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The Interaction of Child-labour and Schooling in Developing Countries: A Theoretical Perspective

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This paper analyses the interaction between child labour and schooling in developing countries. A theoretical framework is developed, where fertility and schooling decisions are made in an environment where children contribute through child labour when young and provide old-age security as adults. The model demonstrates that the child wage rate, which is also the opportunity cost of schooling, is a crucial determinant of total fertility. An increase in the child wage rate leads to lower schooling investments and higher fertility levels. However, changes in schooling costs have no impact on fertility decisions. They only affect the allocation of children's time between schooling and child labour.

I. Introduction

In developing countries, children make substantial contributions to household income through child labour and also act as important sources of old-age security. Children's contributions in rural areas may include performing household tasks such as tending cattle, collecting firewood, taking care of younger siblings, fetching water and helping with cooking. They may also be employed in the urban informal sector.[1] The ILO estimates that there are approximately 120 million children in the age group of 5-14 who work on a full-time basis, and this figure rises to around 200 million including those for whom work is a secondary activity. Further, surveys conducted by the ILO find that over a twelve-month period, the proportion of economically active children in the age group 5-14 years could be as high as 40 per cent. These studies find that children's labour contributions are an important component of household income, in some cases amounting to around one-third of the household income (Lansky (1997)).

Similarly, children are also regarded as sources of financial and emotional security in parents' old-age, particularly in areas where the joint family system is common. Moreover, financial and labour market imperfections combined with a lack of state sponsored social security programs creates an uncertainty about the asset accumulation required for old-age.

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1. For empirical evidence on child labour see Burra (1995), Weiner (1991), Bonnet (1993), and Sharif (1993). See Grootaert and Kanbur (1995) for a review on child labour.

Under these circumstances, children help in filling the gaps left by market failures.[2]

These two economic benefits from children are however linked, as parents face a trade-off between present and future consumption. For instance, greater child labour contributions are consistent with a lower potential for future financial benefits from children. However, despite the scale of child labour in developing countries, the fertility literature has ignored the effect of child labour contributions on fertility decisions. For instance, Becker (1960), Becker and Lewis (1973), Becker and Barro (1991) treat children as consumption goods only. In these models, fertility decisions depend on parental income and the opportunity cost of child- rearing. Similarly, studies by Willis (1980), Nugent (1985), Razin and Sadka (1995) focus mainly on their old-age security contributions and current economic benefits from children are ignored.[3]

In this paper, I propose a theoretical framework that incorporates fertility, consumption and education decisions in a model where there is a child labour market. Although there are many factors that potentially influence fertility decisions, the approach taken here is consistent with several stylised facts from developing countries.

First, children make important economic contributions to their parents, both in their young age and as adults. Second child rearing costs are typically low in developing countries. As women work extensively in the rural sector, the production process is labour intensive and child labour is frequently used (Dasgupta (1993, 1995)). Women have a low opportunity cost of time, as they are more likely to be employed in the informal sector where child-care is compatible with their work (see Oppong (1982), Mueller (1976)). In addition there are opportunities for informal child-care in the form of relatives and friends.

Finally, a number of empirical studies show a trade-off between child labour and schooling. These include Nielsen (1998), Jensen and Nielsen (1997) and Mason and Khandker (1998) for evidence from Africa. Studies by Psacharapoulos (1997) and Patrinos and Psacharopoulos (1997) find evidence of a child labour-schooling trade-off in Latin America. From these studies it is evident that if children are desired for their child labour benefits, there is little incentive for households to invest in their schooling because of its associated opportunity cost. Therefore households with more educated children are also likely to have fewer children.

