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A Bayesian Decision Approach to Deal with the Risks of Exploration and Development of Natural Environments*

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For a two period development project, it is assumed that development is irreversible and the effects of actions are uncertain. The costs and benefits of the proposed project can not be estimated by replacing the unknown random variables with their expectations. A Bayesian decision rule is considered to reflect both irreversibility and uncertainty of the project. Based on the first period experience, expected value of the project for the second period can be revised. If we have option not to develop at the second period, then the conditional expectation of the second period can be restricted to be nonnegative. For the discrete project size case, a more conservative development at the initial period would result with option not to develop. For the continuous project size case, the decision for the second period depends on the realized outcome of the first period. We calculate the option value under the bivariate normal density assumption for the variables characterizing random effects of the period 1 and 2.

I. Introduction

An irreversible decision restricts our future choice set and any decision made in the first period without uncertainty consideration may lead to a reduction in net benefits. Arrow and Fisher (1981) and Kneese (1971) have pointed out in a review of empirical studies of pollution damages, "a general shortcoming of the previous studies has been that they treated a stochastic or probabilistic phenomenon as being deterministic." Therefore when benefits from a project is uncertain, decision should be made with care.

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Henry (1974) has given many examples of irreversible project. For example, the city government of Paris must decide whether the cathedral of Notre-Dame is to be preserved or to be demolished and replaced by a parking lot. A decision to preserve Notre-Dame is not irreversible since, if adopted in 1974, it leaves open the possibility of further choices between Notre-Dame and the parking lot (or other alternative). However, if the parking lot is built in 1974, they will never again be in a position to choose to keep Notre-Dame; this certainly is an irreversible decision. There are numerous other examples. Construction of a highway, change of coastal lines, and global warming of the earth are all irreversible processes so that the net benefit of developing project should be evaluated carefully with full consideration of the costs due to demolision of the nature or cost of breaking the balance of nature.

The uncertainty may not be restricted to the uncertain return of the project. For example, Schmalensee (1972, 1975) and Bohm (1975) considered uncertain taste of consumer. The term option value describes the magnitude, the maximum amount a consumer with uncertain future preferences (and possibly income) would be willing to pay for the option to purchase a particular commodity at a specified price. Such option price is further developed by many researchers. Miller and Lad (1984) introduced a Bayesian approach to characterize a two period decision problem involved in a proposed resource development project. Edwards (1988) estimated option prices for ground water protection. Hartman and Plummer (1987) examined the sign of option value under income and price uncertainty.

In this paper, we introduce a simple two period project. The project can be initiated at the beginning of the first period, or postponed until the beginning of the second period, or may not be done at all. Once the project is developed, it can not be reversed. At the beginning of the first period, a decision must be made but the expected return for the second period is uncertain. The probability density for the second period return can be revised at the end of the first period based on the realized outcome of the first period, and there might be the cases where the expected return for the second period is so bad that it is better not to develop the project at all if we have the option to do so. For simplicity of exposition, we assume a risk netural situation and zero discount rate for future consumption. Therefore the expected benefits of the project can be compared with the cost of the project. We further assume that the returns for two period are random variables whose density function is assumed to be a joint bivariate normal. Then the conditional expectation for the second period return will be a linear function of the first period return. We would initiate the project at the second period if the conditional expected return for the second period is positive. Under the bivariate normal density assumption, we estimate the quasi-option value, which is equivalent to the expected value of information. Even if we do not assume normality, the quasi-option value can be shown to be nonnegative. If a Monte Carlo integration method is applied, the quasi-option value can be calculated for the general form of the bivariate density function.

In section two we discuss the theoretical part of this paper. Both discrete and continuous project sizes are considered. In section three we present a specific example as an example of the theoretical part of section two. Section four concludes the paper.

