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PRODUCT QUALITY, INCOME INEQUALITY AND MARKET STRUCTURE

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This paper analyses the effects of a change in distribution of income on quality choice made by firms producing search goods. It is assumed that willingness to pay for quality of consumers is an increasing function of income. Under the assumption that distribution of income is positively skewed, which is a common characteristic of developing countries, any redistribution of income will induce firms to improve their quality levels in duopoly and monopoly markets if redistribution makes consumers equally or unequally better off. On the other hand, quality levels will deteriorate if poverty is distributed more equally among consumers.

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JEL classification: L15, D31, D42, D43

1. INTRODUCTION

A number of papers in the recent past are concerned with the models of vertical product differentiation (that is differentiation by quality) under different market structures, heterogeneous preferences and income disparities. In this paper we try to find how a change in the distribution of income will affect the quality choice made by firms producing search goods under different market structures with two rather uncommon assumptions, namely, positively skewed income distribution and willingness to pay parameter value expressed as a function of income of individual.

The inequality in the distribution of income is a common and noteworthy feature of developing and under developed economies. In recent years the good development strategy became synonymous with import liberalization and outward orientation (Jalan (1991)) and less government intervention. This idea is reflected in the Structural Adjustment Programme adopted by IMF in different third world countries in the early

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nineties (Jalan (1991)). This particular policy requires a high export growth for the maintenance of balance of payments equilibrium. It has been shown elsewhere that among various factors that influence export of a developing country like India non-price factors seem to be dominant (Marjit and Raychaudhuri (1997)). The most significant non-price factor that determines the competitiveness of the export goods is the quality of products. Unless quality standards are maintained properly, goods cannot be sold in the market. In recent years the problem of improvement in product quality has become a matter of serious concern in different third world and less developed countries. Along with this, the policies adopted by third world countries to initiate economic liberalization are also affecting the distribution of income. Not only that, reduction of absolute poverty and relative income inequality are also a part of government policies of less developed counties. This paper tries to find the impact of a change in relative income inequality on the price and quality levels served by firms, in a situation where income is unequally distributed among the population.

The effect of income disparities on a vertically differentiated industry was first analysed by Gabszewicz and Thisse (1979). They considered the impact of income distribution parameters on a vertically differentiated duopoly model where firms are simultaneously determining their prices with exogenously given quality levels of their product. According to them at low average income level both firms have incentives to support a policy that raises average income. Beyond some level, such an increase is no longer profitable to low-quality producer as consumers will become rich enough to purchase the high quality good. Secondly considering standard deviation as a measure of income inequality the model shows that if income distribution becomes more egalitarian the low quality product will disappear from the market. This entire analysis is based on the assumption that income is following a uniform distribution. The assumption of uniform distribution is present in different papers by Shaked and Sutton (1982, 1983), and Boom (1995).

Given that our basic aim is to analyse the effect of different income inequality reduction measures on quality level served by the firms of under developed countries, we have combined the endogenous quality choice with non-uniform distribution of income. Assumption of non-uniform distribution of income helps as to address different types of changes in the relative income inequality that cannot be captured using the Gabszewicz and Thisse (1979) model with uniform distribution of income with exogenous quality choice. For example, the uniform distribution cannot take account of the impact of any asymmetric improvement in the level of income in favour of any particular income group.

The present paper has widened the scope of this type of analysis in the following manner:

- Non-uniform distribution of income is used for the analysis;
- The product quality choice has been made endogenous;
- The model uses the idea of relative concentration curve as developed by Kakwani

(1977) to measure relative income inequality.
- The willingness to pay parameter value for quality of a representative consumer is defined as a function of his income level.

In this analysis we use Mussa and Rosen (1978) type consumer preference structure. According to them consumers are heterogeneous. They have different willingness to pay for quality. Each consumer is indexed by a willingness to pay parameter value. A consumer with higher willingness to pay for quality will prefer a high quality product. In most of the papers it is assumed that the willingness to pay parameter is uniformly distributed (Choi and Shin (1992), Motta (1993), Wauthy (1996) Ecchia and Lambertini (1997)). In the analysis here, it is assumed that willingness to pay for quality of an individual is determined by her income level. An individual with high income has higher willingness to pay for quality and prefers high quality product. In this case inequality in the distribution of income is the reason for heterogeneity in consumer preferences. This type of interpretation of willingness to pay parameter is present in the analysis of Tirole (1988). There, this parameter is considered as marginal rate of substitution between income and quality, which is also equal to inverse of the marginal utility of income. A wealthy consumer has lower marginal utility of income and thus has higher willingness to pay for quality. Given that willingness to pay parameter is a function of income level of individual, the distribution followed by willingness to pay parameter is conditional upon the distribution of income. Any change in the distribution of income will change the distribution of willingness to pay parameter.

The impact of income redistribution measures on quality is interlinked with structure of markets. For this reason we consider the quality choice of firms in both monopoly and duopoly models using the same set of parameters regarding consumers preference pattern.

First we consider how changes in the distribution of income affects the quality choice made by an imperfectly discriminating monopolist supplying search goods. Here following Mussa and Rosen (1978) we assume that cost function of monopolist is convex in quality and linear in quantity.

Secondly the model considers the effects of a change in income distribution parameters on a two-stage duopoly where firms simultaneously choose their quality levels at the first stage of the game and prices are determined simultaneously at the second stage of the game. It is assumed that the producers are supplying two different varieties of a search good with identical cost function that is convex in quality and linear in quantity. The model assumes that the market is endogenously covered.

On the basis of above mentioned assumptions the model reveals some interesting results, which may seem counterintuitive.

- If income inequality is reduced by making consumers equally worse off, a monopolist reduces his quality levels. However certain degree of inequality is required for the existence of both high and low quality product simultaneously in the market. Under the same set of assumptions in a duopoly market high and low quality producers also reduce their quality levels. However if the degree of inequality is reduced below a certain critical level the low quality producer is driven out of the market. This last result is same as that obtained by Gabszewicz and Thisse (1979), but their model did not focus on the movement in the quality levels for the changes in the degree of income inequality.

- Next the model considers an increase in the average income of the population that also reduces degree of income inequality by making consumers equally rich. In this case the monopolist will always improve high quality level. In the duopoly structure both firms will improve their quality levels.

- Finally another type of redistribution of income is considered where position of low-income group is improved by making the distribution of income more concentrated. Due to this asymmetric change in the distribution of income degree of income inequality actually rises. Under this situation the monopolist for some parameter values first reduces quality levels but later improves quality levels of his product. In the duopoly model high quality producer will always improve his quality level and low quality producer will also improve his quality level provided he operates in the market. Thus in this situation even an increase in the degree of income inequality is driving low quality producer out of the market. This result may seem counter-intuitive. The asymmetric improvement in the distribution of income in favour of the low-income group causes low quality consumers to switch towards high quality product and low quality producer disappears from the market for drastic change in income inequality. This result is in contrast with the result obtained by Gabszewicz and Thisse (1979). According to their analysis as we have already mentioned a systematic reduction in income inequality will drive the low quality producer out of the market. But this result shows that even an increase in the relative income inequality is causing the low quality producer out of the market. Basically with the uniform distribution we cannot take account of all types of measures taken to remove relative income inequality. So our model has helped to make the entire analysis closer to reality due to the presence of non-uniform distribution of income.

Rest of the paper is organised in the following way - in Section 2 the basic model is described. In Section 3 the nature of change in distribution of income is analysed. Section 4 shows how a change in the inequality coefficient of the income distribution will affect the quality choice of a monopolist. Section 5 gives the effect of income distribution change on the quality levels served by firms in a vertically differentiated duopoly. Section 6 gives the conclusion.

2. THE MODEL

2.1. The Preference Structure

There is a continuum of consumers, whose types are identified by, θ which is distributed over the range given as $[\underline{\theta}, \overline{\theta}]$. The distribution function is given as $H(\theta)$ and density function is given as $h(\theta) = H'(\theta)$. θ is the index of willingness to pay for quality of an individual consuming one unit of a search good. The utility function of type θ is defined as

$$u = \theta q - p \quad \text{if the consumer purchases one unit of product with price } p \text{ and quality } q.$$
$$0 \qquad \text{otherwise} \tag{1}$$

Here it is assumed that willingness to pay for quality is a function of income of the individual. Let y be the income and it is distributed over the given range $[\underline{y}, \overline{y}]$. The distribution function is $F(y)$ and density function is $f(y) = F'(y)$.
We further assume that

$$\theta = \theta(y) \quad \theta' > 0,\ \theta'' < 0 \tag{2}$$

with $\underline{\theta} = \theta(\underline{y})$ and $\overline{\theta} = \theta(\overline{y})$

Following Flam and Helpman (1987) the distribution function of θ is defined as follows:

$$H(\theta) = k(\theta - \underline{\theta}) + \delta G(\theta) \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}] \tag{3}$$ [1]

Then the density function of θ is defined as

$$h(\theta) = k + \delta g(\theta) \text{ for } \forall\, \theta \in [\overline{\theta}, \underline{\theta}], \text{ where } h(\theta) = H'(\theta) \text{ and } g(\theta) = G'(\theta) \tag{4}$$

$$= 0 \quad \text{otherwise}$$

2.2. The Income Distribution Function

The distribution function of income is defined as

[1] For detailed explanation regarding the properties of density function see APPENDIX 1.

$$F(y) = k[\theta(y) - \theta(\underline{y})] + \delta G(\theta(y)) \tag{5}$$

$$\frac{dF}{d\delta} = G(\theta(y)) > 0$$

The density function of income is defined as

$$f(y) = [k + \delta g(\theta(y))]\theta'(y) \tag{6}$$

The income follows a positively skewed distribution.[2]

In this case for simplicity of analysis and to avoid fuzziness we will be assuming that

$$G(\theta) = (\theta - \underline{\theta})(\overline{\theta} - \theta) \tag{7}$$

$$g(\theta) = G'(\theta) = (\overline{\theta} + \underline{\theta} - 2\theta) \tag{8}$$

$$G''(\theta) = g'(\theta) = -2 < 0 \text{ for all } [\overline{\theta}, \underline{\theta}]$$

From properties of density function we must have $\sqrt{\delta} \le k$.[3]

3. INCOME DISTRIBUTION CHANGE AND RELATIVE INCOME INEQUALITY

3.1. Nature of Income Distribution Change

In this paper we consider two different types of change in income distribution pattern.