Two important results are derived from this model. First, the child wage rate is shown to be an important determinant of fertility whereas factors such as returns to schooling do not affect fertility decisions. Second, higher schooling costs lead to a substitution away from schooling towards child labour. From a policy perspective, the model raises important issues. It shows the conditions under which fertility levels may be at their biological maximum and there is no investment in children' s schooling. This happens for example, if marginal child benefits are greater than marginal child rearing costs. An important policy implication then is

2. In Asia, the old-age security hypothesis is supported in studies by Cain (1991), Jensen (1990) and Vlassoff (1990). In Africa, it is supported in Nigeria (Caldwell (1982)), Rwanda (Clay and Haar (1993)) and in rural Mexico (Nugent and Gillaspy (1983)).

3. Exceptions include Rosenzweig and Evenson (1977), Rosenzweig (1990) who analyse the children' s labour contributions but ignore their old-age security contributions.

that a decrease in the child wage rate, accompanied by compensation for loss of child labour income, may be a powerful policy combination in fertility and child labour reduction programs.

The rest of the paper is organised as follows. Section II introduces the theoretical structure that examines the interaction between child labour and schooling decisions. The main implications of the model are analysed in Sections III and IV. Section III examines the comparative static properties of the model with respect to changes in the child wage rate in III.1 and changes in the price of schooling in Section III.2. Corner solutions are discussed in Section IV, and the conclusions are presented in Section V.

II. The Model

The model economy consists of parents and children. Parents are self-interested in the sense that consumption is the only direct source of utility. Suppose individuals live for three periods- childhood, middle and old age. Childhood is a period when individuals acquire their human capital through the decisions made by their parents. During their middle years, they work, make transfer payments to their parents and make decisions on fertility, child labour and schooling investments. In their last period, they are retired and consume the old-age transfers made by their children. When children become adults they go through the same cycle as their parents. Therefore, individuals only make economic decisions when they are middle-aged.

Under ILO's classification, the employment of children in the age group of 5-14 constitutes child labour. It seems reasonable to assume that young children in this age group do not have any bargaining power over their parents. Therefore parents make decisions on their children's time allocation and the model argues that children cannot bargain or make decisions on time-allocation until they reach their middle-age.[4]

Let $\tilde{n} = n_u + n_s + l$ where $\tilde{n}$ refers to total child hours, n_1 and n_s refer to child hours spent in labour and schooling respectively and l refers to total leisure hours. Assuming that leisure hours are a fixed proportion of $\tilde{n}$, then the decision variables are n_1 and n_s. Hence, fertility is proportional to child non-leisure hours defined as $n = n_u + n_s$.[5] Further assume $n_u \geq 0$, $n_s \geq 0$ and $0 \leq n_u + n_s \leq \overline{n}$, where $\overline{n}$ is the maximum number of non-leisure hours associated with the biological maximum number of children.

Thus the representative agent chooses n_u and n_s at time t, in order to maximise the following utility function,

$$V_t = u_t(c_{t-1}^{\ t}) + u_t(c_{t-1}^{t+1}) , \tag{1}$$

4. The collective model of bargaining is appropriate, for example, in a setting where we examine bargaining between spouses where each spouse has an outside option.

5. An increase in total child hours is the same as an increase in total child numbers, assuming that total leisure hours per child is constant and the length of the working day is also constant. Hence total child hours may be 24 hours per day multiplied by the number of children. For example, *n* may be 8 hours multiplied by 4 children.

where the subscripts denote time of birth and the superscripts denote the time of consumption. Thus, c_{t-1}^{t} is the consumption during period t of a middle-aged parent born at $t-1$ and c_{t-1}^{t+1} is their consumption in the next period, t+1. The budget constraints are:

$$c_{t-1}^{t} = g(n_{st-1}^{t})(1-\tau_t) + w_t n_{ut}^{t} - f(n_t)^t - h_t n_{st}^{t}, \tag{2}$$

$$c_{t-1}^{t+1} = g(n_{st}^{t+1})\tau_{t+1}. \tag{3}$$

Equation (2) is the parent's budget constraint when they are of working age. Adult i's income flow $g(.)$, depends on schooling investments n_{st-1}^{i} made by their parents in period $t-1$. The consumption of the middle-aged generation is financed through two sources. This includes their schooling returns net of transfers made to elderly parents, $g(n_{st-1})(1-\tau_t)$, and the wage contributions of their working children, $w_t n_{ut}^{t}$. Children's labour income at time t depends on the exogenously given child wage rate w and child hours in the labour force, n_1. Children that spend time in the labour force are presumed to be unskilled and contribute all their child wage earnings to parental income. The proportion of earnings that is transferred to their old parents is τ, which is assumed to be exogenously determined by social norms.

