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Production Linkages and Rural-Urban Migration

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Flexible production linkages between the rural and urban sectors are incorporated into the Harris Todaro (HT) model. Each sector uses a part of the other's output as an intermediate input in a small open economy. Implications of separability, elasticities of substitution, and input complementarity in a three-input production function are examined. The intermediate input in the minimum-wage sector has a more pervasive influence than its counterpart in the rural sector. The well known HT result from a final-goods-only (FGO) specification that an increase in expected urban wage causes an outmigration from the rural sector does not hold under several parametric configurations. Unlike the case of a non-traded intermediate good used in fixed proportions, an urban wage subsidy may lower rural employment, as in the FGO specification.

I. Introduction

In the many extensions, elaborations, and modifications of the Harris -Todaro (HT) model of rural-urban migration, the structure of production has remained extremely simple, essentially consisting of two sectors which produce only final goods. One exception is Marjit's (1991) analysis of agro-based industry in a small open economy (SOE) where agriculture produces an intermediate good, which is not internationally traded, to be used in fixed proportions by the minimum-wage sector (M-sector) for further processing. In input-output (i-o) terminology, this is a one-way, fixed coefficients linkage between the two sectors. Many of the commodities cited by Marjit (jute, cotton, fruit), however, are internationally traded,[1] and i-o data for several developing countries point to flexible backward and forward linkages for quite a few activities. Moreover, intermediate goods can be needed just as easily in the M-sector as elsewhere in the economy. Even in a two-sector framework, say, agriculture and manufacturing, it is common to find the former using

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1. A similar one-way linkage, but with the intermediate good in the minimum-wage sector, is also featured in Beladi and Marjit (1996) for analysing some trade-policy issues.

q for agriculture, both requiring three inputs - are assumed to be linear, homogeneous, and strictly quasi-concave. The first partial derivatives, f_l and q_l, denote the marginal product of labor (MPL), assumed to be positive, and f_{l3} and q_{l3} indicate how the intermediate inputs affect MPL in the two sectors. Workers are hired such that the value of their marginal product is equal to the wage rate, and purchases of intermediate inputs, X_{AM} and X_{MA}, are determined by equating the marginal value product of each to its price. The residual, after paying for the purchased inputs, is the rental for the specific factors of production which are assumed to be fully employed. For the rest, the HT specification is adopted to facilitate comparison with earlier work. There is thus a minimum wage in the urban sector, fixed in terms of the manufactured good, the probability of finding an urban job is given by the actual employment rate, and the equilibrium in the labor market is characterized by equality of expected wages in the two sectors.[2]

