JOURNAL OF ECONOMIC DEVELOPMENT
Volume 23, Number 2, December 1998

Some Extensions of Optimal Commodity Taxation with Special Reference to India

Sudip Kumar Sinha and Ajitava Raychaudhuri*

Studies on Optimal Commodity Taxation so far have adopted welfare maximization definition of *optimality*. The present paper proposes an alternative model for calculation of optimal commodity tax rates by adopting revenue maximization definition of *optimality*, in a federal economy like India with two different levels of government, the Centre and the States, taking independent fiscal decisions. Another important constitutional characteristic of such a federal economy is resource transfer in the form of devolution of some centrally collected taxes. The present paper also attempts, unlike previous studies, to endogenise the resource transfer variables in a federal economy like India.

I. Introduction

Indirect taxes in general and commodity taxes in particular play a vital role in resource mobilisation in developing countries like India. The Central (Union) Government and the State Governments, in a federal country like India, usually have annual specific revenue requirements (Target) to finance their expenditures. As any tax, direct or indirect, affects individual utility and hence social welfare of an economy adversely by decreasing utility, the traditional view of the Economists is that the commodity tax rates should be decided in such a way that the decrement in utility is minimum while fulfilling such revenue requirements. Economists thus tried to calculate such "*Optimal*" commodity taxes by maximising social welfare subject to the revenue requirements of the Government. While the basic model proposed by Ramsey (1927), developed as a full-fledged many person model by Diamond and Mirrlees (1971) and illustrated and empirical work done in India's context by Ahmad and Stern (1984) and Ray (1990), incorporates calculation of Optimal Commodity Taxes in a centralised economy, recent studies by Ray and Murty (1990) have shown effect of federal (decentralised) set up on calculation of Optimal Commodity Taxes by incorporating resource transfer (although partially) in the revenue constraints of the Centre and the States and calculating optimal excise tax and optimal sales taxes separately.

Recent studies on Optimal Commodity Taxation have also investigated effects of rationing, rural-urban differences in consumption pattern and demographic variables on optimal commodity taxes. Studies by Jha (1990) and others have shown that rationing of one or more commodities will have serious implications for calculation of optimal commodity taxes. Studies by Ray (1990) have worked out empirical implications of difference in rural-urban consumption patterns

* Department of Economics, Jadavpur University, Calcutta-700032, India.

and collected taxes, namely, Income Tax and Excise Duties, grants-in-aids and loans. Hence all these resource transfers are to be accommodated in the Centre's and the States' revenue expressions. The share of each State in resource transfer is decided by the Finance Commission appointed every five years. The traditional models of Optimal Commodity Taxes have assumed the amount of States' shares as exogenous to the model and have incorporated the transfers only partially while calculating the optimal tax rates. Since a portion of both Income Tax and Excise Duty collected by the Centre are distributed among the States in certain proportions as recommended by the Finance Commission, these transfers are to be endogenised, if possible, along with grants-in-aids and loans; and their optimal values linked to provisions of local and global public goods.

The plan of the paper is as follows. In Section II optimal commodity tax rates are calculated for the case of a single government. In Section III the model is extended to the case of a federal government with resource transfer where attempts have been made to endogenise the resource transfers variables in the form of devolution of taxes. The paper concludes in Section IV.

II. Revenue Maximisation and Optimal Commodity Taxes: The Case of the Single Government

Derivation of Optimal Commodity Taxes will depend on the definition of optimality. 'Optimality' has been defined by Ramsey in his seminal contribution (1927) which sought to answer the question ".. if a given revenue is to be raised by proportional taxes on some or all uses of income.....being possibly at different rates, how should these rates be adjusted in order that the decrement. of utility may be a minimum?" This definition of optimality has been adopted by almost all economists on this subject.

However the real picture, as has been pointed out earlier, in a federal set-up is quite different and we can reframe the definition of optimality. Optimal commodity tax rates can be defined, therefore, as the rates which maximise the tax revenue of a government subject to some welfare constraint. We shall first develope a model of revenue maximisation for the case of a single government.

The government's aim is to choose tax rates $t_i s$ on commodities $i = 1, 2, \ldots\ldots, n$ in such a way as to maximise its total tax revenue R subject to some social welfare constraint expressed in the Bergson Samuelson form as a function $\phi(v^1, v^2, v^3, \ldots v^H)$ of the utilities of individual households $h = 1, 2, \ldots H$. In other words, the authorities choose $t_i (i = 1, 2, \ldots\ldots, n)$ to maximise its tax revenue $R(=\sum_i t_i X_i)$ subject to its welfare constraint $\phi(\ ) = \phi_0$.

