Estimation of Consumer Expenditure Systems: Case of Korea (1953-1974)

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I. Introduction

This paper considers applications of three consumer demand models to Korean time series from 1953 to 1974. They include the indirect addilog system, the Rotterdam demand model and the linear expenditure system. The purpose of this paper is then to experiment the validity of the consumer demand theories.

The postulates of consumer demand theory are developed for an individual. An individual's preference is assumed to be representable by a well-behaved utility function, $U(x_1, \cdots x_n)$ The maximization of the utility function $U(x_1, \cdots x_n)$ subject to the budget constraint $p'x=m$ gives rise to demand functions of the form;

$$x_i = x_i \quad (p, m) \qquad (i=1, \ldots, n) \qquad (1\text{-}1)$$

where x is the $n \times 1$ vector whose *ith* component x_i represents the quantity of the *ith* commodity consumed, p is the corresponding vector of commodity prices, m is the total expenditure. These demand functions satisfy the budget constraint and are homogeneous of degree zero in all prices and total expenditure. In addition, the matrix of Slutsky substitution terms is symmetric and negative semidefinite.[1]

The data used for the estimation of the three models came from annual observations on prices, quantities, and total consumption in Korea for the period of 1953-1974, tabulated by the Bank of Korea (1975). We are concerned with market or aggregate demand functions. Unfortunately, market demand functions do not necessarily have theoretical plausibility when every individual's demand functions are aggregated. However, it is generally assumed that market demand functions are theoretically plausible for an analysis of aggre-

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1 For a full discussion, See Phlips (1974); Brown and Deaton (1972).

gated behaviour. The original data published by the Bank of Korea are composed of 12 commodity groups. They are regrouped into four broad categories: food, clothing & furniture, household operation, and miscellaneous. The procedures are shown in Table 1. Our reasons for regrouping are trifold: First, the utility functions underlying our demand functions are additive. Second, data observations cover only 22 years and therefore if we should work with original data, we are bound to have insufficient degree of freedom. Third, because we are estimating an interrelated system of equations simultaneously, the computational burden is substantial.

Table 1
Commodity Grouping

Expendiutre Items	Commodity Groups
Food Beverages Tobacco	Food
Clothing and other personal effects Furniture, furnishings and household equipment	Clothing and furniture
Rent and watercharges Fuel and light Household operation Transportation and communication	Household operation
Personal care and health expenses Recreation and entertainment Miscellaneous services	Miscellaneous

II. Estimation of the Demand Models

A. *Rotterdam Differential Demand Model*

Theil (1967) developed a demand model which approaches demand analysis in a probabilistic manner. The main ingredient of Theil's approach is the value share[2], which can be regarded as a probability in view of the fact that it is nonnegative and adds up to one when summed over all commodities.

The demand functions (1-1) derived from utility maximization

2 The value share implies the proportion of total expenditure spent on a particular commodity.

subject to the budget constraint can be expressed in terms of prices and real income. The logarithmic differential of the resulting demand function can then be written;[3]

$$d(\log x) = \eta_{io} \; d(\log m) + \sum_{j=1}^{N} \beta_{ij} \; d(\log p_j) \qquad (2\text{-}1)$$

where m is the real income, β_{ij} is the compensated cross price elasticity of the ith good with respect to the jth price, and η_{io} is the income elasticity of the ith good. Weighting each demand equation by the expenditure share, $w_i = p_i x_i / m$, we obtain;

$$w_i d(\log x_i) = \mu_i \; d(\log m) + \sum_{j=1}^{N} \pi_{ij} \, d(\log p_j). \qquad (2\text{-}2)$$

The parameter μ_i, the marginal value share, is the ith income elasticity weighted by the expenditure share or equivalently the derivative of expenditure on the ith good with respect to income. The parameter π_{ij} is the compensated cross price elasticity β_{ij}, weighted by the expenditure share. The prior information implies restrictions on these parameters. Homogeneity requires $\sum_{j=1}^{N} \pi_{ij} = 0$ for all i; the Slutsky symmetry condition implies the symmetry of the matrix $[\pi_{ij}]$; the adding-up property implies $\sum_{i=1}^{N} \mu_i = 1$; and finally the classical second order conditions require that the matrix $[\pi_{ij}]$ be negative semidefinite. The first three sets of restrictions can be imposed on the estimation procedure.[4] The inequality constraints are more difficult to impose, but they can be used as a check on the validity of the numerical results.