Firstly we consider a change in δ values which implies a redistribution of income keeping domain of income $[\underline{y}, \overline{y}]$ fixed.

Secondly we consider an improvement in absolute income levels of individuals by

[2] See APPENDIX 1 for proof.

[3] $h(\theta) \ge 0$ for $\forall \theta \in [\overline{\theta}, \underline{\theta}]$. Now $g(\theta) = G'(\theta) < 0$ for $\theta > \dfrac{\overline{\theta} + \underline{\theta}}{2}$.

To get $h(\theta) \ge 0$ for all θ we try to show that $h(\overline{\theta}) \ge 0$.

$h(\overline{\theta}) = k + \delta(\overline{\theta} + \underline{\theta} - 2\overline{\theta}) = k - \delta(\overline{\theta} - \underline{\theta}) = k - \dfrac{\delta}{k} \ge 0 \Rightarrow k^2 \ge \delta$ or $k \ge \sqrt{\delta}$.

changing the domain $[\underline{y}, \bar{y}]$. A change in $[\underline{y}, \bar{y}]$ will in turn affect $[\underline{\theta}, \bar{\theta}]$ and k.

In order to find out the effects of a change of above parameters on relative income inequality we will use the idea of relative concentration curve as developed by Kakwani (1977).

3.2. Effects of a Change in δ on Relative Income Inequality

From (5) we have

$$\frac{dF}{d\delta} = G(\theta(y)) > 0$$

Thus in this case if the value of δ is increased, proportion of population below any income group will rise. This implies that an increase in δ puts a higher weight of the probability mass on poorer part of the distribution of the income and consumers become equally poor.

Let us consider two different values of δ, given as δ_1 and δ_2, with $\delta_1 > \delta_2$.
We define

$$f(y) = [k + \delta_1 g(\theta(y))]\theta'(y)$$

$$f^*(y) = [k + \delta_2 g(\theta(y))]\theta'(y)$$

Here $f(y)$ and $f^*(y)$ are density functions corresponding to δ_1 and δ_2 respectively.

Corresponding distribution functions are

$$F(y) = k[\theta(y) - \theta(\underline{y})] + \delta_1 G(\theta(y))$$

$$F^*(y) = k[\theta(y) - \theta(\underline{y})] + \delta_2 G(\theta(y))$$

We have

$$\mu = \int_{\underline{y}}^{\bar{y}} yf(y)dy \quad \text{and} \quad \mu^* = \int_{\underline{y}}^{\bar{y}} yf^*(y)dy$$

The first moment distribution functions are defined as

$$F_1(y) = \frac{1}{\mu}\int_{\underline{y}}^{y} yf(y)dy \quad \text{and} \quad F_1^*(y) = \frac{1}{\mu^*}\int_{\underline{y}}^{y} yf^*(y)dy$$

We draw the relative concentration curve of $F_1(y)$ vs $F_1^*(y)$. This relative concentration curve will pass through (0,0) and (1,1). If it is concave (or convex) then $F_1(y)$ is greater (or less) than $F_1^*(y)$ for all levels of y and Lorenz curve of income distribution function $F_1(y)$ will lie wholly above (below) the Lorenz curve of income with distribution function $F_1^*(y)$.

Lemma 1 : *Given the distribution function of income, a rise in δ leads to a decrease in the relative income inequality.*

Proof:

$$\frac{dF_1}{dy} = \frac{yf(y)}{\mu} \quad \text{and} \quad \frac{dF_1^*}{dy} = \frac{yf^*(y)}{\mu}$$

The slope of relative concentration curve of $F_1(y)$ vs $F_1^*(y)$ is given by

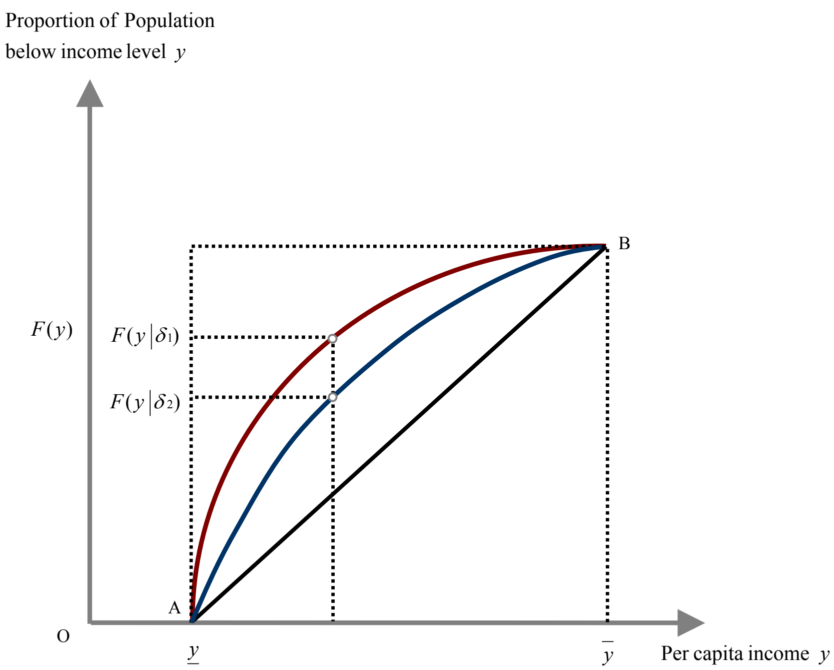
$$\frac{dF_1}{dF_1^*} = \frac{\mu^* f(y)}{\mu f^*(y)} > 0$$

$$\frac{d^2F_1}{dF_1^{*2}} = \frac{\mu^{*2}}{\mu}\frac{f}{f^{*2}y^2}\left[\frac{df}{dy}\frac{y}{f} - \frac{df^*}{dy}\frac{y}{f^*}\right]$$

$$= \frac{\mu^{*2}}{\mu}\frac{f}{f^{*2}y^2}\frac{g'(\theta(y))\theta'(y)yk(\delta_1-\delta_2)}{\{k+\delta_1 g(\theta(y))\}\{k+\delta_2 g(\theta(y))\}}$$

Given $\delta_1 > \delta_2$ and $g'(\theta(y)) < 0$ $\frac{d^2F_1}{dF_1^{*2}} < 0$ [hence proved]

In this case given that $\frac{d^2F_1}{dF_1^{*2}} < 0$, $F_1(y) > F_1^*(y)$ for all levels of income y. So for a rise in δ, Lorenz curve for redistributed income will lie wholly above the initial Lorenz curve. So gini coefficient of income inequality will fall. Thus a rise in δ actually decreases the income inequality.



Note: With $\delta_1 > \delta_2$, as δ falls distribution function moves towards AB line, and proportion of population below any income level y decreases.

Figure 1

3.3. Effect of a Change in $[\underline{y}, \overline{y}]$ on Relative Income Inequality

In this structure a possible change in income distribution pattern can be introduced by change in the domain of income. From properties of distribution function we have

$$k = \frac{1}{(\theta(\overline{y}) - \theta(\underline{y})} \tag{9}$$

Any improvement in income level of individuals, which is affecting $\overline{y}$ and $\underline{y}$ will change k. Here $(\theta(\overline{y}) - \theta(\underline{y}))$ gives that difference between willingness to pay for quality of highest income group and that of lowest income group. It is an index of the size of the market. The parameter k is inversely related to the size of the market.

$$dk = -k^2(\theta'(\overline{y})d\overline{y} - \theta'(\underline{y})d\underline{y}) \tag{10}$$

If $d\overline{y} > 0$ and $dy = 0$ then $dk < 0$. When benefit of change in distribution of income is distributed among the population by widening the domain of income, k will fall.

If $d\overline{y} = 0$ and $dy > 0$ then $dk > 0$. If policy variables increase income of lowest income group so that income distribution is made more concentrated, k will increase.