Maximise $\quad \sum_i t_i X_i \qquad [X_i = \sum_h x_{ih}]$ (1)

Subject to $\quad \phi(v^1, v^2, \ldots\ldots\ldots, v^H) = \phi_0$ (2)

thus will not contribute to the ranking of ϕ_i.

The numerator in the second term is sensitive to the assumption of aggregate demand system and have significant revenue implications.

As a special case if we consider proportional taxes, i.e., $t_k = \alpha p_k$, then we have

$$\sum_k t_k \frac{\partial X_k}{\partial p_i} = \alpha \sum_k p_k \frac{\partial X_k}{\partial p_i} \tag{6}$$

since with fixed producer price $dt_i = dp_i$. Also, from the household budget constraint, we have, the standard adding-up conditions, that

$$\sum_k p_k \frac{\partial X_k}{\partial p_i} = -X_i \tag{7}$$

hence for uniform proportional taxes

$$\phi_i = (1-\alpha)\frac{X_i}{\sum_h \beta_h x_{ih}} \tag{8}$$

and "distributional characteristics" alone will decide the direction of reform. We shall have to raise taxes for which ϕ_i is low and lower taxes for which ϕ_i is high, till all ϕ_is are equal. It can easily be seen from Equation (8) that if all social welfare weights (β_h) are equal, the right hand side of Equation (8) reduces to $(1-\alpha)/\beta$ and hence all ϕ_is will be equal and there will be no improving marginal reform. As has been pointed out by Ahmad and Stern (1984), "this is similar to the result for a one-consumer economy that uniform taxation is optimum provided the necessary revenue does not exceed the lump sum income of the consumer since in that case, proportional taxation at the appropriate rate acts just like the optimum lump sum tax".

It may be noted that the numerator in the second term in (5) may be seen as a weighted sum of the aggregate demand derivatives with the taxes as weights. We know in general from (7) that weighting the demand derivatives by the prices p_k and adding will lead to the number $-X_i$, whatever the demand derivatives happen to be, provided that they are consistent with the adding up property. The condition (7) therefore "acts as a constraint on the sensitivity of ϕ_i to variations in assumptions on the aggregate demand system" (Ahmad and Stern (1984)).

If we assume proportional taxes, i.e., $t_k = \alpha p_k$, it is easily seen from Equations (6) and (7) that ϕ_i becomes insensitive to the demand system - which can therefore be viewed as a limiting case.

Functional forms, particularly the demand functional forms and policy tools available will, however, play important roles on the actual overall impact of reform and optimality.

$$= \sum_j\sum_i\sum_h \theta_i x^j_{ih} - \sum_j (q^j \sum_j\sum_i\sum_h \theta_i x^j_{ih}) + \sum_j\sum_h \tau w^j_h L^j_h - \sum_j (s^j \sum_j\sum_h \tau w^j_h L^j_h).$$

The share of excise duties and income taxes between the Centre and the States and among the States are decided by the Finance Commission appointed every five year. While different Finance commissions have adopted different approaches to calculate the shares in relation to devolution of taxes, they followed two broad principles namely (i) need and (ii) contribution of the States while deciding on the rates of transfer of such taxes.

We shall assume, in accordance with the broad principles of the Finance Commissions, that devolution of excise duties among the States depends on the relative population and the backwardness of the States.

Thus, let us assume,

E_1 = total share of States' of Union excise duties.

M_1 = States' share of excise duties on the basis of relative population.

$1 - M_1$ = States' share of excise duties on the basis of backwardness.

P^j = population of State j with $\sum P^j = P$, the National population.

P^j / P = relative population of State j.

B^j = measure of backwardness of State j, $0 < B^j < 1$, $\sum B^j = 1$.

While considering the devolution of income tax, we shall assume, in line with the Finance Commission, that such devolution to the State will also depend on need (population) and contribution (collection of income taxes from that State). Let us assume:

E_2 = total share of States' of income tax.

M_2 = States' share of income tax on the basis of population.