II. Theory

Weisbrod (1964) suggested that where there is uncertainty in demand for a publicly provided good or service, there may be some benefit ("option value") to the individual in addition to the conventional price-compensating consumer surplus. Arrow and Fisher (1974) raised the question whether the introduction of uncertainty as to the costs or benefits of a proposed development have any effect on an appropriately formulated investment criterion beyond the replacement of unknown values with their expectations? They answered yes. If the development involves some irreversible transformation of the environment, hence a loss in perpetuity of the benefits from preservation, and if information about the costs and benefits of both alternatives realized in one period results in a change in their expected values for the next, the answer is yes — net benefits from developing the area are reduced and, broadly speaking, less of the area should be developed. There are many good examples, Conrad (1980) suggested two period project and Howe (1979) introduced a Bayesian decision strategies for dealing with the risks of exploration and development from the mining firm's viewpoint.

In this paper, we generalize Arrow and Fisher's (1974). Arrow and Fisher considered net benefits of development (z and w) as random variables so that they could show the expected value of benefits under uncertainty is seen to be less than the value of benefits under certainty. However, without precise description of the probability density function of the random variables (z and w), quantitative discussion is difficult. Therefore, in this paper, we introduce random variables Z_1 and Z_2 for the period 1 and 2 whose probability density function is denoted by $f(z_1, z_2)$. At the end of the first period, the state variable Z_1 is observed and net benefit as well as the conditional density function for the second period are determined. For simplicity of exposition, we introduce a bivariate normal density function for $f(z_1, z_2)$ and we derive quasi-option value which will be defined later. To determine optimal project size, Arrow and Fisher suggested introducing assumption of diminishing returns to both development and preservation, but in this paper, we introduce a quadratic cost function and linear utility function. At the beginning of the second period, the conditional expectation of net benefit is revised based on the first period outcome, and we equate the marginal cost with the conditional expected benefit of the project.

A. Optimal Choice When the Project Size Is Fixed

Consider the following model of resource development. If the project is not developed, net benefit is zero. If the project is developed, net benefit depends on the realization of some random variables.

Z_1 = a random variable characterizing all the random effects of period 1
Z_2 = a random variable characterizing all the random effects of period 2
$R_1(z_1)$ = net benefit from period 1 is a increasing function of z_1
$R_2(z_2)$ = net benefit from period 2 is a increasing function of z_2
$f(z_1, z_2)$ = the joint probability density function for random variables, Z_1 and Z_2
$f(z_1)$ = the marginal probability density function of Z_1
$f(z_2|z_1)$ = the conditional probability density function of Z_2 for given Z_1
C = fixed development cost

We consider the following two different cases. In the first case, the project is developed at the beginning of the first period, while in the second case, the decision for the project is postponed until the beginning of the second period. The total expected returns (TER) are respectively,

Case 1:

$$\text{(2.1)} \quad TER(1) = \int R_1(z_1)f(z_1)dz_1 + \int R_2(z_2)f(z_2)dz_2 - C$$

We may rewrite the right hand side second term of (2-1),

$$\int R_2(z_2)f(z_2)dz_2 = \int R_2(z_2) \int [\frac{f(z_1)f(z_1, z_2)}{f(z_1)} dz_1] dz_2$$
$$= \int f(z_1)dz_1 \int R_2(z_2)f(z_2|z_1)dz_2$$

Case 2:

$$(2.2) \quad TER(2) = \int_{z_1^*} f(z_1)dz_1[\int R_2(z_2)f(z_2|z_1)dz_2 - C]$$

where z_1^* is determined by $\int R_2(z_2)f(z_2|z_1)dz_2 = C$ but its value may not be determined uniquely unless $\int R_2(z_2)f(z_2|z_1^*)dz_2$ is an increasing function of z_1. For simplicity, we assume that z_1^* is uniquely determined, so that $\int R_2(z_2)f(z_2|z_1)dz_2 > C$ for $z_1 > z_1^*$ and $\int R_2(z_2)f(z_2|z_1)dz_2 \leqslant C$ for $z_1 \leqslant z_1^*$.

