box Q-statistics also indicate the absence of serial correlation. The only exception is the Ljung-Box Q-statistic at 12 lags for <GDP, X>, which is significant at the 5% level. Otherwise, the diagnostics suggest that the GARCH models assuming constant conditional correlation are adequately specified. In sum, the evidence demonstrates that the correlations among the shocks to the real GDP components are time-invariant.

It is interesting to note that the correlations among the shock to real output and expenditure components are quite low, usually of the order between 0.2 and 0.3. One possible conjecture for the low correlations between demand and real output shocks could be due to the relative predominance of supply-side shocks. This finding is consistent with Cashin and Ouliaris (2001), who observes that the Australian real output is predominantly subjected to the supply-side shocks in the post-war period from 1959 to 2000.

Table 5. Estimation Results of AR(1)-CC-QGARCH(1,1) Model: $y_t = \pi_0 + \pi_1 y_{t-1} + \varepsilon_t ; h_t = \eta + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} + \gamma\varepsilon_{t-1}$

Variable	π_0	π_1	η	α	β	γ	Correlations	Likelihood Value
GDP	0.9166	-0.0468	0.0204	0.1534	0.8422	-0.1403	0.4380	-337.254
	(0.1068)	(0.0784)	(0.0333)	(0.0960)	(0.0792)	(0.2771)	(0.0563)	
I	0.9715	-0.0690	4.9774	0.0476	0.3331	-0.7154		
	(0.2637)	(0.0867)	(2.5046)	(0.0995)	(0.3942)	(0.3390)		
GDP	0.8605	0.0067	0.0311	0.1598	0.8300	-0.1582	0.2854	-414.537
	(1.5159)	(1.4714)	(0.0350)	(0.0989)	(0.0720)	(0.7057)	(0.1116)	
M	1.0628	0.2205	9.2316	0.2566	0.2907	-1.0125		
	(0.5209)	(0.1709)	(3.8223)	(0.1634)	(0.2228)	(0.9011)		
I	0.9028	0.0050	4.8392	0.0581	0.3481	-0.8632	0.0981	-440.173
	(0.2452)	(0.0373)	(1.8241)	(0.0876)	(0.2731)	(0.3329)	(0.0816)	
G	1.3503	-0.3612	0.2513	0.0526	0.8868	0.0656		
	(0.1991)	(0.0884)	(0.6321)	(0.0548)	(0.1622)	(0.1303)		
I	0.9621	-0.0502	5.6597	0.0609	0.2456	-0.9019	0.3259	-546.627
	(0.2610)	(0.0828)	(1.7857)	(0.0923)	(0.2611)	(0.3383)	(0.0738)	
M	1.0824	0.2279	8.5636	0.3166	0.2900	-1.1573		
	(0.3669)	(0.0917)	(3.1438)	(0.1619)	(0.1415)	(1.0546)		
C	0.8393	0.0796	0.0852	0.1272	0.7465	-0.0325	0.3189	-349.032
	(0.1026)	(0.0841)	(0.0902)	(0.0932)	(0.1656)	(0.1001)	(0.0790)	
M	1.0962	0.2260	9.6871	0.2795	0.2642	-1.4554		
	(0.3822)	(0.0881)	(2.7752)	(0.1488)	(0.1059)	(0.9547)		

Note: All standard errors are the heteroskedastic-consistent Bollerslev-Wooldridge standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.

Table 6. Estimation Results of AR(1)-CC-LGARCH(1,1) Model: $y_t = \pi_0 + \pi_1 y_{t-1} + \varepsilon_t ; h_t = \eta + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^2 + \beta h_{t-1}$