For purpose of statistical estimation we use the discrete analog of the basic equation (2-2)[5];

3 See Parks (1969), p. 630 and Theil (1967, 1971).

4 See Deaton (1974), p. 342...."to apply constraints explicitly within the model.... is the peculiar strength of the Rotterdam system."

5 Equation (2-3) was derived by Theil by algebraic manipulation of a first order Taylor linearization of a general demand relation. See Theil (1967), Chapter 7.

$$w_{it}\,Dx_{it} = \sum_{j=1}^{N} \pi_{ij}\,Dp_{jt} + \mu_i\,Dx_t + u_{it} \qquad (2\text{-}3)$$
$$(i=1,\ldots\ldots N;\ t=1,\ldots\ldots T)$$

The operator D represents the log difference; $Dx_{it} = \log x_{it} - \log x_{it-1}$. The weight $w^*_{it} = 1/2(w_{it} + w_{it-1})$ is the average value share in succesive periods. The variable $Dx_t = \sum_{i=1}^{N} w^*_{it}\,Dx_{it}$ is a value weighted average of the logarithmic differences of the quantities demanded. It is thus a volume index of the change in total consumption and can be interpreted as a measure of the change in real income. Finally, u_{it} is a random disturbance term. It is assumed[6] that

$$E(u_{it}) = 0 \text{ for all i and t,}$$
$$E(u_{is}\,u_{jt}) = \begin{cases} 0 & \text{for } s=t;\ i,j=1,\ldots\ldots N \\ w_{ij} & \text{for } s\neq t;\ i,j=1,\ldots\ldots N \end{cases} \qquad (2\text{-}4)$$

that is, the random disturbances are uncorrelated across observations but are correlated across equations for the same observations. The contemporaneous covariance matrix $\Omega = [w_{ij}]$ is then singular (Theil (1971)). Summing equation (2-3) over *i* gives the restriction $\sum_{i=1}^{N} u_{it} = 0$ from which it follows that $\Omega\,\iota = 0$, where $\iota' = (1, 1, \ldots, 1)$.

The estimation technique should take account of this covariance singularity as well as the parameter constraints implied by the homogeneity, adding-up, and symmetry conditions. Consider the sum of the first N-1 equations (2-3);

$$\sum_{i=1}^{N-1} w^*_{it}\,Dx_{it} = \sum_{j=1}^{N} \Big(\sum_{i=1}^{N-1} \pi_{ij}\Big) Dp_{jt} + Dx_t \sum_{i=1}^{N-1} \mu_i + \sum_{i=1}^{N-1} u_{it} \qquad (2\text{-}5)$$

Applying the parameter constraints, this becomes

$$Dx_t - w^*_{it}\,Dx_{Nt} = \sum_{j=1}^{N} -\pi_{Nj}\,Dp_{jt} + (1-\mu_N)\,Dx_t - u_{Nt} \qquad (2\text{-}6)$$

which is equivalent to the Nth demand equation. In other words, this equation is a redundant repetition of the first N-1. Elimination of the redundancy[7] reduces the system to be estimated by one equa-

6 See Zellner (1962).

7 The choice of equation to be deleted is arbitrary.

tion and also avoids the covariance singularity in the reduced system[8]. The homogeneity constraint will be imposed on all computations described below, but the symmetry constraints will be tested and then imposed when the test shows that they are acceptable on the basis of our data. Homogeneity restriction can be written as $\pi_{iN} = -\sum_{j=1}^{N-1} \pi_{ij}$ and it is readily verified that;

$$w^*_{it} Dx_{it} = \sum_{j=1}^{N-1} \pi_{ij} (Dp_{jt} - Dp_{Nt}) + \mu_i \; Dx_t + u_{it} \qquad (2\text{-}7)$$

which means that the price of the *n*th commodity is used as a deflator of the other prices; $Dp_{jt} - Dp_{Nt} = D(p_{jt}/p_{Nt})$.[9]

The system (2-7) is a set of "seemingly unrelated" regression equations in the sense of Zellner (1962). If we disregard the symmetry conditions, the N-1 equations (2-7) fall under the case of identical explanatory variables. Hence Zellner's estimation procedure reduces to ordinary single equation least square, and the resulting estimates are best linear unbiased.[10] The results are shown in the Table 2. The last element of each row is the point estimate of π_{i4} and its standard error. The point estimate is the negative value of the sum of the estimates of π_{i1}, π_{i2}, and π_{i3}. The standard error is computed from the sum of the estimated sampling variances and covariances of these estimates.[11] Since (2-6) represents the sum of the first N-1 demand equations, we can derive the estimates and their standard errors of μ_4 and π_{4j} from μ_i and π_{ij} (i=1,2,3), respectively.