From (5) we have

$$dF = -(\theta(y) - \theta(\underline{y}))\theta'(\overline{y})d\overline{y}(k^2 - \delta) - (\theta(\overline{y}) - \theta(\underline{y}))d\underline{y}(k^2 + \delta)(\theta(\overline{y}) - \theta(y)) < 0 \tag{11}$$

So for improvement in $\overline{y}$ and (or) $\underline{y}$, number of consumers below any income level will fall and overall population will become rich.

Secondly we try to find how the average income level of the economy changes for improvements in $\overline{y}$ or $\underline{y}$

$$\mu = \int_{\underline{y}}^{\overline{y}} y(k + \delta(\theta(\overline{y}) + \theta(\underline{y}) - 2\theta(y))\theta'(y)dy$$

$$\frac{d\mu}{d\overline{y}} = \overline{y}\theta'(\overline{y})\left(\frac{k^2 - \delta}{k}\right) + \int_{\underline{y}}^{\overline{y}} \delta\theta'(\overline{y})y\theta'(y)dy > 0$$

Thus an increase in $\overline{y}$ will improve the average income level of the economy.

$$\frac{d\mu}{d\underline{y}} = -\underline{y}\theta'(\underline{y})\left(\frac{k^2 - \delta}{k}\right) + \int_{\underline{y}}^{\overline{y}} \delta\theta'(\underline{y})y\theta'(y)dy$$

In this case first part is always negative and second part is positive. We find that the effect of a redistribution in income through an increase in $\underline{y}$ is ambiguous in nature.

Next we try to find the nature of change in relative income inequality. Let the initial values of parameters are $(k_1, \overline{y}_1, \underline{y}_1)$ where $k_1 = \dfrac{1}{(\theta(\overline{y}_1) - (\underline{y}_1))}$

$$f(y) = [k_1 + \delta(\theta(\overline{y}_1) + \theta(\underline{y}_1) - 2\theta(y))]\theta'(y)$$

$$F(y) = k_1(\theta(y) - \theta(\underline{y}_1)) + \delta(\theta(y) - \theta(\underline{y}_1))(\theta(\bar{y}_1) - \theta(y))$$

Let the income levels are improved and new parameter values are $(k_2, \bar{y}_2, \underline{y}_2)$

$$f^*(y) = [k_2 + \delta(\theta(\bar{y}_2) + \theta(\underline{y}_2) - 2\theta(y))]\theta'(y)$$

$$F^*(y) = k_2(\theta(y) - \theta(\underline{y}_2)) + \delta(\theta(y) - \theta(\underline{y}_2))(\theta(\underline{y}_2) - \theta(y))$$

The slope of relative concentration curve is

$$\frac{dF_1}{dF_1^*} = \frac{\mu^* f(y)}{\mu f^*(y)} > 0$$

$$\frac{d^2F_1}{dF_1^{*2}} = \frac{\mu^{*2}}{\mu}\frac{f}{f^{*2}y^2}\left[\frac{df}{dy}\frac{y}{f} - \frac{df^*}{dy}\frac{y}{f^*}\right]$$

The sign of the expression $\left[\frac{df}{dy}\frac{y}{f} - \frac{df^*}{dy}\frac{y}{f^*}\right]$ determines the direction of change in the of slope of relative concentration curve.
From the above expression it can be shown that

$$\left[\frac{df}{dy}\frac{y}{f} - \frac{df^*}{dy}\frac{y}{f^*}\right] = \frac{2\delta\theta'(y)y[\theta'(\bar{y}_1)d\bar{y}(k_1^{\ 2} - \delta) - \theta'(\underline{y}_1)d\underline{y}(k_1^{\ 2} + \delta)]}{\{k_2 + \delta(\theta(\underline{y}_2) + \theta(\bar{y}_2) - 2\theta(y))\}\{k_1 + \delta(\theta(\underline{y}_1) + \theta(\bar{y}_1) - 2\theta(y))\}} \qquad \text{(A)}$$

where k_1 is the initial value of k

Lemma 2 : *A redistribution of income that entails an increase in only $\bar{y}$ will reduce relative income inequality and any redistribution with an increase in only $\underline{y}$ will increase income inequality.*

Proof : Obtained by putting $d\bar{y} = 0$ and $d\underline{y} = 0$ respectively in the expression (A) above.

So following Lemma 2, if $\bar{y}$ changes alone, then $F_1(y) < F_1^*(y)$ for all levels of income y. So for rise in $\overline{y}$ with $\underline{y}$ fixed, Lorenz curve for redistributed income will lie wholly above the initial Lorenz curve. So gini coefficient of income inequality will fall. Thus a rise in $\overline{y}$ is actually reducing the income inequality. Basically an increase in $\overline{y}$ expands the domain of distribution of income. This allows upward income mobility of people belonging to all income groups below initial value of $\overline{y}$. Thus number of people belonging to all income groups below initial value of $\overline{y}$ will fall and the distribution becomes less skewed, or relative income inequality is reduced.

On the other hand, if $\underline{y}$ changes alone, then the relative concentration curve is concave and $F_1(y) > F_1^*(y)$ for all levels of income y. So for a rise in $\underline{y}$ with $\overline{y}$ fixed, Lorenz curve for redistributed income will lie wholly below the initial Lorenz curve. So gini coefficient of income inequality will rise. Thus a rise in $\underline{y}$ is actually increasing the income inequality. This result may seem counter intuitive. Suppose $\underline{y}$ is increased to $\underline{y}'$. An increase in $\underline{y}$ narrows the domain of distribution of individual income and this improves the absolute position of people belonging to the lower end of the distribution of income (that is people belonging to the range $\underline{y} < \underline{y}'$) without affecting the position of higher income groups. Thus for this type of redistribution of income proportion of people in the lowest income level (that is in $\underline{y}'$) rises while number of people in the higher income groups remains the same (here we are rejecting the possibility of downward income mobility). So this makes distribution of income more skewed and relative income inequality rises even if the absolute poverty of lower income groups is reduced.

4. THE IMPACT OF CHANGE IN INCOME DISTRIBUTION ON MONOPOLY MARKET

4.1. The Monopoly Structure

In this case we try to find the price and quality levels provided by an imperfectly discriminating monopolist producing two different varieties of a search good where cost function is linear in quantity and convex in quality.

Let the cost function be defined as $C = \alpha q^2 x$ where q is the quality level and x is the quantity level and $\alpha > 0$. We assume that monopolist is imperfectly segmenting his market. He serves two different quality levels of a search good.

Let q_h : quality level of high quality product

q_1 : quality level of low quality product where $q_h > q_1$ at all levels

p_h : price level of high quality product
p_1 : price level of low quality product
x_h : demand level of high quality product
x_1 : demand level of low quality product

Let us consider a value of θ such as θ^* for which an individual is indifferent between purchasing high and low quality products.

$$\theta^* = \frac{p_h - p_1}{q_h - q_1} \tag{12}$$

Here $\underline{\theta} < \theta^* < \overline{\theta}$

An individual with value of $\theta \geq \theta^*$ will purchase high quality product. So $x_h = 1 - H(\theta^*)$. Now an individual will buy low quality product if $\theta q_1 - p_1 \geq 0$. Let us define a value $\theta = \theta_1$ where

$$\theta_1 = \frac{p_1}{q_1} \tag{13}$$

An individual with value $\theta < \theta_1$ is not purchasing anything. Let us define an income level y_1 where $\theta_1 = \theta(y_1)$. So individuals with income below y_1 are facing a purchasing power constraint and are not consuming anything. Under this case demand for low quality product is given by $x_1 = H(\theta^*) - H(\theta_l)$. Now equilibrium value of θ_1 depends on the quality and price choice of the monopolist. We assume the monopolist is maximizing his profit to determine p_h, p_1, q_h, q_1.

The profit of the monopolist is defined as

$$\pi = [p_h - \alpha q_h^2](1 - H(\theta^*)) + [p_1 - \alpha q_1^2](H(\theta^*) - H(\theta_1)) \tag{14}$$

The first order conditions of profit maximization are as follows :

$$\frac{\partial \pi}{\partial p_h} = 0 \Rightarrow \theta^* = \alpha(q_h + q_1) + \frac{1 - H(\theta^*)}{h(\theta^*)} \tag{15}$$

$$\frac{\partial \pi}{\partial p_1} = 0 \Rightarrow \theta_1 = \alpha q_1 + \frac{1 - H(\theta_1)}{h(\theta_1)} \tag{16}$$

$$\frac{\partial \pi}{\partial q_h} = 0 \Rightarrow \frac{p_h - p_1}{q_h - q_1}\left[\frac{p_h - \alpha q_h^2}{q_h - q_1} - \frac{p_l - \alpha q_1^2}{q_h - q_1}\right] h(\theta^*) = 2\alpha q_h (1 - H(\theta^*))$$

Using Equation (15) we get the following equation

$$\theta^* = 2\alpha q_h \tag{17}$$

$$\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow \theta^*(1 - H(\theta^*)) + 2\alpha q_1 (H(\theta^*) - H(\theta_1)) = (1 - H(\theta_1))\theta_1 \tag{18}$$

Solving (15), (16), (17), (18) simultaneously we get the equilibrium value of q_h, q_1, θ_h, θ_1. Given these values prices p_h and p_1 can be found from (12) and (13).[4]

4.2. Effect of a Change in δ

Given the first order conditions, we try to find how the quality levels of the product will change for a change in the income distribution parameter δ.[5]

In this case the computations show that as the value of δ increases from $\delta = 0$ to $\delta = 1$, over the whole range the equilibrium values of q_h, q_1, p_h, p_1, θ^* and θ_1 are declining. In this case, as explained in Section 3, a rise in δ has two implications. Firstly relative income inequality is reduced and secondly, at the same time, number of people below any income level is increased. Due to this second factor the overall population will become poorer for a rise in δ. So monopolist charges lower prices and also serve lower quality levels. Thus as inequality is reduced through cutting income from above, thereby making population poorer, the willingness to pay for quality of the marginal consumers consuming both high and low quality products will fall due to fall in p_h, p_1, q_h, q_1.