Variable	π_0	π_1	η	α	β	γ	Correlations	Likelihood Value
GDP	0.8290	0.0410	0.0144	0.1331	0.8550	0.2026	0.1721	-305.387
	(0.1315)	(0.0904)	(0.0229)	(0.0683)	(0.0695)	(0.1578)	(0.0747)	
G	1.3109	-0.3524	0.1125	0.0523	0.9176	-0.0488		
	(0.1953)	(0.0850)	(0.2540)	(0.0584)	(0.0812)	(0.2616)		
GDP	0.8597	0.0066	0.0163	0.1290	0.8584	0.1360	0.2887	-415.671
	(0.1019)	(0.0482)	(0.0237)	(0.0655)	(0.0674)	(0.1389)	(0.0632)	
M	1.1170	0.2163	9.4193	0.2428	0.2693	0.2551		
	(0.3593)	(0.0874)	(2.8615)	(0.1462)	(0.1282)	(0.2547)		
G	1.3117	-0.3307	0.2475	0.0469	0.8937	0.0111	0.1506	-507.89
	(0.0911)	(0.0662)	(0.0617)	(0.0448)	(0.0435)	(0.1895)	(0.0691)	
M	1.0773	0.2587	8.5082	0.2941	0.2812	0.2758		
	(0.0724)	(0.0692)	(0.0100)	(0.0961)	(0.0612)	(0.0966)		

Note: See Note to table 5.

Table 7. Estimation Results of AR(1)-CC-TGARCH(1,1) Model: $y_t = \pi_0 + \pi_1 y_{t-1} + \varepsilon_t ; h_t = \eta + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1}) + \beta h_{t-1}$

Variable	π_0	π_1	η	α	β	γ	Correlations	Likelihood Value
GDP	0.8164	0.0518	0.0129	0.1332	0.8801	0.1981	0.1549	-306.28
	(0.0038)	(0.0863)	(0.0410)	(0.1036)	(0.1077)	(0.2551)	(0.0804)	
G	1.3906	-0.3595	1.9249	0.0846	0.0078	0.3342		
	(0.1871)	(0.0765)	(0.2384)	(0.0832)	(0.0610)	(0.6025)		
G	1.4230	-0.3678	1.8886	0.0928	0.0185	0.2598	0.0574	-508.70
	(0.1073)	(0.0629)	(0.0300)	(0.0734)	(0.0793)	(0.0561)	(0.0692)	
X	1.9698	-0.0856	0.0450	0.0819	0.9230	-0.1231		
	(0.1150)	(0.0716)	(0.0797)	(0.0784)	(0.0707)	(0.1255)		

Variable	π_0	π_1	η	α	β	γ	Correlations	Likelihood Value
G	1.3630	-0.3330	1.7925	0.0923	0.0655	0.2339	0.1545	-507.47
	(0.1833)	(0.0804)	(0.7772)	(0.0839)	(0.3356)	(0.6144)	(0.0811)	
M	0.8983	0.2943	1.7106	0.2448	0.4242	0.4423		
	(0.4102)	(0.0978)	(0.8098)	(0.1194)	(0.2126)	(0.3188)		
GDP	0.8290	0.0235	0.0133	0.1244	0.8869	0.1333	0.2826	-415.94
	(0.1296)	(0.0927)	(0.0339)	(0.0816)	(0.0854)	(0.2152)	(0.0633)	
M	0.9613	0.2410	1.9154	0.2021	0.4056	0.3914		
	(0.3967)	(0.0903)	(0.9046)	(0.1069)	(0.2130)	(0.3906)		
X	1.8845	-0.0549	0.0281	0.0785	0.9296	-0.1946	0.1455	-621.53
	(0.3782)	(0.0815)	(0.2133)	(0.1885)	(0.1911)	(0.9937)	(0.0867)	
M	0.8969	0.3113	1.8682	0.2341	0.3928	0.3963		
	(0.4076)	(0.0994)	(0.7867)	(0.1221)	(0.2069)	(0.3451)		

Note: See Note to table 5.

Table 8. Estimation Results of CC-APARCH(1,1) Model: $y_t = \pi_0 + \pi_1 y_{t-1} + \varepsilon_t; h_t^{\delta} = \eta + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^{\delta} + \beta h_{t-1}^{\delta}$

Variable	π_0	π_1	η	α	β	γ	δ	Correlations	Likelihood Value
GDP	0.8637	0.0062	0.0124	0.0739	0.8147	0.1272	4.0089	0.2819	-415.296
	(1.3016)	(1.2967)	(0.0269)	(0.1180)	(0.1508)	(0.6800)	(2.6402)	(0.1263)	
M	1.0538	0.2289	3.8836	0.2297	0.3348	0.3198	1.4280		
	(0.5632)	(0.2108)	(6.2196)	(0.1461)	(0.2112)	(0.5535)	(1.0436)		

Note: See Note to table 5.