It is important to distinguish total expected return for the above two cases. In the first case, the project is already initiated at the beginning of the first period so that it is irreversible. At the end of the first period, we may conclude that the project was a mistake with negative expected return for the second period, but there is no way to cancel it. In the second case, we postponed the project until the beginning of the second period, so that we missed positive expected return of the first period $\int R_1(z_1)f(z_1)dz_1$ in (2.1) but we still hold the option not to develop the project if the expected return of the second period is negative.

We define the quasi-option value[1] or equivalent expected value of information (EVI) as

$$(2.3) \quad EVI = \max[TER(2) - TER(1), 0]$$

Since the joint probability density function $f(z_1, z_2)$ is assumed to be known, TER(1), TER(2), and EVI can be calculated.

[1] Conrad (1980) shows that quasi-option value is equivalent to the expected value of information (a more fundamental concept) while option value is equal to the expected value of perfect information. When we purchase of an option or delay of an irreversible action allows an individual to ascertain the true state with certainty, option value is equivalent to the expected value of perfect information. Though Z_2 is a random variable, perfect information means that decision is allowed to vary in accordance with Z_2. See Contrad (1980) to distinguish the option from the quasi-option value. Conrad also provides a numerical example on p. 818 to show the difference between the option value and quasi-option value.

If TER(1)>TER(2), then the quasi-option value is zero. We develop the project at the beginning of the first period because loss of the first period expected return is bigger than the compensation for holding an option not to develop in the second period. By holding an option, we can avoid expected negative return at the second period, but its value can be smaller than the first period expected return.

$$\begin{aligned} &TER(1) - TER(2) \\ &\quad = \int R_1(z_1)f(z_1)dz_1 - \int^{z_1^*} f(z_1)dz_1 [R_2(z_2)f(z_2|z_1)dz_2 - C] > 0 \end{aligned}$$

If TER(1)<TER(2), then the quasi-option value is positive. If we replace the random variables with expected values as shown in (2.1), total expected return (TER) will be smaller compared to the second case (2.2) where stochastic nature is fully considered. Conrad (1980) has given a more or less similar example, but due to discrete probability density function, decisions and stochastic realizations were drawn with a tree diagram. In comparison, a continuous random variable is used in this paper and an explicit solution is established.

In the following, we provide a simple example how to derive EVI based on a joint normal distribution for $f(z_1, z_2)$. See Goldberger (1991) for notation of bivariate density function.

If $f(z_1, z_2)$ has a bivariate normal density function whose parameters are described by a set of numbers ($\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \sigma_{12}$), then the expectations, variances, and covariances are:

$$\begin{aligned} (2.4) \quad & E(Z_1) = \mu_1, \ E(Z_2) = \mu_2, \\ & V(Z_1) = \sigma_1^2, \ V(Z_2) = \sigma_2^2, \ C(Z_1, Z_2) = \sigma_{12} \end{aligned}$$

The marginal distribution of Z_1 is normal:

$$(2.5) \quad f(z_1) \sim N(\mu_1, \sigma_1)$$

The conditional distribution of Z_2 given Z_1 is normal

$$(2.6) \quad f(z_2|z_2) \sim N(\alpha + \beta z_1, \sigma^2)$$

where

$$(2.7) \quad \beta = \sigma_{12}/\sigma_1^2, \quad \alpha = \mu_2 - \beta\mu_1, \quad \sigma^2 = \sigma_2^2 - \beta^2\sigma_1^2$$

We observe that the conditional expectation is a linear function and the conditional variance is constant.

Further assume that the return functions are linear, $R_1(z_1)=z_1$ and $R_2(z_2)=z_2$,

(2.8) $\quad TER(1)= \mu_1+ \mu_2-C$

(2.9) $\quad TER(2)= \int_{z_1^*} f(z_1)(\alpha+ \beta z_1-C)dz_1$

where $z_1^*=-(\alpha-C)/\beta$. We used $\int R_2(z_2)f(z_2|z_1)dz_2= \int z_2f(z_2|z_1)dz_2= \alpha+ \beta z_1=C$. In practice, truncated normal distribution (2.9) is not explicitly integrable so that a Monte Carlo integration method will be discussed in the next section.