However under this structure existence of low quality product in the market depends on certain criterion. When θ_1 and θ^* both are falling, as θ_1 falls below $\underline{\theta}$ we have $H(\theta_1) = 0$. This implies that there is full market coverage endogenously. However as δ increases, if θ^* also moves below $\underline{\theta}$, we have zero demand for low quality product in equilibrium. Thus low quality will not be sold in the market.

[4] In this framework, to solve the simultaneous equations we use the algorithm of "Newton's method for solving the system of nonlinear equations".

[5] For second order conditions see APPENDIX 3.

Our computations with $\overline{\theta} = 2.5$, $\underline{\theta} = 1.5$ and $\overline{\theta} = 2.8$, $\underline{\theta} = 1.5$ show that for .54 $\leq \delta \leq 1$, θ^* and θ_1 will fall below $\underline{\theta} = 1.5$ and low quality product will disappear from the market. Basically as δ increases and the relative inequality is reduced, the monopolist will refrain from serving different qualities to different consumers.

To maximize profit through discrimination certain degree of income inequality is required in the monopoly market. However with $\overline{\theta} = 2.5$ and $\underline{\theta} = 1.8$, the low quality product will disappear from the market for $.27 \leq \delta \leq 1$. So for a given value of $\overline{\theta}$, as $\underline{\theta}$ increases and market size becomes more concentrated, low quality product will disappear for a lower value of δ relative to earlier cases. So we observe that a redistribution of income, which reduces relative income inequality by making consumers poorer, will reduce the quality levels served by the monopolist. However if inequality is reduced below a certain level consumers will only purchase high quality good and low quality product will disappear from the market.[6]

Thus we conclude that a monopolist will be able to discriminate among the consumers (or will offer a separating menu) with heterogeneous preferences if there exists certain degree of inequality in the income distribution, under the assumption that willingness to pay for quality is a positive function of income of the consumers. Hence the model shows more than just a dependence of willingness to pay on income - *the non-uniform distribution of income plays a more vital role here,* which is not really highlighted in other models. Also high quality level falls as inequality is reduced. This leads to the proposition 1 below.

Proposition 1 : *There exists a set of values of $\overline{\theta}$, $\underline{\theta}$, and k such that the monopolist will reduce the quality levels of the products, as inequality in the distribution of income is reduced as a consequence of a rise in δ. Secondly monopolist will serve two different quality levels in equilibrium (offering a separating menu) if and only if there exists certain degree of inequality in the market.*

4.3. Effect of a Change in $\overline{y}$

Now we consider the situation where a redistribution of income is introduced by increasing $\overline{y}$ while $\underline{y}$ is kept constant. In this case distribution of income is made wider. A rise in $\overline{y}$ will increase $\overline{\theta}$ and reduce k. We have already proved that a rise

[6] The result of Mussa and Rosen (1978) shows that the discrimination is always profitable for the monopolist. However that result is based upon a given distribution of willingness to pay parameter value of the consumers, which is exogenously given. In this paper we have linked that distribution to distribution of personal income and relative inequality of income. Hence we show that certain degree of income inequality is required for profitable discrimination among consumers.

in $\overline{y}$ will reduce relative income inequality and make consumers richer. It is also improving average income of the economy. In this case computations show that direction of change in price levels and quality levels for a rise in $\overline{y}$ will depend upon values of δ also.

First we consider the model with $\overline{\theta} = 2.5$ $\underline{\theta} = 1.5$.

In the Figures 2, 3 and 4 values of quality levels, prices, and willingness to pay parameters corresponding to these parameter values of $\overline{\theta}$ and $\underline{\theta}$ are given as (q_h, q_1), (p_h, p_1) and (θ^*, θ_1) respectively. Next we consider the case where $\overline{y}$ has increased so that $\overline{\theta} = 2.8$ while $\underline{\theta}$ remains constant. In the figures, values of quality levels, prices, and willingness to pay parameters corresponding to these new parameter values of $\overline{\theta}$ and $\underline{\theta}$ are given as (q_h1, q_11), (p_h1, p_11) and (θ^*1, θ_11) respectively.[7]

The computations show that as $\overline{y}$ has increased high quality level and price of high quality good has increased over the relevant range of δ values. Basically a rise in $\overline{y}$ has reduced relative income inequality by making consumers richer. It is also increasing average income level of the economy. For this reason the monopolist is charging higher price and also serving better quality to consumers purchasing high quality product. The willingness to pay for the marginal consumer consuming high quality good will also increase.

The monopolist is serving better quality level and charging a relatively higher price to even low quality consumer for a rise in $\overline{y}$ for lower values given as $0 \leq \delta \leq .3$. A lower δ value implies higher degree of income inequality. In this situation if $\overline{y}$ is increased, the overall population will become richer and inequality is also reduced. Under this situation the monopolist is serving better quality good at higher price to low quality consumers also. However as δ is increased ($\delta > .3$) with corresponding rise in $\overline{y}$ (with $\overline{\theta} = 2.8$, $\underline{\theta} = 1.5$) income inequality is reduced further and population is also better off. In this case to sell the low quality product the monopolist changes its strategy and reduces the quality level of low quality good and charges lower price compared to the initial situation of $\overline{\theta} = 2.5$, $\underline{\theta} = 1.5$. However for $\delta > .53$ low quality product will not be sold. (See Figures 2, 3 and 4) In this case as inequality is reduced, all consumers will switch towards high quality product and low quality product disappears from the market.

Proposition 2 : *A reduction in relative income inequality by increasing $\overline{y}$ increases willingness to pay for quality of the consumers and this will induce the monopolist to improve the high quality level for all values of δ. Low quality level is improved for lower*

[7] In this case with $\overline{\theta} = 2.8$ and $\underline{\theta} = 1.5$ the value of $k = 0.769$. From the properties of the distribution function we require that $k^2 \geq \delta$. So values of the variables are calculated till $\delta = .59$.

values of δ. For higher values of δ as inequality is reduced monopolist initially lowers low quality levels, but after a certain level it is not sold in the market at all.

4.4. Effect of a Change in $\underline{y}$

A redistribution of income that entails an improvement of $\underline{y}$ will increase income inequality. A rise in $\underline{y}$ narrows the domain of distribution of income. This in turn improves the position of people belonging to the lower end of the distribution of income without affecting the position of people in higher side of the distribution of income. The effect of an increase in $\underline{y}$ on average income level is also ambiguous in sign. Under the circumstances initially we have $\overline{\theta} = 2.5$, $\underline{\theta} = 1.5$. For a rise in $\underline{y}$, parameter values become $\overline{\theta} = 2.5$, $\underline{\theta} = 1.8$. In the figures given below the values of quality levels, prices and willingness to pay parameter values of marginal consumer corresponding to $\overline{\theta} = 2.5$, $\underline{\theta} = 1.8$ are given as $(q_h 2, q_l 2)$, $(p_h 2, p_l 2)$ and $(\theta^* 2, \theta_l 2)$ respectively. (See Figures 2, 3 and 4)

In this case our computations show some interesting results. At $\delta = 0$ the quality levels, prices and willingness to pay parameter values of the marginal consumer will be unchanged for an increase in $\underline{y}$. As δ is increased gradually, the monopolist will serve lower quality levels and charge lower prices relative to the initial level. Subsequently, willingness to pay for quality levels of the marginal consumer consuming high and low quality product will also fall. Basically at low values of δ, degree of income inequality is high. In this situation a redistribution of income will cause further deterioration in the income inequality pattern and due to this effect, the monopolist reduces quality levels even when some consumers are becoming richer. However for higher values of δ, when degree of inequality is becoming lower, a rise in $\underline{y}$ will lead to improvement in both quality levels because the second effect of a rising $\underline{y}$ (i.e., consumers are becoming richer) will dominate the increase in the degree of income inequality. At higher value of δ, the willingness to pay parameter values for quality of the marginal consumer will also increase leading to higher levels of quality. However low quality product will completely disappear from the market for $.27 \leq \delta \leq 1$, even when $\underline{y}$ increases because population has become rich enough to consume high quality level only.