It is clear from the results in Table 2 that the estimated matrix is negative semidefinite as required by the second order conditions. A statistical test of the symmetry hypothesis can be performed as the test of a linear hypothesis within the jointly estimated system of equations (2-7). We have considered the unconstrained estimator of β;

$$\mathbb{b} = \begin{pmatrix} \mathbb{b}_1 \\ \mathbb{b}_2 \\ \mathbb{b}_3 \end{pmatrix} = \begin{pmatrix} (\mathbb{X}'\mathbb{X})^{-1}\mathbb{X}'\mathbb{Y}_1 \\ (\mathbb{X}'\mathbb{X})^{-1}\mathbb{X}'\mathbb{Y}_2 \\ (\mathbb{X}'\mathbb{X})^{-1}\mathbb{X}'\mathbb{Y}_3 \end{pmatrix} \qquad (2\text{-}8)$$

8 See Theil (1971), p. 333.
9 *Ibid.*, p. 337.
10 *Ibid.*, p. 309.
11 *Ibid.*, p. 338.

where

$$\mathbb{Y}_i = \begin{pmatrix} w^*_{i1}Dx_{i1} \\ w^*_{i2}Dx_{i2} \\ \vdots \\ w_{iT}Dx_{iT} \end{pmatrix} \qquad \beta_i = \begin{pmatrix} \pi_{i1} \\ \pi_{i2} \\ \pi_{i3} \\ \mu_i \end{pmatrix} \qquad u_i = \begin{pmatrix} u_{i1} \\ u_{i2} \\ \vdots \\ u_{iT} \end{pmatrix} \quad (i=1,2,3)$$

$$\overline{\mathbb{X}} = \begin{pmatrix} Dp_{11}-Dp_{41} & Dp_{21}-Dp_{41} & Dp_{31}-Dp_{41} & Dx_1 \\ Dp_{12}-Dp_{42} & Dp_{22}-Dp_{42} & Dp_{32}-Dp_{42} & Dx_2 \\ \vdots & \vdots & \vdots & \vdots \\ Dp_{1T}-Dp_{4T} & Dp_{2T}-Dp_{4T} & Dp_{3T}-Dp_{4T} & Dx_T \end{pmatrix}$$

The symmetry constraint can be written as; $\mathbb{R}\,\beta = \mathbb{0}$

$$(2\text{-}9)$$

where

$$\mathbb{R} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \mu_1 & \pi_{21} & \pi_{22} & \pi_{23} & \mu_2 & \pi_{31} & \pi_{32} & \pi_{33} & \mu_3 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

The symmetry constrained generalized least square estimator is then[12]

$$\hat{\beta}^* = \mathbb{b} - \mathbb{C}\mathbb{R}'(\mathbb{R}\mathbb{C}\mathbb{R}')^{-1}\mathbb{R}\mathbb{b} \qquad (2\text{-}10)$$

where $\mathbb{b}$ is given (2-8), $\mathbb{R}$ in (2-9), and

$$\mathbb{C} = [\mathbb{X}'(\Omega^{-1}\otimes\mathbb{I})\mathbb{X}]^{-1} = \Omega\otimes(\overline{\mathbb{X}}'\overline{\mathbb{X}})^{-1} \qquad (2\text{-}11)$$

The test statistic is now

$$\frac{3T-12}{3} \times \frac{\mathbb{b}'\mathbb{R}'\{\mathbb{R}[\Omega\otimes(\overline{\mathbb{X}}'\overline{\mathbb{X}})^{-1}]\mathbb{R}'\}^{-1}\mathbb{R}\mathbb{b}}{(\mathbb{Y}-\mathbb{X}\mathbb{b})'(\Omega^{-1}\otimes\mathbb{I})(\mathbb{Y}-\mathbb{X}\mathbb{b})} \qquad (2\text{-}12)$$

which has an F distribution. Since the contemporaneous covariance matrix is unknown and must be estimated, it has an approximate F distribution in our case. The value of the test statistic is 1.37. The 5 percent significance limit is about 2.7 and, therefore, the result is not significant.