The term $f(n)$ represents the child rearing cost function. Child rearing costs include resource costs such as expenditures on food, education and medicine. In addition parents also incur time costs in raising children. Therefore, the child rearing cost function implicitly includes both parental time and resource inputs into child rearing. As discussed above, leisure time per child is constant, and so total leisure time increases with the number of children. Hence, child rearing costs are an increasing function of total child hours. Further it is assumed that $f_1 > 0$ and $f_{11} > 0$, due to the fact that there are also fixed input costs in child rearing.

Parents also incur schooling resource costs h, on child hours spent in schooling. These costs include the direct financial costs incurred by parents on tuition fees, books, cost of travel, and so on.

Equation (3) is the constraint faced by parents in their old age. In period t+1, each adult is retired and consumes transfers τ_{t+1}, from their children, which is a function of their children's schooling returns, $g(.)$. The direct schooling costs such as expenditure on tuition fees, transportation, uniforms, books etc, increase with each additional child that is sent to school. They increase with each additional child that is sent to school. Diminishing returns sets in because of fixed inputs of parent's time and resources such as land and income. Therefore it is assumed that $g_1 > 0,\ g_{11} < 0$.

Assume that the utility function is additively separable and let $u_1 > 0,\ u_{11} < 0$. Parents choose the number of child hours to allocate to child labour and schooling, with the aim of maximising utility over their lifetime. As there are no financial markets, individuals attain optimal inter-temporal consumption smoothing only through children.

Substituting (2) and (3) in (1), individual's utility maximisation corresponds to the following problem,

$$\max_{n_{st},n_{ut}} u[g(n^t_{st-1})(1-\boldsymbol{t}_t) + w_t n^t_{ut} - f(n^t_t)f_t - n^t_{st}h_t] + u[g(n^{t+1}_{st})\boldsymbol{t}_{t+1}], \tag{4}$$

where $\boldsymbol{t}_{t+1} \geq 0$, and w_t and h_t are exogenous parameters for all t. Utility maximisation with respect to n_{ut} and n_{st} yields the following first-order conditions,

$$\begin{aligned} V'(n_{ut}) = u_1[w_t - f_1(n_t)] &= 0 \quad \text{if} \quad \overline{n} > n_{ut} > 0; \\ &\leq 0 \quad \text{if} \quad n_{ut} = 0; \\ &\geq 0 \quad \text{if} \quad n_{ut} = \overline{n}, \end{aligned} \tag{5}$$

$$\begin{aligned} V'(n_{st}) = -u_1[h_t + f_1(n_t)] + u_2[g_1(.)\boldsymbol{t}_{t+1}] &= 0 \quad \text{if} \quad \overline{n} > n_{st} > 0; \\ &\leq 0 \quad \text{if} \quad n_{st} = 0; \\ &\geq 0 \quad \text{if} \quad n_{st} = \overline{n}. \end{aligned} \tag{6}$$

Define the steady-state to be a situation where $n_{st-1} = n_{st+1} = n_{st}$ and $n_{ut-1} = n_{ut+1} = n_{ut}$. Hence in a steady-state the model reduces to a two-period model. We restrict attention to steady-state solutions and drop the time-subscripts for notational convenience. Since we are interested in poor developing countries, where there is a persistence of high fertility and child labour levels along with low schooling investments, these parameters do not change. As the steady state is more relevant under these circumstances, for the rest of the paper, the analysis is conducted in terms of a two-period model.