Proposition 3 : *A rise in relative income inequality due to improvement in $\underline{y}$ will induce the monopolist to reduce quality levels for lower values of δ. At higher values of δ both quality levels will be improved. However for a specific range of δ, low quality is not offered at all.*

5. THE IMPACT OF REDISTRIBUTION OF INCOME ON DUOPOLY MARKET

5.1. The Duopoly Structure

In this section we analyse how the change in income distribution parameter will affect the quality choice made by duopolists producing two different varieties of a search good with identical cost functions, and endogenous market coverage. In the models of Shaked and Sutton (1983) and Cremer and Thisse (1999) there is a priori upper bound to the level of quality. In this model there is no such restriction on the quality level. It is only assumed that quality level assumes positive finite values. We assume that firm 1 is producing high quality variety of product with quality level q_h, price level p_h and demand for commodity x_h. Firm 2 is producing the low quality variety of the same product with quality level q_1, price level p_1 and demand for commodity x_1. Here we assume that $q_h > q_1$ at all levels. The cost function of each firm is identical, and it is linear in quantity and convex in quality.

Given heterogeneous preference structure, there is a value of θ given as θ^* for which a consumer is indifferent between purchasing from high and low quality producers.

θ^* is defined as $\theta^* = \dfrac{p_h - p_1}{q_h - q_1}$

$$p_h = p_1 + \theta^*(q_h - q_1) \tag{19}$$

Now consumers with value of $\theta \in [\theta^*, \bar{\theta}]$ purchase the high quality product.

So $x_h = \Pr \text{ob } [\theta \geq \theta^*] = 1 - H(\theta^*)$.

Let θ_1 be the willingness to pay for quality of the marginal consumer consuming low quality product where $\theta_1 = \dfrac{p_1}{q_1}$ or,

$$p_1 = \theta_1 q_1 \tag{20}$$

Consumers with willingness to pay parameter value below θ_1 will not purchase anything. In this case size of market is endogenously determined. Thus, consumers with value of $\theta \in [\theta_1, \theta^*]$ will purchase low quality product. So demand faced by low quality producer is given as

$x_1 = \Pr ob\ [\theta_1 < \theta < \theta^*] = H(\theta^*) - H(\theta_1)$.

Here we consider a two-stage game. In the first stage of the game firms are simultaneously choosing their quality levels. In the second stage of the game firms, given the quality chosen in the first stage of the game, simultaneously determine price levels. The profits are determined as

$$\pi_h = [p_h - \alpha q_h^2] x_h \tag{21}$$

$$\pi_1 = [p_1 - \alpha q_1^2] x_1 \tag{22}$$

The game is solved by backward induction method. Initially profits are maximized with respect to prices given quality levels, and optimal prices are found as functions of quality levels. In this case we try to find a subgame perfect Nash equilibrium, so in the second step profits are maximized simultaneously with respect to quality levels, given the prices as function of quality levels. Thus Nash equilibrium quality levels are found.

$$\frac{\partial \pi_h}{\partial p_h} = 0 \Rightarrow p_h = \alpha q_h^2 + \frac{\left(1 - H(\theta^*)\right)\left(q_h - q_1\right)}{h(\theta^*)} \tag{23}$$

Using (19) and (20) we have

$$(1 - H(\theta^*))(q_h - q_1) - h(\theta^*)(\theta^*(q_h - q_1) + \theta_1 q_1 - \alpha q_h^2) = 0 \tag{24}$$

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow (H(\theta^*) - H(\theta_1)) - (p_1 - \alpha q_l^2)\left(\frac{h(\theta^*)}{q_h - q_1} + \frac{h(\theta_1)}{q_1}\right) = 0 \tag{25}$$

Using (20) we have

$$(H(\theta^*) - H(\theta_1))(q_h - q_1) - (\theta_1 - \alpha q_1)(h(\theta^*) q_1 + h(\theta_1)(q_h - q_1)) = 0 \tag{26}$$

Solving (23) and (25), Nash equilibrium prices are found as functions of quality levels. For simplicity of analysis we replace expression for prices and get (24) and (26). Differentiating totally (24) and (26) with respect to q_h will give $\frac{d\theta^*}{dq_h}$ and $\frac{d\theta_1}{dq_h}$ and that of q_1 will give $\frac{d\theta^*}{dq_1}$ and $\frac{d\theta_1}{dq_1}$.

Given (23) and (25) reduced form profits are given as follows

$$\pi_h = \frac{(q_h - q_1)}{h(\theta^*)} x_h^2 = \frac{(q_h - q_1)}{h(\theta^*)}(1 - H(\theta^*))^2 \tag{27}$$

$$\pi_1 = \frac{x_1^2}{\left[\dfrac{h(\theta^*)}{(q_h - q_1)} + \dfrac{h(\theta_1)}{q_1}\right]} = \frac{\left(H(\theta^*) - H(\theta_1)\right)^2 (q_h - q_1) q_1}{[h(\theta^*) q_1 + h(\theta_1)(q_1 - q_1)]} \tag{28}$$

$$\frac{\delta \pi_h}{\delta q_h} = 0 \Rightarrow -2(1 - H(\theta^*))(q_h - q_1)\frac{d\theta^*}{dq_h} + \frac{(1 - H(\theta^*))^2}{h(\theta^*)} - \frac{(1 - H(\theta^*))^2}{h^2(\theta^*)}(q_h - q_1) h'(\theta^*)\frac{d\theta^*}{dq_h} = 0 \tag{29}$$

$$\frac{\delta \pi_1}{\delta q_1} = 0 \Rightarrow \frac{(H(\theta^*) - H(\theta_1))^2 (q_h - 2q_1)}{(h(\theta^*) q_1 + h\delta(\theta_1)(q_h - q_1))} + \frac{2(\delta H(\theta^*) - H(\theta_1))(q_h - q_1) q_1}{(h(\theta^*) q_1 + h(\theta_1)(q_h - q_1))}\left\{h(\theta^*)\frac{d\theta^*}{dq_1} - h(\theta_1)\frac{d\theta_1}{dq_1}\right\}$$

$$-\frac{2(\delta H(\theta^*) - H(\theta_1))^2 (q_h - q_1) q_1}{(h(\theta^*) q_1 + h(\theta_1)(q_h - q_1))^2}\left\{(h(\theta^*) - h(\theta_1)) + h'(\theta^*) q_1 \frac{d\theta^*}{dq_1} + h'(\theta_1)(q_1 - q_1)\frac{d\theta_1}{dq_1}\right\} = 0 \tag{30}$$

Given (23) and (25), Nash equilibrium quality levels can be obtained by simultaneously solving (29) and (30)

Next we try to find how quality levels served by firms will change for change in different income distribution parameters.[8]

5.2. Effects of a Change in δ

Computations show that for different values of $\overline{\theta}$ and $\underline{\theta}$ a rise in δ, which leads to a fall in relative income inequality, will induce both firms to reduce quality levels along with the price levels. Willingness to pay for quality of marginal consumer consuming both low and high quality product will also fall for a rise in δ. Basically a rise in δ is reducing relative income inequality but number of people below any income level increases. This implies that overall population is becoming poorer for this type of redistribution of income. For this reason willingness to pay for quality for marginal consumer consuming the high and low quality products will decline with a rise in δ, so that both firms will reduce the price levels and supply poor quality goods to the market.

[8] Computations show that second order conditions of the duopoly case are always satisfied for given parameter values.

Proposition 4 : *A redistribution of income that makes overall population poorer by improving the relative income inequality through an increase in δ will cause both firms to reduce quality levels in a duopoly market under the assumption of endogenous market coverage.*

5.3. Effect of Change in $\bar{y}$

As we have already mentioned an increase in $\bar{y}$ reduces relative income inequality by making overall population richer and it also increases average income level of the economy. A rise in $\bar{y}$ that increases $\bar{\theta}$ will induce both firms to serve better quality levels at higher prices. Initially we have considered the case with $\bar{\theta}$ = 2.5, $\underline{\theta}$ = 1.5 and next we have considered the case with $\bar{\theta}$ = 2.8, $\underline{\theta}$ = 1.5 (i.e., parameter values are the same as in the monopoly case). With an increase in $\bar{\theta}$, firms are charging higher prices and serving better quality goods as overall population has become richer. In this case a reduction in income inequality has caused an increase in the willingness to pay for quality of the marginal consumer consuming both high and low quality goods.

5.4. Effects of an Increase in $\underline{y}$

An increase in $\underline{y}$ implies a corresponding increase in $\underline{\theta}$. In this case income inequality increases and the consumers in the lower end of the distribution of income become richer. Because of this second effect both firms will improve the quality and charge higher prices relative to the initial level and willingness to pay for quality of the marginal consumer will improve. However our computations with $\bar{\theta}$ = 2.5, $\underline{\theta}$ = 1.8 show an interesting result. In this case both qualities will be available in the market for $\delta = 0$. For $\delta > 0$ high quality producer will drive low quality product out of the market for $\underline{\theta}$ = 1.8. A rise in $\underline{\theta}$ is increasing the degree of competition by reducing the size of the market. Hence low quality product is driven out of the market. It also induces the lower income group to switch towards high quality product due to an increase in their willingness to pay for quality.