We now impose the symmetry constraints and estimate β according to (2-10). We use

12 *Ibid.*, p. 340.

$$\mathbb{C} - \mathbb{C}\mathbb{R}'(\mathbb{R}\mathbb{C}\mathbb{R}')^{-1}\mathbb{R}\mathbb{C} \tag{2-13}$$

as a covariance matrix of this estimator.[13] The point estimates and their standard errors are given in Table 3. Again, for the constrained estimates, the diagonal elements of π_{ij} are negative as required and the matrix is negative semidefinite. Considerable gains in the efficiency of the point estimates were achieved by adding symmetry restrictions. The second term in (2-13) introduces the gain in efficiency of the point estimation due to the inclusion of prior information. It is apparent that even incorrect prior information brings a reduction in the standard errors of the point estimates.

Further examination reveals that adjustment is taken up by the cross price elasticities and these tend to change substantially. It appears that this adjustment of the cross price elasticities occurs partly because of the number of restrictions on them, and also due to the fact that many of the unrestricted cross elasticities were non-significant and further changes in these parameters would be less likely to reduce the objective function than changes in statistically significant parameters (Byron (1970)).

It should be noted that the use of the Rotterdam equations (2-6) amounts to a first differencing of the data, which seems to eliminate serial correlation of the time series data. The Durbin-Watson test for auto-correlated residuals was not significant.

Table 2

Rotterdam Demand Model without Symmetry Constraint

T=21

Commdity group	μ_i	π_{i1}	π_{i2}	π_{i3}	π_{i4}	R^2	D.W.
Food	.478 (.053)	−.048 (.025)	.001 (.028)	−.009 (.031)	.056 (.027)	.72	2.18
Clothing & furniture	.215 (.029)	.041 (.014)	−.014 (.016)	−.021 (.017)	−.047 (.015)	.68	2.18
Household operation	.315 (.022)	−.004 (.010)	.026 (.012)	−.034 (.013)	.012 (.011)	.20	2.13
Miscellaneous	.173 (.025)	.01 (.011)	−.012 (.013)	.022 (.015)	−.021 (.013)	.57	1.75

13 *Ibid.*, p. 344.

Table 3

Rotterdam Demand Model with Symmetry Constraint T=21

Commdity group	μ_i	π_{i1}	π_{i2}	π_{i3}	π_{i4}
Food	.490 (.053)	−.034 (.024)	.028 (.012)	−.007 (.009)	.013 (.011)
Clothing & furniture	.213 (.029)	.028 (.012)	−.021 (.012)	.018 (.009)	−.025 (.008)
Household operation	.131 (.022)	−.007 (.009)	.018 (.009)	−.034 (.010)	.023 (.007)
Miscellaneous	.166 (.025)	.013 (.011)	−.025 (.008)	.023 (.007)	−.011 (.005)

B. *The Indirect Addilog Model*

Under the strong assumption of additive utility, Houthakker (1960) derived indirect addilog model from the indirect additive utility function;

$$V\left(\frac{m}{p}\right)=\sum_{i=1}^{N} a_i \left(\frac{m}{p_i}\right)^{b_i} \qquad (2\text{-}14)$$

where p and m are prices and income, respectively.[14] The resulting demand functions are

$$X_i=\frac{a_i b_i m^{b_i} p^{-b_i}}{\sum_{j=1}^{N} a_j b_j m^{b_j-1} p_j^{-b_j}} \qquad (i=1,\cdots,N) \qquad (2\text{-}15)$$

These functions satisfy the homogeniety, adding-up, and the Slutsky symmetry conditions. Negative semidefinite condition requires that $b_i>-1$ for all *i*. The logarithm of the ratio of that expression for two distinct commodities produces an equation that is linear in the b_i parameters, which will facilitate the estimation procedure.[15]

For estimation, equations (2-15) can be written in stochastic expenditure form as;

$$Y_{it}=\frac{a_i b_i m_t^{b_i} p_{it}^{-b_i} e^{\varepsilon_{it}}}{\sum_{j=1}^{N} a_j b_j m_t^{b_j-1} p_{jt}^{-b_j}} \qquad (i=1,2,\cdots,N) \qquad (2\text{-}16)$$

14 Both a_i and b_i are parameters which can be interpreted as the preference indicators and reaction coefficients respectively. See Wit and Somermeyer (1956)).