Tables 5-8 report the estimation results for asymmetric conditional volatility based on the four bivariate asymmetric GARCH models as specified in equations (23) and (24). The estimated values of the asymmetry parameter γ for Gross Domestic Product (GDP), Government Expenditure (G), Consumption (C), Exports (X), and Imports (M) are generally insignificant at the 5% level. Our results indicate that the Australian National Accounts do not exhibit highly significant asymmetric conditional volatility. This is similar to the findings of several researchers on asymmetries in the Australian business cycle, such as Olekalns (1998), Cashin and Ouliaris (2001), and Bodman and Crosby (2002). In particular, Olekalns (1998) tests for the presence of asymmetric steepness and deepness in various Australian macroeconomic variables, such as Gross Non-farm Domestic Product and Private Consumption Spending, and concludes that these variables do not display any detectable asymmetries. Similarly, Cashin and Ouliaris (2001) note that in terms of duration and amplitude, the Australian growth cycle (periods of above-trend and below-trend rates of economic growth) is relatively symmetric. Bodman and Crosby (2002) find that, unlike recent evidence for the US and Canada, the Australian GDP growth does not manifest asymmetry in the responses of output growth to positive and negative shocks. Apparently, the absence of asymmetry applies not only to the conditional mean, but also to the conditional variance.

The only exception to the general absence of asymmetric volatility is the investment component (I). As shown in Table 5, the estimated value of coefficient of asymmetry for the investment component (I) from the CC-QGARCH model is negative and significant at the 5% level. This suggests that negative shocks to the investment variable engender higher levels of volatility in future periods compared with positive shocks of the same magnitude.

Moreover, several studies find that volatility is higher during the recessionary phase of the business cycle (see French and Sichel (1993) and Ho and Tsui (2001, 2003 and 2004)). In particular, French and Sichel (1993) note that volatility asymmetry is concentrated in the cyclically sensitive sectors of the US economy, such as the investment components including non-residential structures, residential structures and inventories. Additionally Ho and Tsui (2001, 2003 and 2004) find support for association of volatility asymmetry with the contractionary phase in output for the United States, Canada and greater China by using exponential GARCH models. In the case of Australia, we notice that even though the overall statistical evidence for asymmetric volatility is weak, the conditional volatility of GDP and its components is usually higher around periods of recession (business cycle troughs). This is corroborated by the estimated business cycle dates published by various sources. Figures 1-2 highlight the recessionary phase of the Australian business cycles[4], alongside with the estimated

[4] It should be noted that this is mainly qualitative evidence. In fact, the estimated dates of the peaks and troughs published by various institutions do differ slightly. As such, we use the dates of troughs estimated by Economic Cycle Research Institute (ECRI), which are generally consistent with other sources such as Melbourne Institute and CIBCR. The ECRI data set is mounted on webpage http://www.businesscycle.com/

values of conditional standard deviations of GDP from the CC-GARCH and CC-QGARCH models. Our findings are robust to different specifications of the conditional variance equation.