Until now, we have assumed the knowledge of joint probability density function. In general, the density function is not known and must be estimated. Howe (1979) shows how to use the Bayes' theorem to identify and quantify new oil reserves to replace those used up. Upon exploration, we spend some cost R_1 (negative if cost) and get a summary of the outcome of on-site evaluation z_1. We want to find the conditional probability of Z_2 (size of oil reserve) given that Z_1 is observed. The conditional densities are

(2.10) $\quad f(z_2|z_1)=\dfrac{f(z_1, z_2)}{f(z_1)}$

and

(2.11) $\quad f(z_1|z_2)=\dfrac{f(z_1, z_2)}{f(z_2)}$

Geologists can estimate the values of $f(z_1|z_2)$, the probability of the various on-site outcomes z_2, given the real nature of the deposit. Multiplying (2.11) through by $f(z_2)$ produces a computable expression for the joint probability of Z_2 and Z_1.

(2.12) $\quad f(z_1, z_2)=f(z_1|z_2)f(z_2)$

Assuming that geologists estimated the reserve density $f(z_1)$ and $f(z_2)$ from general geological exploration data and the geologists have estimated the conditional probabilities of the various on-site outcomes given the deposit, $f(z_1|z_2)$, we can construct the joint density (2.12) and the conditional probability of reserve $f(z_2|z_1)$ given the on-site evaluation summary z_1.

If we assume that the joint density function has a bivariate normal distribution, then it is convenient because $f(z_1)$, $f(z_2)$ and $f(z_1|z_2)$ are all univariate normal density functions. We need to specify mean and variance of these functions. In the following subsection, we relax the fixed project size assumption so that the project size should be determined to maximize expected return.

B. Optimal Choice When the Project Size Is A Continuous Variable

Case 1: No Option

The decision for the second period project should be made at the beginning of the first period. Let us introduce the following notation.

v = Total expected net benefit from the projects of the period 1 and 2
d_1 = The size of project at period 1
d_2 = The size of project at period 2
Z_1 = The random variable characterizing all the random effect of period 1
Z_2 = The random variable characterizing all the random effect of period 2
$R_1(z_1)$ = z_1 is benefit at period 1 for a unit project developed
$R_2(z_2)$ = z_2 benefit at period 2 for a unit project developed either at period 1 or 2
$C_1(d_1)$ = d_1^2C is developing cost at period 1 for project size d_1
$C_2(d_2)$ = d_2^2C is developing cost of period 2 for project size d_2
$f(z_1, z_2)$ = joint probability density function for random variables Z_1 and Z_2
$f(z_1)$ = marginal probability density function of Z_1
$f(z_2|z_1)$ = conditional probability density function of Z_2 for given Z_1

We have introduced a quadratic cost function. If it is a linear function of the project size, the optimal size becomes either zero or infinity so that we needed some penalty for becoming large.

Solve a maximization problem:

$$(2.13)\quad v = \max_{d_1, d_2} d_1 \int z_1 f(z_1)dz_1 + (d_1 + d_2) \int z_2 f(z_2)dz_2 - (d_1 + d_2)C$$

Let

$A \equiv \int z_1 f(z_1)dz_1$: marginal expected benefit of the project at period 1

$B \equiv \int z_2 f(z_2)dz_2$: marginal expected benefit of the project at period 2

$$(2.14)\quad \frac{\partial v}{\partial d_1} = A + B - 2d_1C = 0 \Rightarrow d_1^* = \frac{A+B}{2C}$$

$$(2.15)\quad \frac{\partial v}{\partial d_2} = B - 2d_2C = 0 \Rightarrow d_2^* = \frac{B}{2C}$$

Introducing d_1^* and d_2^* into (2.13)

$$(2.16)\quad v^* = \frac{(A+B)^2 + B^2}{4C}$$

For period 1 development, marginal expected benefits of 1 and 2 are equated with marginal cost. For period 2 development, marginal expected benefits of 2 is equated with marginal cost.