15 It is the technique developed by Parks (1969).

where it is assumed for the stochastic factor $e^{\varepsilon_{it}}$ that $E(\varepsilon_{it})=0$ for all *i* and that;

$$E(\varepsilon_{it}\varepsilon_{js})=\begin{cases} O & \text{for} \quad t=s \\ \omega_{ij} & \text{for} \quad t\neq s \end{cases} \qquad (i,j=1,\cdots N) \tag{2-17}$$

Taking the logarithm of the ratio of the pairs of equations (2-16) for different commodities, the estimation equations become

$$(\log Y_{it}-\log Y_{jt})=A_{ij}+b_i \log\left(\frac{m}{p_{it}}\right)-b_j \log\left(\frac{m}{p_{jt}}\right)+u_{ijt} \tag{2-18}$$
$$(i=1,2,\cdots\cdots N;\ j=i+1,i+2,\cdots,N\ ;\ t=1,\cdots,T)$$

where $A_{ij}=\log(a_i b_i/a_j b_j)$; $u_{ijt}=\varepsilon_{it}-\varepsilon_{jt}$ is a random disturbance term with $E(u_{ijt})=0$. Equations (2-18) are symmetric with respect to subscripts *i* and *j*. Hence, there are a total of N(N-1)/2 different equations, one for each pair of commodities.

If all of the equations (2-18) are estimated separately, we obtain N-1 distinct estimates for each of the b_i parameters. However, according to the specification of the addilog model, the parameter b_i takes the same value in every equation wherever it appears. When this restriction is imposed on the estimators, all but N-1 of the equations (2-18) become redundant. If we arbitrarily select the equations

$$(\log Y_{1t}-\log Y_{2t})=A_{12}+b_1 \log\left(\frac{m}{p_{1t}}\right)-b_2 \log\left(\frac{m}{p_{2t}}\right)+u_{12t} \qquad (t=1,\cdots,T)$$
$$(\log Y_{1t}-\log Y_{Nt})=A_{1N}+b_1 \log\left(\frac{m}{p_{1t}}\right)-b_N \log\left(\frac{m}{p_{Nt}}\right)+u_{1Nt} \tag{2-19}$$

subject to the restriction of the same b_1, all of the remaining equations (2-18) can be described as exact linear combinations of the N-1 in (2-19).[16]

The addilog specification can be tested as test of the hypothesis that the separate estimates of the b_i's are equal across all equations in (2-18). We test in the set of equations (2-19) the hypothesis that b_1 is equal across equations. Rejection of the latter implies rejection of the former. Equations (2-19) are a system of "seemingly unrelated" regression equations and Zellner's two stage method is again appropriate. Table 4 and 5 presents the generalized least square es-

16 See Parks (1969), p. 639.

timates by Zellner's method without and with the restriction of the same b_I, respectively. Since the covariance matrix is unknown and must be estimated, the approximate test of the hypothesis that the b_I estimates are equal across equations gives $F_{2,57}=6.95$, which implies a rejection of the hypothesis.

Table 4

Estimate of the Indirect Addilog Model without Constraint

T=22

Commodity group	$\log\left(\frac{a_i b_i}{a_j b_j}\right)$	$\log\left(\frac{m}{p_{it}}\right)$	$\log\left(\frac{m}{p_{jt}}\right)$	R^2
Miscellaneous vs. food	−.208 (.161)	−.215 (.084)	−.011 (.082)	.81
Miscellaneous vs. clothing & furniture	−.086 (.216)	.292 (.124)	.294 (.143)	.26
Miscellaneous vs. household operation	.385 (.311)	.105 (.094)	.193 (.131)	.15

Table 5

Estimate of the Indirect Addilog Model with Constraint

T=22

Commodity group	$\log(a_i b_i/a_j b_j)$	$\log\left(\frac{m}{p_{it}}\right)$	$\log\left(\frac{m}{p_{jt}}\right)$
Miscellaneous vs. food	−.266 (.159)	.001 (.060)	.198 (.059)
Miscellaneous vs. clothing & furniture	−.323 (.198)	.001 (.060)	−.035 (.073)
Miscellaneous vs. household operation	.029 (.250)	.001 (.060)	.034 (.083)