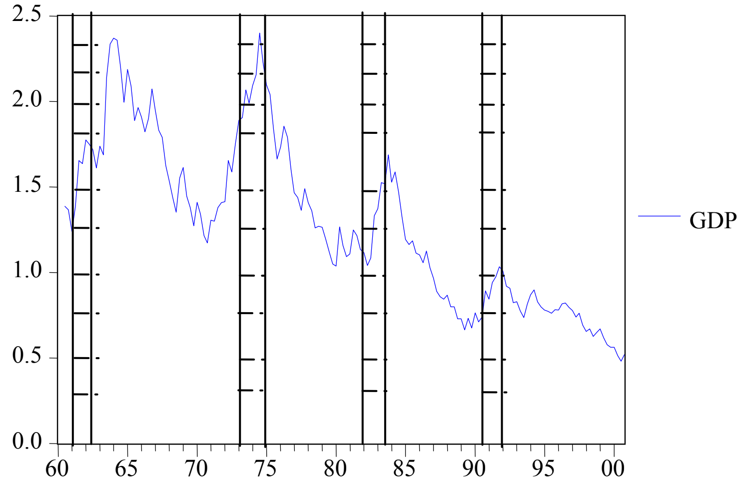

Figure 1. Conditional Volatility of GDP (CC-GARCH)

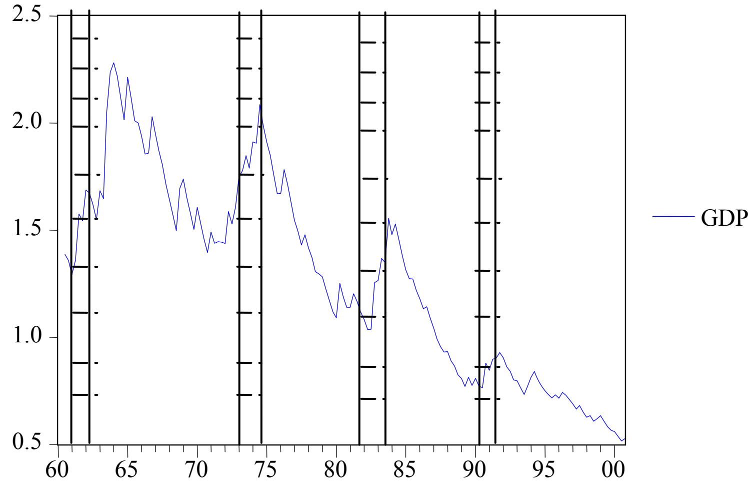

Figure 2. Conditional Volatility of GDP (CC-QGARCH)

pdfs/cycles/encrypted/08212007_bc_x_free.pdf. Further details are available from the authors upon request.

We mention in passing that with the exception of a few cases of significant test statistics, most diagnostic checks suggest that most of the constant-correlation asymmetric GARCH models are adequately specified. Both parametric and non-parametric tests generally indicate the absence of serial correlation in the standardised residuals. Regarding the Jarque-Bera test for normality, the null hypothesis of conditional normality cannot be rejected at the 5% level for almost all models. The test results are available from the authors upon request.

5. CONCLUSION

In this paper we have employed the bivariate asymmetric GARCH models to examine the conditional volatility of the Australian real GDP and expenditure components using quarterly data set covering the period from 1960Q1 to 2000Q4. We first test the plausibility of the constant-correlation assumption by employing Tse's (2000) LM test and Bera and Kim's (2002) IM test. The results indicate that the correlations among the shocks to real GDP and its various expenditure components are invariant over time, and these shocks are not highly correlated with one another. It seems that the constant-correlation GARCH model is adequately specified. We then estimate the four bivariate asymmetric GARCH specifications to determine the existence of volatility asymmetry in the Australian business cycle. Some weak evidence is found for the asymmetric conditional volatility in the growth rates of the Australian real GDP and the other components with the exception of gross fixed capital formation. Despite the weak evidence of asymmetric volatility, we notice that higher volatility is generally associated with the contractionary phase of the Australian business cycle. The finding is robust to alternative specifications of the conditional variance equation.

Our findings have important implications for macroeconomic policy and forecasting for business cycle. First, models that explicitly include the variance of income or activity should incorporate time-varying variance of income and activity over the business cycle. Second, macroeconomic forecasting of Australia should take into consideration the possibility of time-varying volatility to develop more realistic confidence intervals of forecasts.

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Mailing Address: Zhaoyong Zhang, School of Accounting, Finance and Economics, Edith Cowan University, 100 Joondalup Drive, Joondalup, WA 6027, Australia. Tel: 61-8-6304-5266. Fax: 61-8-6304-5271. E-mail: zhaoyong.zhang@ecu.edu.au.

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