Case 2: With Option

V = Total expected net benefit from the projects of the period 1 and 2
D_1 = The size of project at period 1
D_2 = The size of project at period 2

We assume the random variables, density functions, benefit functions, and cost functions are same to those of the previous no option case.

Solve a maximization problem:

$$(2.17)\quad V = \max_{D_1} D_1 \int z_1 f(z_1)dz_1 - D_1^2C + \int f(z_1)dz_1 \left[\max\{(D_1+D_2)\int z_2 f(z_2|z_1)dz_2 - D_2^2C\}\right]$$

Assuming a bivariate normal density for $f(z_1, z_2)$,

$$(\partial/\partial D_2)[(D_1+D_2)\int z_2 f(z_2|z_1)dz_2 - D_2^2C] = (\partial/\partial D_2)[(D_1+D_2)(\alpha + \beta z_1) - D_2^2C] = 0$$

Hence optimal project size for the period 2 is

(2.18) $D_2^* = \dfrac{\alpha+\beta z_1}{2C}$

If the realized summary random variable z_1 is large, then larger project will be developed at period 2. Since we have the option not to initiate a development at period 2, the optimal project size D_2^* will be zero if $z_1 \leqslant z_1^* = -\alpha/\beta$. Note that D_2^* does not depend on D_1^*. To choose an optimal project size for the period one, introduce D_2^* into (2.17).

$$(\partial/\partial D_1)[D_1 \int z_1 f(z_1)dz_1 - D_1^2 C + D_1 \int z_2 f(z_2)dz_2 + \text{Terms without } D_1] = 0$$

Hence

(2.19) $D_1^* = \dfrac{A+B}{2C}$

The project size for the first period is the same either we have option or not, i.e., $D_1^* = d_1^*$.

Introducing D_1^* and D_2^* into (2.17) and using the definition of A and B below (2.13), we get

(2.20) $V^* = \dfrac{(A+B)^2}{4C} + \int_{z_1^*} \dfrac{(\alpha+\beta z_1)^2}{4C} f(z_1)dz_1$

Therefore the quasi-option value is

(2.21) $V^* - v^* = \dfrac{1}{4C}[\int_{z_1^*} f(z_1)(\alpha+\beta z_1)^2 dz_1 - B^2]$

If Z_1 and Z_2 are independent variables with $f(z_1, z_2) = f(z_1)f(z_2)$, then $V^* = v^*$. The information of Z_1 is of no use for the second period. We have $\beta = 0$, $z_1^* = -\infty$, and $\alpha = \mu_2 = B$. In general, $z_1 > -\infty$, but if $f(z_1)$ is very close to zero (i.e., measure zero in the probability theory) for all $z_1 \leqslant z_1^*$, then we can approximate $\int_{z^*}$ with $\int_{-\infty}$. Under this assumption, second period investment is always positive. Now introduce $\alpha = \mu_2 - \beta\mu_1$, $E(Z_1) = \mu_1$, and $E(Z_1^2) = \mu_1^2 + \sigma_1^2$

(2.22) $V^* - v^* = \dfrac{1}{4C}(\alpha^2 + 2\alpha\beta\mu_1 + \beta^2(\sigma_1^2 + \mu_1^2) - \mu_2^2) = \sigma_1^2\beta^2 > 0$

where = means approximately equal to.