$1 - M_2$ = States' share of income tax on the basis of collection.

$$\frac{\sum_h \tau w^j_h L^j_h}{\sum_j\sum_h \tau w^j_h L^j_h} = \text{relative collection of income tax of State } j.$$

For notational simplicity we are assuming identical number of individuals in each State. The Centre's revenue expression thus, becomes,

$$R_0 = \sum_j\sum_i\sum_h \theta_i x^j_{ih} - \sum_j (q^j \sum_j\sum_i\sum_h \theta_i x^j_{ih}) + \sum_j\sum_h \tau w^j_h L^j_h - \sum_j (s^j \sum_j\sum_h \tau w^j_h L^j_h)$$

$$= \sum_j\sum_i\sum_h \theta_i x^j_{ih} + \sum_j\sum_h \tau w^j_h L^j_h$$
$$- \sum_j [E_1 M_1 (\sum_j\sum_i\sum_h \theta_i x^j_{ih}) \frac{P^j}{P} + E_1(1 - M_1)(\sum_j\sum_i\sum_h \theta_i x^j_{ih}) B^j]$$
$$- \sum_j [E_2 M_2 (\sum_j\sum_h \tau w^j_h L^j_h) \frac{P^j}{P} + E_2(1 - M_2)(\sum_h \tau w^j_h L^j_h)] \tag{10}$$

1. Calculation of Optimal E_1 and E_2

a. Calculation of Optimal E_1

Optimal values of E_1 and E_2 are calculated by maximising the Centre's revenue expression subject to the welfare constraint of the nation and setting partial derivatives with respect to E_1 and E_2 equal to zero.

Thus, we want to maximize

$$R_0 = [1-E_1]\sum_j\sum_i\sum_h \theta_i x^j_{ih} + [1-E_2]\sum_j\sum_h \tau w^j_h L^j_h \tag{13}$$

Subject to the national welfare constraint,

$$\psi(W^1, W^2, \ldots\ldots\ldots, W^J) = \psi_0. \tag{14}$$

The Lagrangean equation will be

$$\begin{aligned} L &= R_0 + \zeta(\psi - \psi_0) \\ &= [1-E_1]\sum_j\sum_i\sum_h \theta_i x^j_{ih} + [1-E_2]\sum_j\sum_h \tau w^j_h L^j_h \\ &\quad + \zeta[\psi(W^1, W^2, \ldots\ldots\ldots W^J) - \psi_0]. \end{aligned} \tag{15}$$

Setting $(\partial L/\partial E_1) = 0$ to find optimal E_1, i.e., $\hat{E}_1$, we get,

$$\partial L/\partial E_1 = -\sum_j\sum_i\sum_h \theta_i x^j_{ih} + (1-\hat{E}_1)\sum_j\sum_i\sum_h \theta_i \frac{\partial x^j_{ih}}{\partial E_1} + \zeta\left[\frac{\partial \psi}{\partial E_1}\right] = 0. \tag{16}$$

Rearranging,

$$(1-\hat{E}_1)\sum_j\sum_i\sum_h \theta_i \cdot \frac{\partial x^j_{ih}}{\partial \mu^j_h} \cdot \frac{\partial \mu^j_h}{\partial E^j_1} \cdot \frac{\partial E^j_1}{\partial E_1} = \sum_j\sum_i\sum_h \theta_i x^j_{ih} - \zeta\left[\frac{\partial \psi}{\partial E_1}\right]. \tag{17}$$

The Lagrangean parameter ζ, which is basically $((\triangle R_0/\triangle E_1)/(\triangle\psi/\triangle E_1))$, can be interpreted as the revenue cost on the margin of generating an extra unit of welfare by an increase in E_1. An increase in E_1 will reduce R_0, tax rates remaining unchanged and increase ψ by an increase in the amount of devolution of taxes which will benefit the States. However, if the Centre decides to make up for the losses in R_0 by increasing θ_i, it will

b. *Calculation of Optimal* E_2

The Lagrangean equation remains, from (15), as

$$L = [1 - E_1]\sum_j\sum_i\sum_h \theta_i x^j_{ih} + [1 - E_2]\sum_j\sum_h \tau w^j_h L^j_h + \vartheta[\psi(W^1, W^2, \dots\dots\dots, W^J) - \psi_0]. \tag{20}$$

Setting $\partial L/\partial E_2 = 0$, we get

$$-\sum_j\sum_h \tau w^j_h L^j_h + (1 - \hat{E}_2)\sum_j\sum_h \tau\left(\frac{\partial w^j_h}{\partial E_2} L^j_h + w^j_h \frac{\partial L^j_h}{\partial E_2}\right) + \vartheta\left(\frac{\partial \psi}{\partial E_2}\right) = 0. \tag{21}$$

The derivative $\frac{\partial w^j_h}{\partial E_2}$ will exist because an increase in E_2 will lead to an increase in the amount of devolution of income tax to the States. Assuming a balanced budget expenditure, if the whole of such amount of income tax is spent, it will lead to an increase in wage rate due to creation of new jobs and increased demand for skilled labour.

The derivative $\frac{\partial L^j_h}{\partial E_2}$ exists because L^j_h is the number of hours worked by an individual or the work effort. It is usually affected by the wage rate w^j_h. If E_2 affects w^j_h then it will affect L^j_h through w^j_h.