C. *The Linear Expenditure System*

The linear expenditure system (LES) is based on the Stone-Geary utility function (Stone (1954));

$$U=\prod_{i=1}^{N}(x_i - r_i)^{\beta_i} \tag{2-20}$$

The resulting expenditure functions can be written as;

$$y_i = p_i\ x_i = r_i\ p_i + b_i\ (m - \sum_{j=1}^{N} r_j\ p_j\ )\quad (i=1, \cdots, N) \qquad (2\text{-}21)$$

where, as before, x_i and p_i are the quantity and price of the i th commodity y_i is the expenditure on the i th commodity; and m is total expenditure or income. The parameter b_i is the derivative of expenditure on ith good with respect to income. According to equations (2-21), expenditure on each commodity is determined by two terms. Samuelson (1948) interpreted the parameter r_i as the subsistence or permanent level of demand for the ith good; hence the first term $r_i\ p_i$ represents the basic expenditure. After making all of the basic expenditures, the consumer allocates the remainder of his income, his supernumerary income, to the various commidities in proportions given by the b_i's. The second term on the right hand side of the equation (2-21) thus represents the amount of supernumerary income spent on the ith good.

The demand equations are homogeneous of degree zero in prices and income; they satisfy the adding-up criterion and the Slutsky symmetry condition. Negativity conditions will be sastisfied if $m - \sum_{j=1}^{N} r_j\ p_j > 0$ and $0 < b_j < 1$ hold for all j. The estimation of the LES has been discussed by Stone (1954). His method involves the assumption of a set of initial values for the b_i's; then, after a suitable transformaion, a set of estimates for the r_i's can be obtained. The estimated r_i's are then used to estimate the b_i's and this iterative process continues until some convergence is achived.

For statistical purposes the LES can be written in the following form;[17]

$$y_i = r_i\ p_i + b_i\ (m - \sum_{j=1}^{N} r_j\ p_j\ ) + u_i \qquad (i=1, \cdots, N) \qquad (2\text{-}22)$$

Where u_i is the vector of unobserved random disturbances. The constraint on the b_i's and the fact that total expenditure m is the sum

17 See Parks (1971), p. 900.

of the y_i's imply that $\sum_{i=1}^{N} u_i = 0$. Thus, one of the equations (2-22) is redundant in the sense that we can obtain the Nth equation by an appropriate linear combination of the remaining N-1 equations. We shall consider the reduced system which consists of the equations (2-22) with the Nth equation deleted.

A regression equation for r_i's given the b_i's can be expressed;

$$(y_i - b_i m) = \sum_{j=1}^{N} r_j (\delta_{ij} - b_i) r_j + u_i$$

$$\text{where } \delta_{ij} = \begin{cases} 1 \text{ for } i=j \\ 0 \text{ for } i \neq j \end{cases} \quad (i=1, \cdots, N) \tag{2-23}$$

Alternatively, a typical equation may be written as follows to provide an equation for the b_i's given the r_i's;

$$(y_i - r_i p_i) = (m - \sum_{i=1}^{N} r_j p_j) + u_i \tag{2-24}$$

Finally, the complete statistical system can be written as;

$$\begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{N-1} \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_{N-1} \end{pmatrix} r + \begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_{N-1} \end{pmatrix} \tag{2-25}$$

or as

$$\begin{pmatrix} W_1 \\ W_2 \\ \vdots \\ W_{N-1} \end{pmatrix} = \begin{pmatrix} Z & & \\ & Z & \\ & & \ddots \\ & & & Z \end{pmatrix} b + \begin{pmatrix} U_1 \\ U_2 \\ \\ U_{N-1} \end{pmatrix}$$

The random disturbances, $(U_{1t}, \ldots\ldots, U_{N-1t})$ for observation t are assumed to come from a multivariate normal distribution with mean zero and covariance matrix Σ. Disturbances for different observations are assumed to be uncorrelated. Thus the disturbance vector U has $E(U) = O$ and $E(UU') = \Omega = \Sigma \otimes I$, where Σ is the $(N-1)$

$\times$ (N−1) covariance matrix of the multivariate normal distribution, where I is a T$\times$T identity matrix and where $\otimes$ denotes the Kronecker–product operation. Table 6 presents results of the fitted LES for the Korean consumer expenditure data.