From (5) at an interior optimum child labour hours are determined by the condition $w = f_1(n_u + n_s)$. Thus optimal child labour hours are determined at the point where the marginal benefits from child labour (w), equal the marginal cost (f_1) of an additional child. This condition also determines total fertility levels $(n = n_u + n_s)$.

Equation (6) characterises the optimal level of human capital investment. Thus in an equilibrium:

$$\frac{U_2}{U_1} = \frac{(h+w)}{[g_1(n_s)\boldsymbol{t}]}. \tag{7}$$

Accordingly, an interior solution is characterised by the tangency between "full prices" and the marginal rate of substitution between consumption in the respective periods. The full price of schooling in children includes both the monetary cost (h) and the opportunity cost of human capital investment (w). An increase in schooling costs or a decrease in the discounted marginal returns to schooling leads to a fall in MRS_{C_1,C_2}.

III. Analysis of the Model: Comparative Statics

The comparative static results (See Appendix) show that changes in the child wage rate effect both child labour and schooling decisions. While the child wage rate represents direct benefits from child labour, it is also the opportunity cost of schooling. Hence, w has an ambiguous effect on choice variables n_u and n_s.

III.1. Changes in the Child Wage Rate

Consider first the impact on child labour of changes in the child wage rate, $\partial n_u / \partial w$. From the Appendix, the partial derivative of n_u with respect to w is:

$$\frac{\partial n_u}{\partial w} = \frac{-u_1(C_1)U_{22} + u_{11}(C_1)n_u(-h - f_1)U_{12}}{|J|} \gtrless 0 \tag{8}$$

and the cross-price effect of a change in the child wage rate on child schooling is given by:

$$\frac{\partial n_s}{\partial w} = \frac{U_{11}(-u_{11}(C_1)n_u(-h - f_1)) + U_{21}(u_1(C_1))}{|J|} \gtrless 0. \tag{9}$$

In Equations (8) and (9), the terms U_{22} and U_{12} are the second- order partial derivatives of (5) and (6) with respect to n_s and n_u respectively, both of which are negative. However, the effect of a change in the child wage rate on child labour and child schooling is ambiguous. Below we analyse three possible ways in which changes in the child wage rate may affect child labour and schooling decisions. First, an increase in the child wage rate reduces demand for child hours in schooling, but increases the supply of child labour. Second, child schooling and labour hours may both rise. Third, schooling hours may increase whereas child labour hours may fall. Each of these cases is analysed below.

Case 1: Suppose $\partial n_u / \partial w > 0$ and $\partial n_s / \partial w < 0$. From (5) total fertility level, $n = n_u + n_s$ must increase if w rises, so $\partial n / \partial w > 0$. Thus $\partial n_u / \partial w$ must be greater than $\partial n_s / \partial w$.

From (A.7 in the Appendix) $\partial n_u / \partial w > 0$ if

$$-u_1(C_1)U_{22} > u_{11}(C_1)n_u(h + f_1)U_{12} \tag{10a}$$

and from (A.8) $\partial n_s / \partial w < 0$ if

$$-u_1(C_1)U_{21} > u_{11}(C_1)n_u(h + f_1)U_{11}. \tag{10b}$$

Note that the RHS of (10a) and (10b) are identical because $U_{12} = U_{11}$. It is also known (See the Appendix) that $|U_{22}| > |U_{21}|$, so if (10b) holds then (10a) must also hold. Thus (10b) is a sufficient condition for Case 1. The direction of the inequality in (10a) indicates either that the marginal utility of period 1 income, $u_1(C_1)$, is relatively high, or that child costs, $(h + f_1)$ are relatively low. Intuitively, at low levels of period one consumption, the opportunity cost of schooling in terms of forgone period one consumption is high. Hence, for an increase in the child wage rate, there is a substitution away from schooling towards higher child labour. From these results follow:

Proposition 1: *If (10b) holds, then a rise in the child wage rate (i) increases total fertility (ii) leads to a decline in child hours in schooling, and (iii) increases child hours in the labour force.*

Part (i) follows directly from (5) since $w = f_1(n_u + n_s)$. Thus, for a given child-rearing cost function $f(n)$, the child wage rate alone determines total fertility levels. To see this, consider a situation where parents wish to increase children's schooling hours either by substituting out of child labour or by having more children. However note that $f_{11} > 0$, hence if child schooling hours increase, marginal child rearing costs must rise as well. Since the wage rate is determined exogenously, child numbers will not change. Thus an increase in child schooling is only possible by substituting away from child labour. As shown before, it is not possible for both child labour and schooling hours to fall when the child wage rate rises. In other words, $\partial n_u / \partial w < 0$ and $\partial n_s / \partial w < 0$ cannot happen because $|U_{22}| > |U_{21}|$. Now consider the alternative possibility.

Case 2: If both $\partial n_u / \partial w > 0$ and $\partial n_s / \partial w > 0$, then an increase in the child wage rate raises total fertility levels.

This condition holds if in addition to (10a) it is also the case that

$$-u_1(C_1)U_{21} < u_{11}(C_1)n_u(h + f_1)U_{11}\,. \tag{10c}$$

This solution holds for intermediate values of $u_1(C_1)$. Finally consider Case 3 below,

Case 3: For $\partial n_u / \partial w < 0$ and $\partial n_s / \partial w > 0$ to hold, a sufficient condition is that $u_{11}(C_1)n_u(h + f_1)U_{12} > -u_1(C_1)U_{22}$. This also implies (10c) because $|U_{22}| > |U_{21}|$. From these results follow:

Proposition 2: *If child rearing costs are relatively high then a rise in the child wage rate implies: (i) an increase in child hours in schooling and (ii) a decline in child hours in the labour force. Alternatively,* $\partial n_u / \partial w < 0$ and $\partial n_s / \partial w > 0$.

If child rearing costs are high, then the marginal utility of period one consumption must be low. Under these conditions, parents do not require children's labour income to increase current consumption. Therefore total fertility *n* does not rise in this case, making it similar to the Becker model. This comparative static result, however, seems more relevant to developed countries. In these countries child rearing costs both in terms of resources and parental time cost are high.

What are the implications of these results in the context of developing countries? The comparative static results with regard to changes in the child wage rate are summarised in the three cases above. The most interesting situation from a developing country' s perspective is Case 1, where a rise in the child wage rate implies a substitution towards more child labour hours (and hence more unskilled children). Fertility levels increase in this case. In developing countries, low incomes mean that marginal utility of period one consumption $u_1(C_1)$, is likely to be relatively high and hence consumption levels are relatively low. On the other hand, since women' s opportunity cost of time and schooling costs are low, $(h+f_1)$ are also low. Under such conditions, small changes in the child wage rate lead to higher child labour hours and consequently we observe a decline in child schooling levels. This result is different from Becker' s model where the emphasis is on parental opportunity cost of time. Hence in that model an increase in the (adult) wage rate leads to a substitution towards more skilled children. Importantly, as Case 2 and Equation (5) demonstrate, an increase in the child wage rate always raises total fertility.

Finally, Case 3 is the only one where child labour hours fall in response to a rise in the child wage rate. Thus, in an institutional setting where child labour is prevalent, the only situation when child labour hours do not increase in response to a rise in the child wage rate is if both consumption levels and child costs are relatively high. From this we can deduce that income levels must be high so that parents do not need their children' s labour income to increase period one consumption.

These results also have implications for schooling investments, since a rise in the child wage rate increases children' s schooling only under very restrictive conditions. It requires either that child costs or consumption levels be low. Empirically studies by Rosenzweig and Evenson (1977), Kanbargi and Kulkarni (1991), and Vemuri and Sastry (1991), attribute a rise in fertility and child labour levels to an increase in the child wage rate. It is this aspect that policy makers should be concerned about, particularly in those countries that are characterised by low levels of schooling investment.