Rearranging, we get,

$$(1 - \hat{E}_2)\sum_j\sum_h \tau\left(\frac{\partial w^j_h}{\partial E_2} L^j_h + w^j_h \frac{\partial L^j_h}{\partial E_2}\right) = \sum_j\sum_h \tau w^j_h L^j_h - \vartheta\left(\frac{\partial \psi}{\partial E_2}\right).$$

$$\text{or,}\quad (1 - \hat{E}_2) = \frac{\sum_j\sum_h \tau w^j_h L^j_h - \vartheta\left(\frac{\partial \psi}{\partial E_2}\right)}{\sum_j\sum_h \tau\left(\frac{\partial w^j_h}{\partial E_2} L^j_h + w^j_h \frac{\partial L^j_h}{\partial E_2}\right)}$$

$$= \frac{\sum_j\sum_h \tau w^j_h L^j_h - \vartheta\left(\frac{\partial \psi}{\partial W^j}\frac{\partial W^j}{\partial \mu^j_h}\frac{\partial \mu^j_h}{\partial E^j_2}\frac{\partial E^j_2}{\partial E_2}\right)}{\sum_j\sum_h \tau\left(\frac{\partial w^j_h}{\partial E_2} L^j_h + w^j_h \frac{\partial L^j_h}{\partial w^j_h}\frac{\partial w^j_h}{\partial E_2}\right)}$$

$$= \frac{\sum_j\sum_h \tau w^j_h L^j_h - \vartheta\left(\frac{\partial \psi}{\partial W^j}\frac{\partial W^j}{\partial \mu^j_h}\frac{\partial \mu^j_h}{\partial E^j_2}\frac{\partial E^j_2}{\partial E_2}\right)}{\sum_j\sum_h \tau \frac{\partial w^j_h}{\partial E_2}\left(L^j_h + w^j_h \frac{\partial L^j_h}{\partial w^j_h}\right)}.$$

Differentiating L $w.r.t.$ θ_i and setting the partial derivatives to 0, we get,

$$\partial L/\partial \theta_i = (1 - \hat{E}_1)[\sum_j \sum_i \sum_h (x^j_{ih} + \theta_i \frac{\partial x^j_{ih}}{\partial \theta_i})] + \phi_i \left(\frac{\partial \psi}{\partial \theta_i}\right) = 0, \tag{24}$$

$$\text{or,} \quad \phi_i = \frac{(1 - \hat{E}_1)\left[\sum_j \sum_i \sum_h (x^j_{ih} + \theta_i \frac{\partial x^j_{ih}}{\partial \theta_i})\right]}{\left(\frac{\partial \psi}{\partial \theta_i}\right)}. \tag{25}$$

Now, $\psi = \psi[(W^1(v^1(p^1, w^1, \mu^1)), \ldots, W^J(v^J(p^J, w^J, \mu^J))]$ from Equation (12).

Hence,

$$\frac{\partial \psi}{\partial \theta_i} = \frac{\partial \psi}{\partial W^j} \frac{\partial W^j}{\partial v^j_h} \frac{\partial v^j_h}{\partial \mu^j_h} \frac{\partial \mu^j_h}{\partial \theta_i}$$

$$= \frac{\partial \psi}{\partial W^j} \frac{\partial W^j}{\partial v^j_h} \alpha^j_h \frac{\partial \mu^j_h}{\partial p^j_i} \tag{26}$$

(as $\partial \theta_i = \partial p_i$ when t^j_i constant and α^j_h = private marginal utility of income of individual h in State j.)

$$= \sum_j \sum_h \sum_k \left(\frac{\partial \psi}{\partial W^j} \beta^j_h x^j_{kh}\right). \tag{27}$$

β^j_h (i.e., $\partial W^j / \partial \mu^j_h$) is the gross social marginal utility of income of individual h in State j.

Putting this value of $\partial \psi / \partial \theta_i$ in Equation (24) and setting $\partial \theta_i = \partial p_i$ due to the assumption of full shifting of excise duties,

$$\phi_i = \frac{(1 - \hat{E}_1)\left[\sum_j \sum_i \sum_h (x^j_{ih} + \theta_i \frac{\partial x^j_{ih}}{\partial \theta_i})\right]}{\sum_j \sum_k \sum_h \left(\frac{\partial \psi}{\partial W^j} \beta^j_h x^j_{kh}\right)}. \tag{28}$$