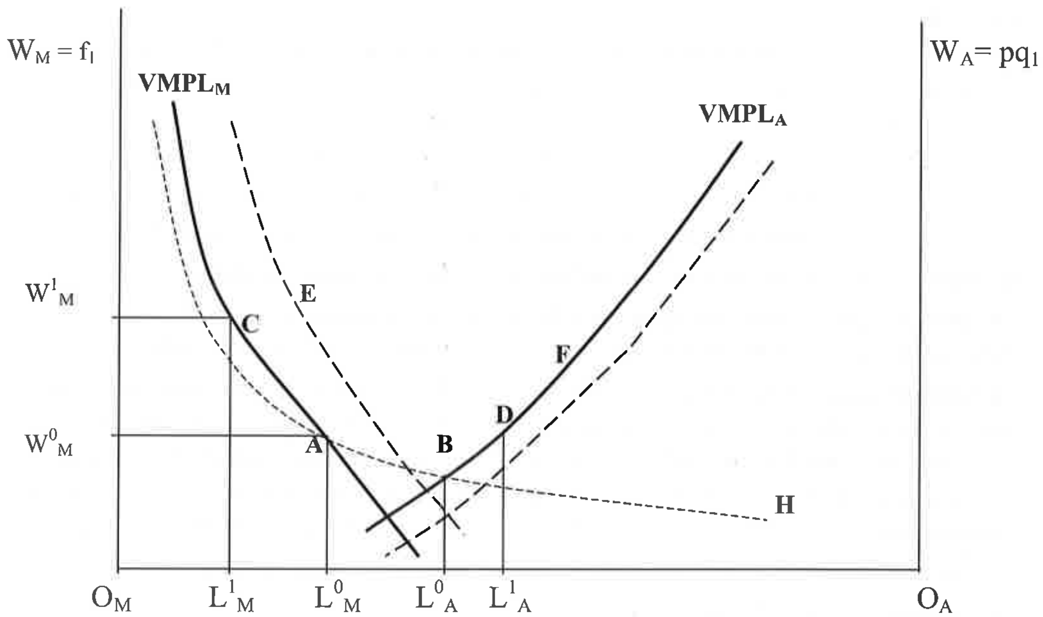

Figure 1 Employment and Labor Market Equilibrium

2. These assumptions, although criticized sometimes in the literature,are retained for comparability with earlier work. The present analysis is mainly concerned with structure of production anyhow, rather than with alternative specification of employment probabilities, rates of labor turnover, and so on.

III. The Model and Its Comparative Statics

The key Equations needed for deriving analytical results are the labor market equilibrium, those which determine the input-output coefficients, and the overall labor constraint. The HT labor market equilibrium is given by:

$$E = p \cdot q_1 - f_1 \cdot L_M /(\bar{L} - L_A) = 0 \tag{1}$$

where, as defined above, q_1 is the MPL in agriculture and f_1 in manufacturing, the first partial derivatives of the corresponding production functions. The term post-multiplying f_1 is the urban employment rate, the perceived probability of finding an urban job. Since $p = p_A / p_M$, the equation signifies equality of expected wages between the two sectors, expressed in comparable units, and it appears in almost all versions of the HT model. Some flexibility in production coefficients is essential for Equation (1) to be meaningful; otherwise, if all input ratios are fixed, f_1 and q_1 will be zero.

Input-output coefficients involving intermediate goods ($a_{MA} = X_{MA} / X_A$ and $a_{AM} = X_{AM} / X_M$) are determined by all input prices, and that can be expressed by:

$$a_{MA} = a_{MA}(w_A, r_A, p_M) \tag{2}$$

$$a_{AM} = a_{AM}(w_M, r_M, p_A) \tag{3}$$

Both Equations (2) and (3) are assumed to be homogeneous of degree zero in input prices. Lastly, if L_U is the number of unemployed workers in the urban sector, the labor constraint may be written as:

$$L_A + L_M + L_U = \bar{L} \tag{4}$$

Equilibrium conditions for employers of labor in the two sectors, respectively, are $w_A = pq_1$, and $w_M = f_1$. Agricultural wage, w_A, is assumed to be flexible, so there will be no unemployment in the rural sector.

For deriving comparative-static results, it is convenient to follow a technique adopted by HT, namely, differentiate the equilibrium condition Equation (1) with respect to L_A and L_M ("small" changes in the vicinity of the initial equilibrium) to determine its slope in $L_A L_M$ space (dL_A / dL_M) while taking the rest of the model into account. The new element here will be the interaction between labor and the intermediate inputs - in particular, dX_{MA} / dL_A and dX_{AM} / dL_M.

1. Effects of Changing Manufacturing Labor

Differentiating Equation (1) partially with respect to L_M, holding L_A constant, sufficient for that). Therefore, considering both theory and potential applications, especially those involving actual numbers, cases in which $dX_{AM}/dL_M \geq 0$ are more likely, perhaps more rewarding too because at least for one well-known FGO result noted in the Introduction ($|\eta_M| < 1$), the sign of Equation (6) can be switched from positive to negative.