III.2. Changes in Price of Schooling

The next comparative static exercise is to consider the effect of a change in *h* on child labour and schooling. From the *conjugate-pairs theorem*, the parameter *h* only enters the first-order condition with respect to n_s with a negative sign. Hence, an increase in schooling costs, *h*, unambiguously reduces schooling investments. These results are summarised in Proposition 3 below:

Proposition 3: *An increase in schooling costs reduces the demand for child hours in schooling and leads to higher child labour hours, its substitute. That is,* $\partial n_s / \partial h < 0$; $\partial n_u / \partial h > 0$.

Proof: See Appendix.

A rise in schooling costs has the same effect as a decline in the household's budget. Hence the own-price effect of an increase in h, leads to a reduction in child schooling hours, n_s. The higher demand for n_u is met by substituting away from n_s as children's time in schooling and labour are considered to be substitutes. This result is also consistent with empirical studies from developing countries which find that a decline in the cost of child schooling or health (in the form of greater availability of schools and medical facilities), raises child schooling hours. Indeed low schooling enrolment in developing countries is often attributed to the high cost of schooling (see studies by Duraisamy and Malathy (1991), Rosenzweig and Evenson (1977), Rosenzweig (1990)). As decisions on child schooling hours are unilaterally imposed on the child, it represents an intergenerational externality to the child. If parents were completely altruistic and cared about their children's welfare, they might allocate the right amount of schooling. Therefore there is a missing market in schooling allocation across generations.

IV. Corner Solutions

Cross-sectional studies from several developing countries show that there are positive economic returns to high fertility in poor areas of developing countries (Cain (1991), Caldwell (1982), Boserup (1984)). In environments where children are valued as assets, increasing the number of children helps in achieving the twin objectives of higher current period income and also greater old-age security. Thus, households may choose to have their biological maximum number of children (Caldwell (1982), Clay and Vander Haar (1993), Sharif and Saha (1993)). In the context of developing countries these include situations where parents either have the biological maximum number of children, $\overline{n}$, or do not educate any of their children.

Using Equations (5) and (6) we discuss below four possible corner solutions. Consider first a situation where marginal child benefits are greater than marginal child-rearing costs, $w > f_1$. With each additional child, the rise in total child benefits is greater than the increase in child costs. This condition implies that fertility levels are at their biological maximum and all the children are employed in child labour activities, so that $n_u = \overline{n}$. As a sub-set of the above condition, consider the following two possibilities. First suppose,

$$w > f_1 \text{ and } g_1 \boldsymbol{t} > (h + f_1)\,. \qquad \text{(11a)}$$

In this case in addition to higher net marginal benefits from children in the first period, net returns from educating children are also greater than the costs. However, it is not clear from this condition if parents choose to send more children to school or to the labour force. Alternatively consider,

$$w > f_1 \text{ and } g_1 \boldsymbol{t} < (h + f_1)\,. \qquad \text{(11b)}$$

These conditions imply that total benefits from child labour are higher than the cost of child rearing and in addition, returns to schooling investments are low relative to their costs.

Hence there is no incentive for households to invest in the schooling of their children. Therefore, $n_s = n_s^{\min}$ and $n_u = \bar{n}$. The term $n_s^{\min}$ refers to minimum child schooling hours. However since children are endowed with some minimum human capital even if they do not go to school, we assume that although this is close to zero, it is not zero. This ensures that period two consumption is still positive. It is also the most interesting case from a policy point of view particularly for developing countries with population problems. When parental decisions on schooling are motivated by their own utility, there is the possibility of low investment in schooling from society' s point of view.