Under the assumption of bivariate normal density for $f(z_1, z_2)$, we derived a simple expression for the quasi-option value. However, without normality assumption, we can show that the quasi-option value is positive using the Jensen's inequality. Replacing $\alpha + \beta z_1$ with $E[Z_2|Z_1]$, (2.21) becomes

$$(2.23)\quad V^* - v^* = \frac{1}{4C}\left[\int_{z_1^*} f(z_1)[E(Z_2|Z_1)]^2 dz_1\right] - \frac{1}{4C}B^2$$

$$-\frac{1}{4C}\left[\int_{z_1^*} f(z_1[E(Z_2|Z_1)]^2 dz_1\right.$$

$$-\frac{1}{4C}\left[\int f(z_1)[E(Z_2|Z_1)]dz\right]^2 = \frac{1}{4C}(P - Q]$$

where we used $B = \int z_2 f(z_2)dz_2 = \int f(z_1)[E(Z_2|Z_1)]dz_1$. Also note that the cutoff level z_1^* is determined by $E(Z_2|Z_1) \geqslant 0$ for $z_1 \geqslant z_1^*$ and $E(Z_2|Z_1) \leqslant 0$ for $z_1 \leqslant z_1^*$.

Now define

$$(2.24)\quad P \equiv \int_{z_1^*} f(z_1)[E(Z_2|Z_1)]^2 dz_1$$

$$(2.25)\quad Q \equiv \left[\int f(z_1)[E(Z_2|Z_1)]dz_1\right]^2$$

$$(2.26)\quad E^*(\cdot) \equiv S \int_{z_1^*} f(z_1)dz_1(\cdot)$$

$$(2.27)\quad g(z_1) \equiv E(Z_2|Z_1)$$

$$(2.28)\quad \zeta \equiv E^*[g(Z_1)] = E^*[E(Z_2|Z_1)]$$

where $S > 1$ is introduced in (2.26) for normalization because $\int_{z_1^*} f(z_1)dz_1 < 1$. We will use the following Jensen's inequality, see Goldberger (1991).

The Jensen's inequality: Suppose $Y = h(g)$ is a convex function of g, then $E^*(Y) \geqslant h[E^*(g)]$.

In particular, we consider the case of $Y = h(g) = g^2$ so that $E^*(Y) \geqslant [E^*(g)]^2$.

$$(2.29)\quad E^*(Y) = SP$$

$$(2.30) \qquad \geqslant \zeta^2$$
$$= (E^*[g(Z_1)])^2$$
$$= [S\int_{z_1^*} f(z_1)[E(Z_2|Z_1)]dz_1]^2$$
$$\geqslant S[_{z_1^*} f(z_1)[E(Z_2|Z_1)]dz_1]^2$$
$$(2.31) \qquad \geqslant S[\int_{-\infty} f(z_1)[E(Z_2|Z_1)]dz_1]^2$$
$$(2.32) \qquad = SQ$$

where (2.30) is due to the Jensen's inequality. For (2.31), see $E(Z_2|Z_1) \geqslant 0$ for $z_1 \geqslant z_1^*$ and $E(Z_2|Z_1) \leqslant 0$ for $z_1 \leqslant z_1^*$. Therefore EVI in (2.23) is nonnegative.

To summarize this section, we assumed a simple linear utility function with zero discount rate. The project can be initiated at the first period or at the second period, but once intiated, it is not reversible. When the project size is fixed (only one size project is available) and if the quasi-option value [EVI in (2.3)] is positive, then we better postpone the project until the beginning of the second period. If the quasi-option value is zero, then no need to wait for the second period. As an alternative case, we considered how to determine the project size when it is not fixed. We introduced a linear benefit function, but the cost function is assumed to be quadratic. Without such restriction, the optimal project size becomes either zero or infinity. The optimal project size for the first period is same either we have an option not to develop or not. If we do not have option, then we decide the second period project size at the beginning of the first period by equating the marginal cost with the expected benefit of the project at the second period. However, if we have option, we decide the second period project size at the beginning of the second period based on the realized outcome of the first period, i.e., the conditional expectation of net benefit is revised based on the first period outcome z_1. We equate the marginal cost with the conditional expected benefit of the project at the second period. Under the bivariate normal density assumption for z_1 and z_2, the expected value of information (EVI) is explicitly derived. When the density function is not known, EVI is shown to be positive based on Jensen's inequality.