2. Effects of Changing Agricultural Labor

Differentiating Equation (1) partially with respect to L_A, holding L_M constant, we get

$$\frac{\partial E}{\partial L_A}\bigg| L_M const. = p\frac{dq_1}{dL_A} - \frac{L_M}{(\bar{L}-L_A)}\frac{df_1}{dL_A} - \frac{L_M}{(\bar{L}-L_A)^2}\cdot f_1 . \tag{8}$$

Now, q_1 is a function of L_A and X_{MA}, and $df_1/dL_A = 0$, so Equation (8) can be rewritten as Equation (9):

$$\frac{\partial E}{\partial L_A}\bigg| L_M const. = p(q_{11} + q_{13}dX_{MA}/dL_A) - \frac{L_M}{(\bar{L}-L_A)^2}\cdot f_1 . \tag{9}$$

Because $q_{11} < 0$ and $f_1 > 0$, Equation (9) will be negative so long as its middle term is not positive; its sign, therefore, is of critical importance, for only this term can make $\partial E/\partial L_A$ positive. Analogous to Equation (7), we have:

$$\frac{dX_{MA}}{dL_A} = \frac{a_{MA}q_1[1+(\sigma^A_{LM} - \sigma^A_{KM})/\eta_A]}{1-\rho_{MA}[1+q_{13}L_A(\sigma^A_{LM} - \sigma^A_{KM})/q_3]}, \tag{10}$$

where ρ_{MA} is the share of the intermediate good in agriculture, the σs are Allen-Uzawa elasticities of substitution between the intermediate and the primary input, and η_A is the wage elasticity of demand for labor in agriculture. This expression, as in the case of Equation (7), can have any sign, and it will be unambiguously positive in the fixed-proportions case. Again, parallel to Equation (7), if $(\sigma^A_{LM} - \sigma^A_{KM})$ is positive and greater in absolute value than η_A, and $q_{13} = 0$, $dX_{MA}/dL_A < 0$, but the X_{MA} terms get deleted from Equation (9).

It is worth noting here that Figure 1 illustrates a special case of Equation (10) in which both q_{13} and dX_{MA}/dL_A are negative. The derivation in Equation (10), based on the complete model, shows that there is no necessary correlation between the two; in fact, even q_{13} and the elasticities of substitution need not have the same sign. This well-known property of a three-input production function will figure prominently in the analytical results presented in the next section. An analogous argument applies to Equation (7) (recall that Figure 1 was also based on the assumption that $f_{13} > 0$ and $dX_{AM}/dL_M < 0$).

condition for these outcomes will be $f_{13} = q_{13} = 0$, or, more generally, $dX_{AM}/dL_M = dX_{MA}/dL_A = 0$.

It should be clear from the full expressions in Equations (6) and (9), however, that the migration outcomes based mainly on η_M would be affected by the intermediate-good terms, and when they are positive, a serious challenge to the established results might arise, for both Equations (6) and (9) can then switch signs. Fixed production coefficients involving intermediate inputs, a hallmark of many modern technologies (the Leontief production function, for instance) is an obvious case in point, because both Equations (7) and (10) will be positive. To further explore this idea, unless specified otherwise, it will be assumed that f_{13} and q_{13} are positive, i.e., labor and the intermediate input are technical complements in both sectors.[5] A common point of initial equilibrium is also assumed in all cases, and we shall consider "small" minimum-wage changes in its vicinity.

1. The Results

To compare and contrast with the two results highlighted in the Introduction, and to simplify the derivations, we shall take up one intermediate input at a time, starting with the M-sector.

a. Intermediate Input in the M-Sector

Let us set q_{13} equal to zero so that the intermediate input used in agriculture is relegated to a secondary role, and it does not affect the sign of Equation (9), which is negative. Actually, Equation (9) will be reduced to the case of a one-way linkage considered by Marjit (possibly agro-based manufacturing or something similar). A convenient starting point is a case in which the intermediate good is separable from K and L in the M-sector production function $(\sigma^M_{LA} = \sigma^M_{KA})$:

Lemma 1: If $\sigma^M_{LA} = \sigma^M_{KA}$, $dX_{AM}/dL_M > 0$.