Now consider the least likely cases in the context of developing countries. This includes a situation where $w < f_1$. Here total child costs are greater than child benefits, hence parents are at a corner where $n_u = 0$. Suppose however that, $w < f_1$ and $g_1 t < (h + f_1)$. In this case, total child costs are greater than child benefits and therefore parents have no children, $n_u = n_s = 0$.

Alternatively consider the possibility that, $w < f_1$ and $g_1 t > (h + f_1)$. If this happens, total benefits from children in period one (child labour benefits) are less than their costs, hence $n_u = 0$. However, since net returns from schooling are greater than their costs, parents educate all their children and $n_s = n_{s\max}$ where $n_{s\max} \le \bar{n}$ refers to maximum child schooling, which may be less than or equal to the biological maximum.

Thus if the child wage rate is high, this may result in a corner solution where fertility is at its maximum level and all children are employed in child labour levels. These results indicate that from a policy perspective, human capital development and fertility reduction programs should aim at reducing incentives for child labour.

V. Conclusion

In this paper a theoretical framework is developed where the interaction between child labour and schooling determines fertility levels. The model is relevant to developing countries, where schooling and fertility decisions are often made in an environment where children contribute to household income through child labour and also provide for their parents' old-age security. The paper highlights an aspect of the literature that has been extensively discussed in empirical studies but has received very little theoretical attention.

It is shown that changes in the child-wage rate affect fertility, child labour and schooling decisions. In poor households characterised by high marginal utility of period one consumption, an increase in the child wage rate leads to higher fertility levels as households employ more children in the labour force. As the child wage rate is also the opportunity cost of schooling, this leads to a decline in schooling investments. However, changes in schooling costs do not affect fertility decisions, but only affect the child labour-schooling allocation. The model shows that a rise in schooling costs leads to lower levels of schooling, and higher levels of child labour.

The analysis also discusses possible corner solutions. If the child wage rate is relatively high, and child rearing costs and returns from educating children are relatively low, then fertility may reach the biological maximum with no investment in schooling.

From a policy perspective, the model raises several interesting issues. The model demonstrates that in situations where income from child labour forms an integral part of the household income, fertility and schooling decisions are also affected by changes in the child wage rate. Therefore, a reduction in the child wage rate along with compensation for parents affected by the loss in children's labour income, can serve as useful tools in fertility and child labour reduction programs. Moreover, efforts to expand schooling opportunities should also take into consideration its interaction with child labour, and the possibility that non-altruistic parents may underinvest in children's schooling from society's point of view.

Mathematical Appendix

This mathematical appendix describes the second-order conditions and derives the comparative static results essential to prove the propositions.

1. Second-order Sufficient Conditions

Define:

$$\frac{\partial^2 U}{\partial n_u^2} = U_{11} = u_{11}(C_1)(w - f_1)^2 + u_1(C_1)(-f_{11}) = u_1(C_1)(-f_{11}) \,, \tag{A.1}$$

$$\frac{\partial^2 U}{\partial n_u \partial n_s} = \frac{\partial^2 U}{\partial n_s \partial n_u} = U_{12} = u_1(C_1)(-f_{11}) \,, \tag{A.2}$$

$$\frac{\partial^2 U}{\partial n_s^2} = U_{22} = u_{11}(C_1)(-h - f_1)^2 + u_1(C_1)(-f_{11}) + \boldsymbol{r} u_{11}(C_2)(g_1)^2 \boldsymbol{t} + \boldsymbol{r} u_{11}(C_2)(g_{11})\boldsymbol{t} \,. \tag{A.3}$$

The second-order sufficient conditions are satisfied if,

$$U_{11} < 0, \; U_{22} < 0, \; U_{11}U_{22} - U_{12}^2 > 0 \,.$$

From (A.3) $U_{22} = u_{11}(C_1)(-h - f_1)^2 + u_1(C_1)(-f_{11}) + \boldsymbol{r} u_{11}(C_2)(g_1)^2 \boldsymbol{t} + \boldsymbol{r} u_1(C_2)(g_{11})\boldsymbol{t} < 0$, since $g_{11} < 0$. $U_{11} < 0$ because $f_{11} > 0$. The Jacobian, $|J|$, for the interior first-order conditions of (6) and (7) in n_s and n_u is

$$|J| = \begin{vmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{vmatrix},$$

which is positive as the second-order sufficient conditions are satisfied.