It is worthy of special mention that all but one of the commodity groups show a negative r_i. This is clearly not satisfactory from the interpretation of Samuelson (1948). However, if r_i is negative, the demand for the *i*th good is elastic with respect to its own price, which seems probable for the consumption data we are dealing with.

Table 6

Estimated Coefficients of Linear Expenditure System

N=22

Commodity group	r_i	b_i	R^2
Food	−161.06	.66	.94
Clothing & furniture	10.60	.10	.84
Household operation	−14.83	.14	.62
Miscellaneous	−2.45	.10	.86

III. Performance Comparisons of the Models

The preceding sections examined the empirical validity of the three consumer demand models. The usefulness of the demand models should also be based on the good fit of the models. Table 7 presents the R^2 values. The results are somewhat mixed. On the whole the LES seems to show the best performance. It completely dominates the two other models. However, the fact that the LES has been fitted by a procedure that involves minimizing sum of square residuals gives an advantage to that model based on R^2. Theil (1967) has applied the techniques of information theory to the evaluation of the share predictions of demand models. The model's prediction can be considered as prior probabilities and then we can evaluate the expected gain in information that we obtain from the posterior shares. If our predicted shares for period *t* are W_{it} , then the information inaccuracy of the predictions is measured by;

$$I_t = \sum_{i=1}^{N} W_{it} \log \frac{W_{it}}{\hat{W}_{it}} \qquad (3\text{-}1)$$

where $\hat{W}_{it}$ is the actual share of the ith good in the tth period.

However, formulation (3-1) can not be established if W_{it} is smaller than $\hat{W}_{it}$. Accordingly, this paper considers the modification of (3-1) as follows by taking the absolute value of $W_{it}/\hat{W}_{it}$ (Lee (1977)).

$$I_t = \sum_{i=1}^{N} W_{it} \left| \log \frac{W_{it}}{\hat{W}_{it}} \right| \tag{3-2}$$

This information measure gives each commodity its appropriate weight in the measure of fit. Table 8 gives values of the information inaccuracy for the demand models, which confirms the impression given by the R^2 results. Neither of the Rotterdam demand model and the LES dominates the other. But the indirect addilog model shows the best performance.

Table 7

R^2 Values of the Three Models

Commodity groups \ Models	Indirect Addilog Model (vs. food)	Rotterdam Model	linear Expenditure System
Food		.72	.94
Clothing and furniture	.81	.68	.84
Household operation	.26	.20	.62
Miscellaneous	.15	.57	.86

IV. Conclusions

This paper, on the basis of the classical demand postulates, obtained the parameter estimates and certain test statistics of the three demand models. The classical postulates, that is, homogeneity, adding-up, symmetry, and negative semidefiniteness imply restrictions on the demand functions. As many of these restrictions as possible are used for empirical work in order to simplify the demand model and improve the efficiency of the parameter estimates. However, their empirical usefulness depends on whether they allow adequate representation of the data. According to the estimated results of this paper, the Rotterdam model passed the test, but the indirect addilog

Table 8

Information Inaccuracy

Period \ Models	Indirect Addilog Model	Rotterdam Demand Model	LES
1953	.0512		.0596
1954	.0156	.0232	.0416
1955	.0440	.0159	.0293
1956	.0374	.0081	.0269
1957	.0558	.0361	.0248
1958	.0577	.0194	.0177
1959	.0731	.0217	.0266
1960	.0532	.0359	.0235
1961	.1201	.0173	.0237
1962	.0345	.0346	.0386
1963	.0564	.0461	.0551
1964	.1312	.0347	.0629
1965	.0842	.0316	.0546
1966	.0825	.0137	.0580
1967	.0278	.0143	.0286
1968	.0584	.0214	.0447
1969	.0405	.0177	.0350
1970	.0231	.0151	.0180
1971	.0482	.0104	.0224
1972	.0397	.0087	.0197
1973	.0736	.0231	.0414
1974	.0276	.0097	.0399

model failed. As for the estimation procedure generalized least square methods were used instead of the ordinary least square method. Comparisons of the predictive ability were made by R^2 and the information inaccuracy. According to these criteria, Rotterdam model and the linear expenditure system performed well but the indirect addilog model did not.

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