As an illustration, rubber products are manufactured in many areas of Malaysia and Indonesia, and latex collected in agriculture is the principal raw material for them. Lemma 1 implies that a decrease in the number of M-sector workers will lead to a lower quantity of

5. Neary (1988) shows that this is one of several sufficient conditions for the stability of the model with a three-input production function. Technical complementarity implies that production is "normal", the stability condition discussed by Funatsu (1988). The third input in both these studies is land, not an intermediate good produced in the other sector, and it is not clear if the condition still applies. The important point for the ensuing analysis, however, is that restrictions on these cross derivates of the production function do not restrict the Allen-Uzawa elasticities of substitution. For example, $f_{13} = 0$ does not imply that σ^M_{LA} is zero, and so on. The stability condition for factor demands in terms of the σs, for, say, manufacturing (Allen (1967, pp. 503-505)), is: $\rho_{LM}\sigma^M_{LA} + \rho_{KM}\sigma^M_{KA} > 0$.

In Corollaries 2, 3, and 4, the numerator of Equation (7) is obviously positive, and a bit of manipulation of its denominator shows that it too is positive. Such restrictions on various σ's would be called for in many settings, as in the latex example mentioned above. Often more latex can be processed only by employing more workers (the labor-latex ratio is fixed, as in Corollary 2), and so on. In all cases of this type, $dX_{AM}/dL_M > 0$, and the explanation following Result 1 would apply.

Another interesting scenario occurs where changes in urban employment have no effect on intermediate usage, and that outcome is often associated with production technologies in which one or the other primary factor is a better substitute for the intermediate input. To illustrate, consider an M-sector that produces steel from iron ore mined in the primary sector. Modern processes typically rely on fixed input-output ratios, but China experimented with backyard furnaces some decades ago. These were comparatively small units to be operated by teams of workers. In the present context, that set-up would imply $(\sigma_{LA}^M > \sigma_{KA}^M)$, and the latter elasticity of substitution might be close to zero. A particular grouping of such parameters will generate the somewhat surprising outcome that extra labor has no effect on intermediate usage:

Lemma 2: *If* $(\sigma_{LA}^M - \sigma_{KA}^M) = |\eta_M|$, $dX_{AM}/dL_M = 0$.

And a special case of this lemma $(\sigma_{KA}^M = 0)$ will lead to Result 2:

Result 2: *If the intermediate good cannot be substituted for capital, but its elasticity of substitution with respect to labor is equal to the wage elasticity of demand for labor in the minimum-wage sector, the intermediate input will have no effect on the direction of migration flows.*

With reference to Equations (6) and (7), $\sigma_{KA}^M = 0$ and $\sigma_{LA}^M = |\eta_M|$; therefore, any change in M-sector employment will not affect its intermediate usage. The intermediate-good term drops out of Equation (6), and its sign, along with the direction of migration flows, will depend only on the magnitude of η_M, as in the FGO specification.

An interesting corollary to Result 2 emerges when σ_{LA}^M is unity. This implies a unit-elastic demand for labor in the M-sector because σ_{KA}^M has been assumed to be zero. Both terms in Equation (6) are now zero; therefore, the $E = 0$ line in Figure 2 is horizontal, and a minimum-wage increase simply moves the equilibrium point towards *F*, which indicates a higher rate of unemployment in the M-sector, but no migration will take place.

b. Intermediate Input in the A-Sector

In order to give fuller play to the second linkage (M-sector output used as an intermediate input in agriculture), a production function with $q_{13} > 0$ is postulated. Analogous to Lemma 1 and 2, several settings can be recognized where $dX_{MA}/dL_A \geq 0$. For