Now, since $\theta_i (= \frac{-\partial R/\partial \theta_i}{\partial V/\partial \theta_i})$ is the revenue cost on the margin of an extra unit of welfare via a reduction in the excise duty of the i-th commodity, the set of taxes for which ϕ_is are equal will be the set of optimal excise duties.

$$\lambda_i^j = -\frac{\left[\sum_h x_{ih}^j + \sum_i\sum_h t_i^j \frac{\partial x_{ih}^j}{\partial t_k^j}\right] + \left[\hat{E}_1 M_1 \frac{p^j}{p} + \hat{E}_1(1-M_1)B^j\right]\sum_i\sum_h \theta_i \frac{\partial x_{ih}^j}{\partial t_k^j}}{(\partial W^j/\partial t_k^j)}$$

$$= -\frac{X_i^j + \sum_i\sum_h\left(t_i^j + \left[\hat{E}_1 M_1 \frac{p^j}{p} + \hat{E}_1(1-M_1)B^j\right]\hat{\theta}_i\right)\frac{\partial x_{ih}^j}{\partial t_k^j}}{\sum_h \beta_h^j x_{ih}^j}$$

$$= -\frac{p_i^j X_i^j + \sum_k\left(t_k^j + \left[\hat{E}_1 M_1 \frac{p^j}{p} + \hat{E}_1(1-M_1)B^j\right]\hat{\theta}_i\right)e_{ki}^j X_k^j}{\sum_h \beta_h^j p_i^j x_{ih}^j}. \tag{31}$$

The set of values of t_k^j (incorporating optimal values of E_1 and θ_k) for which λ_i^js are equal will be the set of optimal sales tax rates for State j.

Calculation of optimal commodity tax rates, as have been proposed in this paper, takes note of the simultaneous dependence of taxes, expenditure and price levels/responses on one another. Given the estimates of the demand system, welfare weights of individuals (β_h^j), the states' welfare weights ω^j and the observed set of commodity demand, commodity prices and commodity taxes in the economy, the vector of marginal social costs ϕ_is and λ_i^js can easily be calculated using relations (28) and (31). To get optimal tax rates for which ϕ_is and λ_i^js are equal, we shall have to run an iterative procedure, separately for the two sets of taxes, by increasing the tax rates for which such marginal costs are low and by decreasing the tax rates for which the marginal costs are high - till values of ϕ_is and λ_i^js converge i.e., until the ϕ_is are all equal and the λ_i^js are all equal.

The demand system can be estimated on a standard nine-commodity disaggregation of consumer expenditures, data for which is available from National Sample Survey (NSS) budget surveys published for various rounds. Such demand systems were estimated by Ahmad and Stern (1984) and Ray (1987) by using NSS data. Welfare weights can be calculated using standard expressions used for such terms in optimal taxation literatures.

IV. Conclusion

Calculation of optimal commodity tax rates using revenue maximisation definition of optimality and endogenising resource transfer variables shows that the optimal value of excise duties depends - apart from demand, prices, demand elasticities and welfare weights of the States and the individuals - on the optimal value of E_1, i.e., total share of States of union excise duties. Optimal value of State sales taxes depends, in addition to the above, on M_1, i.e., the State's share of union excise duties on the basis of population, and B^j, measure

References

Ahmad, E., and N.H. Stern (1984), "The Theory of Reform and Indian Indirect Taxes," *Journal of Public Economics*, 25, 259-98.

Atkinson, A.B., and J.E. Stiglitz (1980), *Lectures on Public Economics*, New York, McGraw Hill.

Diamond, P.A., and J.A. Mirrlees (1971), "Optimal Taxation and Public Production II: Tax Rules," *American Economic Review*, 61, 261-78.

Feldstein, M.S. (1972), "Distributional Equity and the Optimal Structure of Public Prices," *American Economic Review*, 62, 32-36.

Jha, R., M.N. Murty, and R. Ray (1990), "Dual Pricing, Rationing and Ramsey Commodity Taxation: Theory and an Illustration," *The Developing Economics*, XXVIII-3, September.

Murty, M.N., and R. Ray (1990), "Fiscal Federalism and Optimal Commodity Taxes: A Comparison of Centralised and Decentralised Decisions," Delhi School of Economics Working Paper No. E/147/90, November.

Ramsey, F.P. (1927), "A Contribution to the Theory of Taxation," *Economic Journal*, 37, 47-61.

Ray, R., (1990), "Optimal Commodity Taxes and Consumer Behaviour in India: Theory and Empirical Evidence," in *Development Policy and Economic Theory*, eds. by K. Basu and P. Nayak, Oxford University Press.

_____ (1993), "Optimal Demogrants and Taxes in a Federal Welfare State," *Journal of Population Economics*, 6, 199-214.