Proposition 5 : *In a duopoly structure a rise in $\bar{y}$ or $\underline{y}$ will induce both firms to improve quality levels.*

5.5. Comparison with Monopoly Case

A comparison of results under two different market structures but identical parameter values reveals certain interesting facts.

Firstly in case of monopoly certain degree of income inequality is required for the existence of both high and low quality goods in the market. Our computations show that with $\overline{\theta} = 2.5$, $\underline{\theta} = 1.5$ and $\overline{\theta} = 2.8$, $\underline{\theta} = 1.5$ for $.54 \leq \delta \leq 1$, θ^* and θ_1 will fall below $\underline{\theta} = 1.5$ and low quality product will disappear from the market. So both goods are sold for $0 \leq \delta < .54$. With $\overline{\theta} = 2.5$, $\underline{\theta} = 1.8$, the low quality product will disappear from the market for $.27 \leq \delta \leq 1$.

However in case of duopoly the situation is totally different. With $\overline{\theta} = 2.5$, $\underline{\theta} = 1.5$ the low quality producer will face zero demand for his product for $\delta \geq .73$. So both qualities are sold even with lower degree of income inequality compared to monopoly case. In this case competition between the firms allows the low quality product to exist in the market. Under this situation for same values of δ, monopolist is serving lower quality levels compared to the duopoly market. Thus monopolist is under providing quality compared to the duopolists.

In case of monopoly, quality levels are falling more rapidly with respect to δ compared to the duopoly case. This is possible because in case of duopoly competition between firms acts as a check on the falling quality levels and the monopolist thus extracts greater amount of consumers' surplus compared to duopolists.

Secondly our computation with the parameter values of $\overline{\theta} = 2.8$, $\underline{\theta} = 1.5$ gives a different result. In this case in a duopoly situation low quality producer will leave the market for $\delta > .29$. This is because here a reduction in the relative income inequality and increase in the average income level are making consumers richer. Due to this effect consumers prefer to purchase high quality product. So low quality producer cannot compete with the high quality one and leaves the market for a lower value of δ compared to the monopoly case.

However our computation with $\overline{\theta} = 2.5$, $\underline{\theta} = 1.8$ shows that both qualities will be sold only for $\delta = 0$ in a duopoly market. A rise in $\underline{\theta}$ reduces the size of the market and increases the degree of competition between firms. Because of this effect low quality will be driven out of the market. In case of monopoly, in the same situation, both qualities will be sold for $0 \leq \delta < .27$. In a monopoly situation profit maximization policy of the monopolist will determine whether a single quality level or more than one quality levels will be sold in the market. In case of duopoly competition between the firms determines whether a single producer or both high and low quality producer will operate in the market for different values of $\overline{\theta}$ and $\underline{\theta}$ when market is not exogenously covered. In general given the same set of parameter values, in a monopoly market both quality levels can be sold while in a duopoly market, low quality may disappear from the market at a much higher relative income inequality compared to the monopoly market.

6. CONCLUDING REMARKS

In this paper, in the backdrop of skewed income distribution in a typical developing country, we have tried to find how a change in the income distribution parameter will affect product qualities served by firms under different market structures. First we have considered a monopoly case and secondly we analyse the case of a duopoly where market is endogenously covered. We have considered different types of changes in the distribution of income.

We find that when consumers are made poorer (thus reducing income inequality), a monopolist reduces quality levels of his products. Under the same set of assumptions, in a duopoly market, high and low quality producers are also reducing their quality levels. Given that index of willingness to pay is a function of income of consumers, a reduction in inequality of income where consumers are equally worse off reduces willingness to pay for quality of the consumers and producers serve lower quality levels in both types of markets. It is also shown that certain degree of inequality is always required for the existence of both high and low quality goods in the monopoly market.

Secondly we have considered another type of change in the distribution of income. In this case distribution of income is made wider and average income level is improved along with a reduction in the degree of relative income inequality. In this case a monopolist always improves high quality level as improvement in average income level and reduction in the degree of relative income inequality increase willingness to pay for quality. However, for some parameter values the monopolist reduces low quality level. But this type of redistribution of income will induce both firms to improve quality levels in a duopoly market as overall population becomes richer.

Thirdly we have considered another types of redistribution of income where distribution is made more concentrated and low-end consumers become richer, although the degree of income inequality increases. In this case for some values, the monopolist first reduces quality levels but later improves quality of its products. However under similar situations, high quality producing duopoly firm always improves quality level. Low quality producer also improves quality level simultaneously provided he operates in the market. Basically for this type of asymmetric change in the distribution of income in favour of low income group, consumers switch towards high quality level and low quality producer will disappear from the market for drastic change in income inequality.

Thus, the paper highlights the asymmetric response of producers under alternative market structures under similar set of values for income distribution and willingness to pay parameters. The key difference of behaviour in the two markets basically is a result of the degree of competition in the markets. The more competitive the market structure is the less is the impact of inequality of income distribution on the quality spectrum of goods. Also, worsening income inequality does not necessarily guarantee higher quality levels if it does not make consumers richer, there by increasing willingness to pay for quality of the marginal consumers.

APPENDIX 1

Following Flam and Helpman (1987) the distribution function of θ is defined as follows:

$$H(\theta) = k(\theta - \underline{\theta}) + \delta G(\theta) \qquad \forall\, \theta \in [\underline{\theta}, \overline{\theta}]$$

There exists a value

$\theta^0 \in (\underline{\theta}, \overline{\theta})$ such that $G'(\theta^0) = 0$ and $G'' < 0$ for $\forall\, \theta \in [\underline{\theta}, \overline{\theta}]$

Further, $g(\theta) = G'(\theta)$ for $\forall\, \theta \in [\underline{\theta}, \overline{\theta}]$

Then the density function of θ is defined as

$$h(\theta) = k + \delta g(\theta) \text{ for } \forall\, \theta \in [\overline{\theta}, \underline{\theta}]$$

$$= 0 \text{ otherwise.}$$

For different values of δ, θ will follow different distribution functions.
For $\delta > 0$, $h'(\theta) = \delta g'(\theta) = \delta G''(\theta) < 0$
For $\delta = 0$, $h'(\theta) = 0$, and θ follows a uniform distribution.
Given the distribution of θ, underlying density function of income is defined as

$$f(y) = h(\theta)\frac{d\theta}{dy}$$

$$= h(\theta(y))\theta'(y)$$

$$= [k + \delta g(\theta(y))]\theta'(y)$$

$$f'(y) = \theta''(y)h(\theta(y)) + h'(\theta(y))\{\theta'(y)\}^2$$

For $\delta > 0$, $f'(y) < 0$ for $\theta''(y) < 0$ and $h'(\theta(y)) < 0$

For $\delta = 0$, $f'(y) = \theta''(y)h(\theta(y)) < 0$ for $y \in [\underline{y}, \overline{y}]$

Thus density function of income is always negatively sloped. So income follows a positively skewed distribution.

APPENDIX 2

The density functions of income are defined as

$$f(y) = [k + \delta_1 g(\theta(y))]\theta'(y)$$

$$f^*(y) = [k + \delta_2 g(\theta(y))]\theta'(y) \quad \text{for} \quad y \in [\underline{y}, \overline{y}] \quad \text{where} \quad \delta_1 > \delta_2$$

The first moment distribution function is defined as

$$F_1(y) = \frac{1}{\mu}\int_{\underline{y}}^{y} yf(y)dy \quad \text{and} \quad F_1^*(y) = \frac{1}{\mu^*}\int_{\underline{y}}^{y} yf^*(y)dy$$

The slope of relative concentration function curve is given as

$$\frac{dF_1}{dF_1^*} = \frac{\mu^* f(y)}{\mu f^*(y)} > 0$$

The second derivative is

$$\frac{d^2F_1}{dF_1^{*2}} = \frac{d}{dF^*{}_1}\left(\frac{dF_1}{dF_1^*}\right) = \frac{d}{dy}\left(\frac{dF_1}{dF_1^*}\right)\frac{dy}{dF_1^*}$$

$$\frac{dy}{dF_1^*} = \frac{\mu^*}{yf^*(y)} > 0 \quad \text{and} \quad \frac{d}{dy}\left(\frac{dF_1}{dF_1^*}\right) = \frac{\mu^*}{\mu}\frac{d}{dy}\left(\frac{f(y)}{f^*(y)}\right) = \frac{\mu^*}{\mu}\frac{f^*f' - ff^{*'}}{f^{*2}}$$

$$\frac{d^2F_1}{dF_1^{*2}} = \frac{\mu^{*2}}{\mu y f^{*3}}\left[f^*\frac{df}{dy} - f\frac{df^*}{dy}\right] = \frac{\mu^{*2}}{\mu y f^{*3}}\left[\frac{ff^*}{y}\frac{df}{dy}\frac{y}{f} - \frac{f^*f}{y}\frac{df^*}{dy}\frac{y}{f^*}\right]$$

$$= \frac{\mu^{*2}}{\mu}\frac{f}{y^2 f^{*2}}\left[\frac{df}{dy}\frac{y}{f} - \frac{df^*}{dy}\frac{y}{f^*}\right]$$

$$\frac{df}{dy} = [k + \delta_1 g(\theta(y))]\theta''(y) + \delta_1 g'(\theta(y))\{\theta'(y)\}^2$$

$$\frac{df}{dy}\frac{y}{f} = \frac{[[k + \delta_1 g(\theta(y))]\theta''(y) + \delta_1 g'(\theta(y))\{\theta'(y)\}^2]y}{[k + \delta_1 g(\theta(y))]\theta'(y)} = \frac{y\theta''(y)}{\theta'(y)} + \frac{\delta_1 g'(\theta(y))\theta'(y)y}{[k + \delta_1 g(\theta(y))]}$$