III. Application: A Monte Carlo Integration

To estimate the quasi-option value in (2.9), we need to calculate truncated normal distribution. To get an explicit answer is impossible, but we can approximate it based on Monte Carlo integration method. Consider

$$(3.1) \qquad \int_{z_1^*} f(z_1)h(z_1)dz_1 \doteq \frac{1}{T}\sum_{i=1}^{T} h[z_1(i)]R(i)$$

where h is an arbitrary function. Suppose we choose T random samples $(i=1,2,\ldots,T)$ from the density $f(z_1)$. Let $R(i)=1$ if $z_1(i) \geq z_1{}^*$ and zero otherwise. This means if the generated sample is larger than the cutoff level $z_1{}^*$, then we use this sample in the summation but discard it if it is smaller than the cutoff level. See Ripley (1987) for more details of this method. In (2.9), we need to calculate

$$(3.2) \qquad D_0(z_1{}^*) \equiv \int_{z_1^*} f(z_1)dz_1$$

$$(3.3) \qquad D_1(z^*) \equiv \int_{z_1^*} z_1 f(z_1)dz_1$$

Suppose $z_1 \sim N[\mu_1, \sigma_1^2]$, then with $z=(z_1 - \mu_1)/\sigma_1$, f(z) has a standard normal distribution. The lower cutoff is

$$(3.4) \qquad z^* \equiv \frac{z_1^* - \mu_1}{\sigma_1}$$

Define standard normal distribution functions,

$$(3.5) \qquad F_0(z^*) \equiv \int_{z^*} f(z)dz$$

$$(3.6) \qquad F_1(z^*) \equiv \int_{z^*} zf(z)dz$$

Rewrite D_0 and D_1 using these[2]

$$(3.7) \qquad D_0(z_1{}^*) = F_0(z^*)$$

$$(3.8) \qquad D_1(z_1{}^*) = \mu_1 F_0(z^*) + \sigma_1 F_1(z^*)$$

[2] To derive $D_1(z_1{}^*)$,

$$\int_{z_1^*} z_1 f(z_1)dz_1 = \int_{z^*}(\mu_1+\sigma_1 z)\frac{1}{\sqrt{2\pi\sigma_1^2}}e^{-(z_1-\mu_1)^2/2\sigma_1}dz_1$$
$$= \int_{z^*}(\mu_1+\sigma_1 z)\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz$$
$$= \mu_1 F_0(z^*) + \sigma_1 F_1(z^*)$$

Hence, using (3.7) and (3.8), TER(2) in (2.9) is

$$TER(2) = (\alpha - C)D_0(z_1^*) + \beta D_1(z_1^*)$$
$$= (\alpha - C + \beta\mu_1)F_0(z^*) + \beta\sigma_1 F_1(z^*)$$

Assuming standard normal density for f(z) and using 100,000 randomly generated samples, $F_0(z^*)$ and $F_1(z^*)$ are calculated for $z^* = -3$, -2,5, -2.0, -1.5, -1.0, -0.5, 0.0, +0.5, +1.0, +1.5, +2.0, +2.5; +3.0 in Table 1.

Example 1: Suppose $TER(1) = \mu_1 + \mu_2 - C = 0$. Since expected net benefit of the project is zero, we are indifferent. However, if we consider the uncertainty of the project, then $TER(2) \geqslant 0$. We postpone the project until the beginning of the second period, and we will initiate it only if the conditional density function is favorable so that conditional expected benefit exceeds the project development cost. For example,