2. Comparative Statics

a. Changes in Wages

Totally differentiating Equations (6) and (7) with respect to n_u, n_s w and h, we get:

$$U_{11}dn_u + U_{12}dn_s + [u_1(C_1) + u_{11}(C_1)n_u(w - f_1)]dw$$
$$+[-u_{11}(C_1)(w - f_1)n_s]dh = 0, \qquad \text{(A.4)}$$

$$U_{21}dn_u + U_{22}dn_s + [u_{11}(C_1)n_u(-h - f_1)]dw$$
$$+[-u_{11}(C_1)n_s(-h - f_1) - u_1(C_1)]dh = 0. \qquad \text{(A.5)}$$

$\partial n_u / \partial w$ is evaluated as,

$$\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{pmatrix} \dfrac{\partial n_u}{\partial w} \\ \dfrac{\partial n_s}{\partial w} \end{pmatrix} = \begin{pmatrix} -u_1(C_1) \\ -u_{11}(C_1)n_u(-h - f_1) \end{pmatrix}, \qquad \text{(A.6)}$$

$$\frac{\partial n_u}{\partial w} = \frac{\left(-u_1(C_1)U_{22} + u_{11}(C_1)n_u(-h - f_1)U_{12}\right)}{|J|} \gtrless 0. \qquad \text{(A.7)}$$

In Equation (A.7), $u_1(C_1)$ is the marginal utility of consumption in period 1, which is positive. U_{22} is negative for an interior solution. $u_{11}(C_1)n_u$, which is the change in marginal utility of consumption in period 1 multiplied by the number of children in child labour, is negative since $u_{11}(C_1) < 0$. Child cost in period 1, $(-h - f_1)$ is also negative. $U_{12} = U_{11} < 0$. The sign of the numerator is therefore ambiguous given the positive Jacobian determinant in the denominator, or $\dfrac{\partial n_u}{\partial w} < (>)\ 0$.

Again using Cramer' s rule,

$$\frac{\partial n_s}{\partial w} = \frac{\left(U_{11}(-u_{11}(C_1)n_u(-h - f_1)) + U_{21}(u_1(C_1))\right)}{|J|} \gtrless 0. \qquad \text{(A.8)}$$

In (A.8), the sign of the numerator is ambiguous since $u_{11}(C_1) < 0$. U_{11} and U_{21} are negative as is $(-h - f_1)$.

b. Proof of Proposition 3

From (A.4) and (A.5) we get

$$\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{pmatrix} \frac{\partial n_u}{\partial h} \\ \frac{\partial n_s}{\partial h} \end{pmatrix} = \begin{pmatrix} 0 \\ u_{11}(C_1) n_s (-h - f_1) + u_1(C_1) \end{pmatrix}, \qquad \text{(A.9)}$$

so using Cramer' s rule,

$$\frac{\partial n_u}{\partial h} = \frac{\left(-U_{12}(u_{11}(C_1) n_s(-h - f_1) + u_1(C_1))\right)}{|J|} > 0 \,. \qquad \text{(A.10)}$$

Since both the numerator and the denominator, $|J|$ are positive in Equation (A.10), $\frac{\partial n_u}{\partial h} > 0$. Furthermore,

$$\frac{\partial n_s}{\partial h} = \frac{U_{11}\left(u_{11}(C_1) n_s(-h - f_1) + u_1(C_1)\right)}{|J|} < 0 \,, \qquad \text{(A.11)}$$

as the numerator in (A.11) is negative whereas $|J|$ is positive.

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