Similarly

$$\frac{df^*}{dy}\frac{y}{f^*}=\frac{y\theta''(y)}{\theta'(y)}+\frac{\delta_2 g'(\theta(y))\theta'(y)y}{[k+\delta_2 g(\theta(y))]}$$

$$\frac{df}{dy}\frac{y}{f}-\frac{df^*}{dy}\frac{y}{f^*}=\frac{\delta_1 g'(\theta(y))\theta'(y)y}{[k+\delta_1 g(\theta(y))]}-\frac{\delta_2 g'(\theta(y))\theta'(y)y}{[k+\delta_2 g(\theta(y))]}$$

$$=yg'(\theta(y))\theta'(y)\left[\frac{\delta_1}{k+\delta_1 g(\theta(y))}-\frac{\delta_2}{k+\delta_2 g(\theta(y))}\right]$$

$$=yg'(\theta(y))\theta'(y)\left[\frac{k(\delta_1-\delta_2)}{(k+\delta_1 g(\theta(y)))(k+\delta_2 g(\theta(y)))}\right]<0$$

For $\delta_1>\delta_2$ and $g'<0$, thus $\dfrac{d^2F_1}{dF_1^{*2}}<0$

APPENDIX 3

Second Order Conditions of the Monopoly Case

$$\frac{\partial^2\pi}{\partial p_h^2}<0\ \Rightarrow\frac{-[2h^2(\theta^*)+h'(\theta^*)(1-H(\theta^*))]}{(q_h-q_l)h(\theta^*)}<0$$

$$\frac{\partial^2\pi}{\partial p_l^2}<0\ \Rightarrow\frac{-[2h^2(\theta^*)+h'(\theta^*)(1-H(\theta^*))]}{(q_h-q_l)h(\theta^*)}-\frac{[2h^2(\theta_l)+h'(\theta_l)(1-H(\theta_l))]}{q_l h(\theta_l)}<0$$

$$\frac{\partial^2\pi}{\partial q_h^2}<0\ \Rightarrow-2\alpha(1-H(\theta^*))-\frac{\theta^{*2}}{(q_h-q_l)}[2h^2(\theta^*)+h'(\theta^*)(1-H(\theta^*))]-\frac{2\theta^*(1-H(\theta^*))}{(q_h-q_l)}<0$$

$$\frac{\partial^2\pi}{\partial q_l^2}<0\ \Rightarrow-\frac{\theta^{*2}h'(\theta^*)(1-H(\theta^*))}{(q_h-q_l)}-\frac{2\theta^*(1-H(\theta^*))}{(q_h-q_l)}-2\alpha(H(\theta^*)-H(\theta_l))$$

$$-2\alpha q_l\left[\frac{h(\theta^*)\theta^*}{(q_h-q_l)}+\frac{h(\theta_l)\theta_l}{q_l}\right]-h'(\theta_l)\frac{\theta_l^2}{q_l}\frac{(1-H(\theta_l))}{h(\theta_l)}-\frac{2\theta_l}{q_l}(1-H(\theta_l))<0$$

Our computation shows that the second derivatives are negative at the relevant range of δ values.

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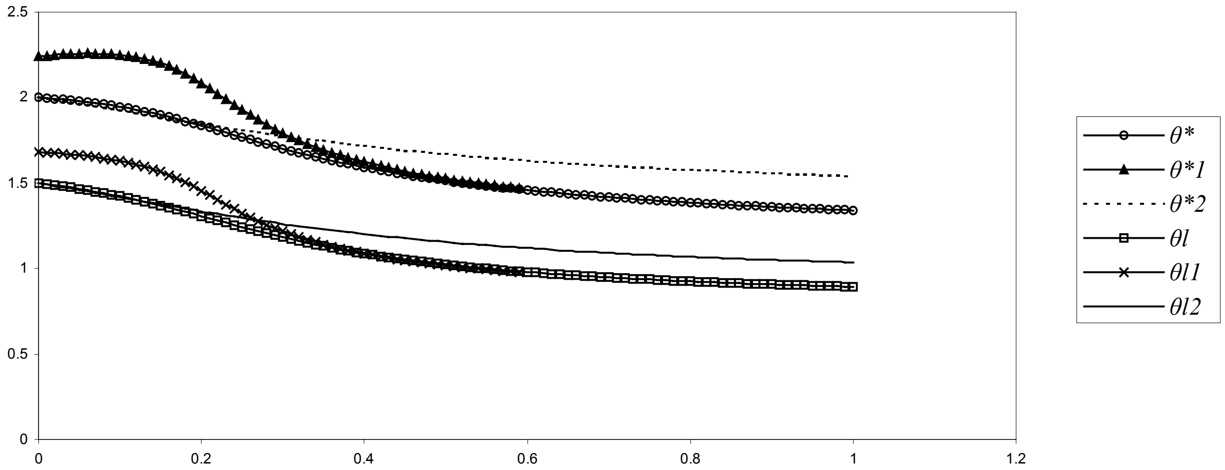
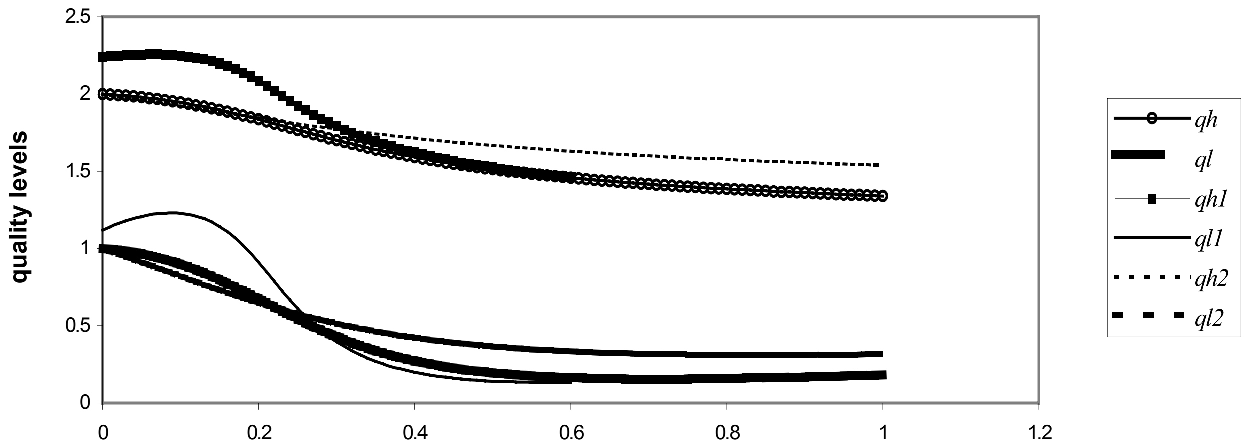


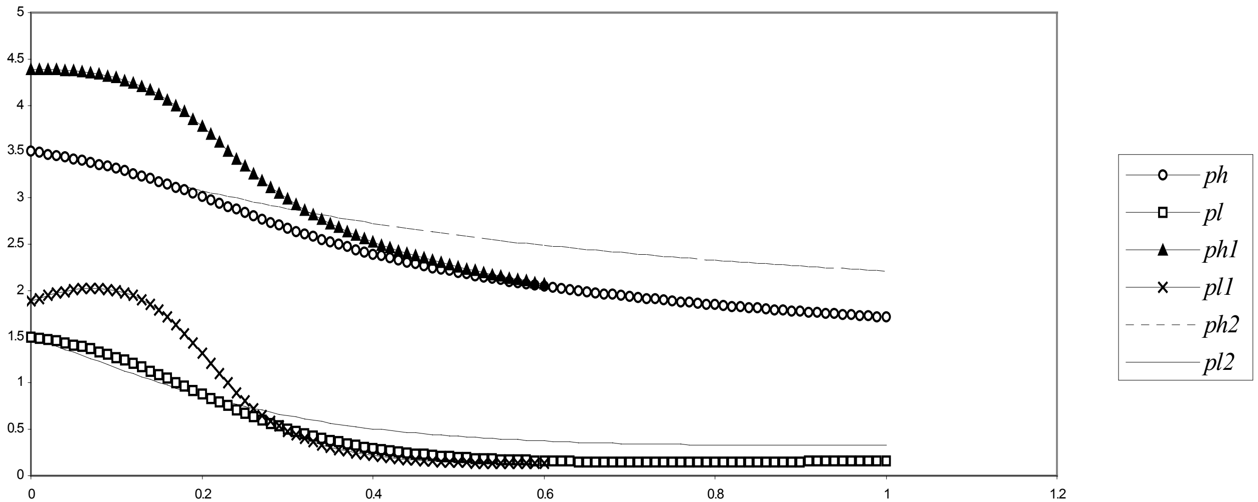
Note: In this case the θ^* and θl correspond to $\overline{\theta}$ = 2.5 and $\underline{\theta}$ = 1.5 and $\theta^* 1$ and $\theta l1$ correspond to $\overline{\theta}$ = 2.8 and $\underline{\theta}$ = 1.5 and $\theta^* 2$ and $\theta l2$ correspond to $\overline{\theta}$ = 2.5 and $\underline{\theta}$ = 1.8