2. Comparison with Earlier Results

Although the case of an inelastic demand for urban labor has not been explicitly considered above, the result mentioned in the Introduction about outmigration from agriculture in response to an increase in the minimum wage has to be qualified. For instance, one additional (sufficient) condition might be $f_{13} = q_{13} = 0$ (marginal productivity of labor is not affected by intermediate inputs), or $dX_{AM}/dL_M = 0$ and $dX_{MA}/dL_A \leq 0$ (changing employment does not cause intermediate usage to move in the same direction). Elaborating further, the first term in Equation (6) is positive when $|\eta_M| < 1$, and the second term will be zero or positive under the additional conditions just mentioned. Referring to Equation (9), only the middle term can be positive, but that is ruled out if $q_{13} = 0$ or $dX_{MA}/dL_A \leq 0$. With Equation (6) positive and Equation (9) negative, the $E = 0$ line in Figure 2 will be positively sloped. Therefore, starting from, say, the full employment point *E*, after the minimum wage is raised, the new equilibrium will be in the direction of point *G*. The same logic will apply if there is a one-way linkage (intermediate good only in the M-sector). Result 1 and its corollaries, however, identify several plausible settings in which $dX_{MA}/dL_A > 0$; the original HT conclusion, therefore, might not hold even when the expected urban wage goes up in the first instance.

Turning to Marjit's proposition, note that an urban subsidy leads to an increase in expected urban wage as well as greater urban employment and output. Marjit's assumptions of fixed input-output ratios and a non-traded intermediate good then ensure an increase in rural output, and since he further assumes the intermediate good to be relatively labor intensive, rural employment also goes up. In the present framework, because of the possibilities of input substitution, manufacturing demand for the agricultural intermediate good may not increase. The higher expected wage, of course, will lead to outmigration from agriculture, as in the original HT model.

V. Conclusion and Summary

This paper has recast the Harris Todaro model of rural urban migration into an input-output framework with flexible production coefficients. Each sector, thus, produces an intermediate input for the other, in addition to a final good, in a small open economy. This enables us to go beyond the FGO or one-way-linkage specifications in the HT literature. The three-input production functions required for this purpose have many interesting properties - separability, different elasticities of substitution, input complementarity - that enrich the analysis and deepen our understanding of how production technologies might affect the migration process. Considering the vast array of intermediate inputs in actual use, these properties are closer to the mark than the staid assumption of fixed proportions, and they drive most of the analytical results. Because of the internal logic of the HT model, the intermediate good in the M-sector plays a more important role than the one in agriculture. Results for the former, accordingly, are also sharper, although one simple condition on various elasticities does emerge under which neither production linkage will have any effect on the qualitative outcomes.

$$= a_{MA}(q_1 + q_3\, dX_{MA}/dL_A) + X_A[(\partial a_{MA}/\partial w_A)\, dw_A/dL_A)$$

$$+(\partial a_{MA}/\partial r_A)(dr_A/dL_A)]. \quad \text{(A.2)}$$

The notation is the same as in the paper. For production functions of the type used here, σ^A_{LM}, the partial elasticity of substitution between labor and the intermediate good in agriculture, can be defined as:

$$(\partial a_{MA} / \partial w_A)(w_A / a_{MA}) = \rho_{LA}\sigma^A_{LM},$$

where ρ_{LA} is the share of labor in agriculture. Other σs are analogously defined. The term between the square brackets in (A.2) then becomes

$$(\rho_{LA}\sigma^A_{LM} a_{MA} / w_A)(dw_A / d_{LA}) + (\rho_{KA}\sigma^A_{KM} a_{MA} / r_A)(dr_A / dL_A).$$

For further simplification, it is necessary to derive

$$dw_A/dL_A \quad \text{and} \quad dr_A/dL_A.$$

Now, $w_A = p_A q_1$ and $q_1 = q_1(L_A, K_A, X_{MA})$. Therefore,

$$dw_A/dL_A = p_A \cdot (q_{11} + q_{13}\, dX_{MA}/dL_A). \quad \text{(A.3)}$$

In this model, r_A is determined as a residual: i.e.,

$$K_A r_A = p_A X_A - w_A L_A - p_M X_{MA}.$$

Full employment of K_A is always assumed, so it is set equal to unity by a choice of units. Also, p_A and p_M are constant because of the SOE assumption. Therefore,

$$dr_A/dL_A = p_A\, dX_A/dL_A - L_A\, dw_A/dL_A - w_A - p_M\, dX_{MA}/dL_A. \quad \text{(A.4)}$$

Substituting (A.4), (A.3) etc. into (A.2), we get Equation (10) after simplification. A similar procedure leads to Equation (7).

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