Table 1

TRUNCATED MOMENT DISTRIBUTION

Assuming standard normal density for f(z), generate 100,000 randomly generated samples to calculate $F_0(z^*)$ and $F_1(z^*)$,

$$F_0(z^*) \equiv \int_{z^*} f(z)dz$$
$$F_1(z^*) \equiv \int_{z^*} zf(z)dz$$

	$F_0(z^*)$	$F_1(z^*)$
$z^* = -3.0$	0.99878	0.00065
$z^* = -2.5$	0.99402	0.01349
$z^* = -2.0$	0.97749	0.04988
$z^* = -1.5$	0.93264	0.12675
$z^* = -1.0$	0.84172	0.23826
$z^* = -0.5$	0.69112	0.34877
$z^* = 0.0$	0.49812	0.39634
$z^* = +0.5$	0.39634	0.34945
$z^* = +1.0$	0.15726	0.23955
$z^* = +1.5$	0.06607	0.12792
$z^* = +2.0$	0.02266	0.05351
$z^* = +2.5$	0.01658	0.01658
$z^* = +3.0$	0.00117	0.00380

let $\mu_1 = \mu_2 = \sigma_1^2 = \sigma_2^2 = 1$, $C=2$, and $\sigma_{12}=0.5$. Then from (2.7), $\alpha = \beta = 0.5$. Furthermore, $z_1^* = 3$, $z^* = (3 - \mu_1)/\sigma_1 = 2$. From the values of Table 1, $F_0(z^*=2) = 0.02266$ and $F_1(z^*=2) = 0.05351$. Hence $EVI = TER(2) = 0.004095$. The value of information is quite small in this case because we develop only if $z_1^* \geqslant 3$, i.e., $z^* \geqslant 2$ for standard normal density. The probability of z_1^* bigger than 3, is small and furthermore, expected net benefit $\alpha + \beta z_1^* - C$ is also small.

Example 2: Let $\mu_1 = 1$, $\mu_2 = 5$, $\sigma_1 = 0.1$, $\sigma_2 = 20$, $C = 5$, and $\sigma_{12} = 1$. Then from (2.7), $\beta = 100$ and $\alpha = -95$. Furthermore, $z_1^* = 100/100 = 1$, $z^* = (1 - \mu_1)/\sigma_1 = 0$. From the values of Table 1, $F_0(z^* = 0) = 0.49812$ and $F_1(z^* = 2) = 0.39634$. Hence $TER(1) = 1 + 5 - 5 = 1$ and $TER(2) = 3.9634$. From these values, $EVI = TER(2) - TER(1) = 2.9634$.

IV. Conclusion

We have considered a two period irreversible project with a simple linear utility function and zero discount rate. When the costs and benefits of a proposed development have uncertainty, replacement of random variables with their expectations is not appropriate if the development involves some irreversible transformation of the environment. We described the two period benefits with a joint normal density. At the end of the first period, the random variable characterizing all the random effect of the first period is observed, and the conditional expectation for the second period can be calculated. If the project is reversible and if the conditional expectation for the second period net return is negative, we will cancel the project. However, if the project is irreversible, we can make the decision based on two different criteria. The first decision rule is to compare the expected return for the two periods with development cost while the second decision rule is to postpone the project until the beginning of the second period to hold the option not to develop the project if the conditional net benefit is negative. In this case, we lose the first period return. Arrow and Fisher (1974) and others have shown how the first decision rule is different from the second decision rule. In this paper, by introducing a bivariate normal density function to describe two period random return, we tried to show the difference between the first and second decision rules.

For simplicity of exposition, we assumed that the utility function is risk neutral and discount rate for future consumption is zero. However, relaxation of these assumptions will not affect the fundamental aspects of our paper. We defined total expected returns (TER) as expected value of return function, R(z), but we did not restrict the functional form of R(z). Therefore, if a risk aversive utility function is desired, we may impose quasi-concavity restriction to the return function R(z) and interpret R(z) as a utility function. Computation of expected utility is difficult, but a numerical integration method can be utilized. Alternatively, if we want to derive an explicit expression for total expected utility, we may choose a linear utility function for $R(z)=z$ and a normal density function for the random variable z. Similarly, if the discount rate is positive, net present value of the second period return (or utility function) will decrease. Conrad (1980) derived similar result to ours based on the concave wealth function and positive discount rate, but the random variable z has only two discrete outcomes to simplify calculation.

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