Figure 2. This Figure Shows the Relationship between Willingness to Pay Parameter Values and δ Under Monopoly Case



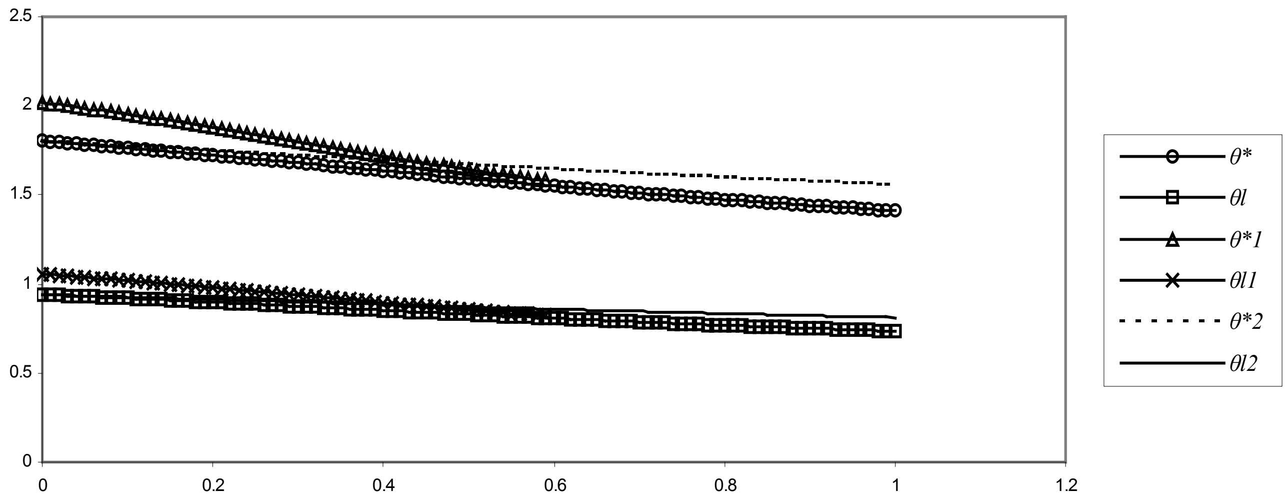
Note: In this case the *qh* and *ql* correspond to $\overline{\theta} = 2.5$ and $\underline{\theta} = 1.5$ and *qh1* and *ql1* correspond to $\overline{\theta} = 2.8$ and $\underline{\theta} = 1.5$ and *qh2* and *ql2* correspond to $\overline{\theta} = 2.5$ and $\underline{\theta} = 1.8$

Figure 3. This Figure Shows the Relationship between Quality Levels and δ Under Monopoly Case



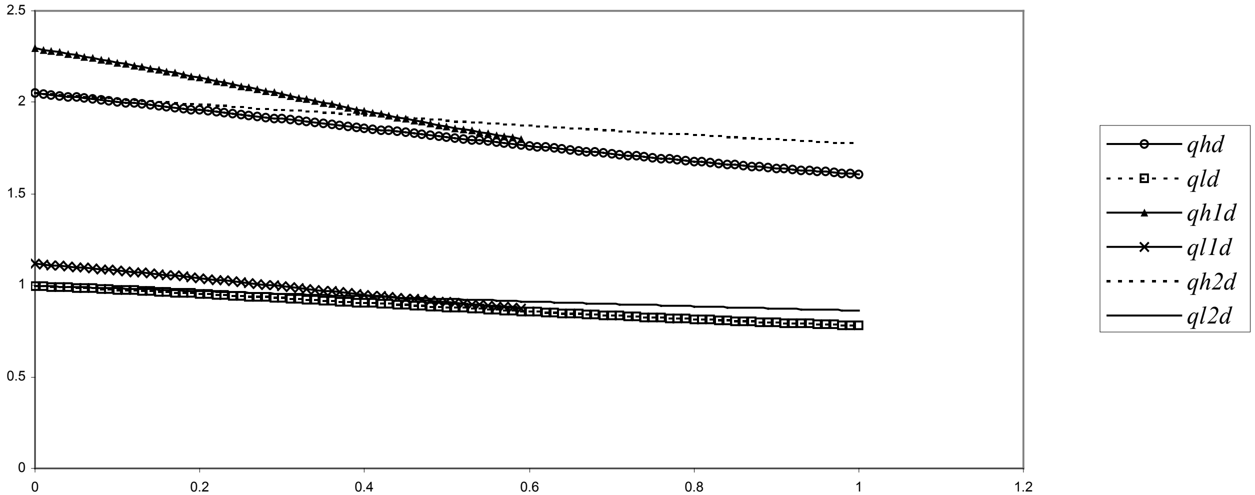
Note: In this case the *ph* and *pl* correspond to $\overline{\theta} = 2.5$ and $\underline{\theta} = 1.5$ and *ph1* and *pl1* correspond to $\overline{\theta} = 2.8$ and $\underline{\theta} = 1.5$ and *ph2* and *pl2* correspond to $\overline{\theta} = 2.5$ and $\underline{\theta} = 1.8$

Figure 4. This Figure Shows the Relationship between Prices and δ Under Monopoly Case



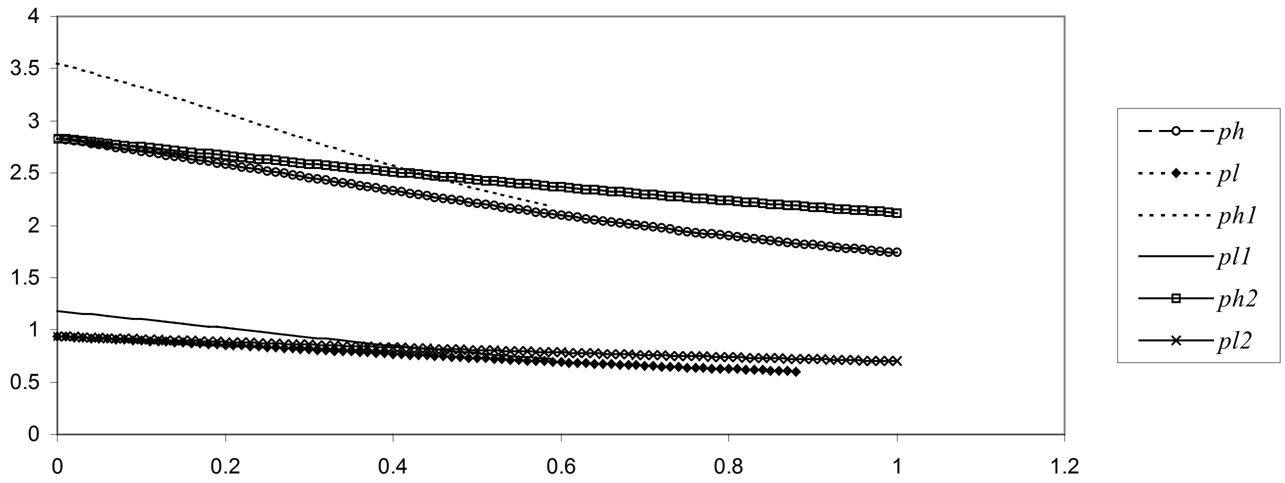
Note: In this case the $\theta*$ and θl correspond to $\overline{\theta}$ = 2.5 and $\underline{\theta}$ = 1.5 and $\theta*1$ and $\theta l1$ correspond to $\overline{\theta}$ = 2.8 and $\underline{\theta}$ = 1.5 and $\theta*2$ and $\theta l2$ correspond to $\overline{\theta}$ = 2.5 and $\underline{\theta}$ = 1.8

Figure 5. This Figure Shows the Relationship between Willingness to Pay Parameter Values and δ Under Duopoly Case



Note: In this case the *qh* and *ql* correspond to $\overline{\theta} = 2.5$ and $\underline{\theta} = 1.5$ and *qh1* and *ql1* correspond to $\overline{\theta} = 2.8$ and $\underline{\theta} = 1.5$ and *qh2* and *ql2* correspond to $\overline{\theta} = 2.5$ and $\underline{\theta} = 1.8$

Figure 6. This Figure Shows the Relationship between Quality Levels and δ Under Duopoly Case



Note: In this case the *ph* and *pl* correspond to $\overline{\theta} = 2.5$ and $\underline{\theta} = 1.5$ and *ph1* and *pl1* correspond to $\overline{\theta} = 2.8$ and $\underline{\theta} = 1.5$ and *ph2* and *pl2* correspond to $\overline{\theta} = 2.5$ and $\underline{\theta} = 1.8$

Figure 7. This Figure Shows the Relationship between Price Levels and δ Under Duopoly Case