JOURNAL OF ECONOMIC DEVELOPMENT
Volume 16, Number 2, December 1991

Research Note

Optimal Growth with Exhaustible Resources and Foreign Technology in a Small Open Economy

A. Aydin Cecen*

This paper develops an infinite horizon optimal growth model whereby the growth dynamics of a small resource economy is scrutinized. The country is assumed to extract its exhaustible resource for both domestic production and export and it imports foreign technology to enhance its domestic productivity. The optimal paths of resource depletion, capital accumulation and productivity growth reveal that while output, capital and consumption per capita all increase exponentially at a constant rate, the resource is depleted asymptotically.

I. Introduction

This paper attempts to analyze the optimal growth of a small open economy that extracts its resource for domestic production and export purposes, and simultaneously imports foreign technology in order to increase domestic productivity.

Various models of long-term international capital movements have been set forth in order to describe a country's patterns of growth (see, for instance, Borts (1964), Hanson and Neher (1967), Onitsuka (1974). These studies ignored the technological aspects of international capital movements. Koizumi and Kopecky (1977) analyzed the role of technological transfers with the help of a descriptive growth model by assuming that these transfers depend on the ownership of a country's capital stock: they found a negative relationship between a country's indebtedness and its capital intensity. On the other hand, the optimal foreign borrowing literature including Hamada (1965, 1966), Bardhan (1965),

* Associate Professor, Central Michigan University, Department of Economics, Mt. Pleasant, MI 48859, U.S.A.

Hochman and Razin (1970), Clifton (1980) among others did not discuss the role of foreign capital borrowed as a vehicle of technical change.

In the context of exhaustible resources a number of economists tackled the question of the feasibility of steadily-growing per capita consumption in closed economies with technical progress. More specifically, it has been shown by Solow (1974) and Stiglitz (1974) that growing per capita consumption is possible in a closed economy where the resource is essential for production and the population grows exponentially. Yet these results brought about by substitution effects between capital and the resource are critically dependent on the assumption of exogenous technical progress. Suzuki (1976), Kamien and Schwartz (1978), and Chiarella (1980) criticized this assumption of cost-free technical progress and studied the same problem for closed economies in the case of endogenous technical progress that requires some fraction of domestic output to be invested in research programs. Dasgupta, Eastwood and Heal (1978) generalized the previous results by combining the effects of exogenous technical change, international capital movements and optimal extraction rates in the framework of an intertemporal growth model. More recently Moussavian (1985) examined the growth of a resource exporting economy with a traded and a non-traded sector having exogenous technical progress.

The model we propose to analyze in this paper is in the same tradition in that it strives to analyze the optimal growth of a resoruce economy with technical change. Yet it differs from previous work in the resource literature in that it attempts to scrutinize the impact of "foreign goods" imported (in order to enhance the technological level) on the optimal growth of the resource economy. In other words the vehicle of technical change is not, as it is usually assumed, the stock of foreign capital imported or borrowed by the economy in question but the *flow* of imported foreign (high tech, for instance) goods used as an input in the domestic production of the country.

II. The Model

Following Kemp and Long (1980), we assume that the aggregate output Y of the economy is represented by a Cobb-Douglas production function, suppressing the time argument t for all variables.

(1) $$Y = T^{\alpha_1} K^{\alpha_2} R_1^{\alpha_3} L^{\alpha_4}$$

where T is the level of technology at a certain point in time, K, R_1, and L are capital, resource and labor inputs respectively. Labor is assumed to be stationary and, by choice of units, equated to one. There are no restrictions on returns to scale with respect to all four factors jointly but it is assumed that there are diminishing returns to capital and resource with the possibility of increasing returns to technology. Hence, $\alpha_i > 0$, $i = 1, 4$ and $1-\alpha_2-\alpha_3 > 0$.

The economy exports part of the resource extracted R_2 and imports foreign goods that are used to improve the technological level of the country. Therefore the total flow of the investment-consumption good Y plus the proceeds from resource exports should cover domestic investment, consumption and total foreign imports. Assuming away depreciation we can write the instantaneous budget constraint as[1]

(2) $$Y + p'(t)R_2 = \dot{K} + C + p(t)F$$

where p(t) and p′(t) are respectively the relative world price of the foreign goods imported, and that of the resource; $\dot{K}$ is investment, F the total flow of foreign goods and C denotes consumption.

The foreign goods imported enhance the technical level of the country so that

(3) $$\dot{T} = g(F)$$

where T is simply dT/dt and g(F) is the function that relates the flow of foreign goods imported to the change of the technological level of the economy. It is assumed that the principle of diminishing returns holds as well for technological progress, hence,

(4) $$g' > 0, \; g'' < 0$$

On the other hand, the resource is extracted at a rate R_1 for domestic production and at R_2 for exports and the remaining stock is denoted by S. Hence,

(5) $$\dot{S} = -R_1 - R_2, \quad S(0) = S_0 > 0$$

Depending on the adopted institutional framework, the planning organization of the economy or the representative consumer uses the utilitarian criterion to maximize the discounted utility of per capita con-

[1] Capital depreciation at a constant exponential rate can be easily introduced without changing the results of the model.

sumption over an infinite horizon. Given a positive discount rate ρ, the optimization problem of the economy can be written as:

$$\underset{\{C,\, R_1,\, R_2,\, F\}}{\text{Max}} \quad \int_0^{\infty} e^{-\rho t}\, U(C)\, dt$$

subject to

(6a) $\dot{K} = Y - C - p(t)F + p'(t)R_2$

(6b) $\dot{S} = -R_1 - R_2$

(6c) $\dot{T} = g(F)$

given K(0), S(0), T(0), ρ and p(t).

Furthermore, we assume for analytical convenience that the welfare of the economy can be represented by a logarithmic utility function, although the results will not be altered if a more general constant elasticity utility function of the form $U(C) = C^{\varepsilon}/\varepsilon$, $0 < \varepsilon < 1$ is adopted.[2]

(7) $U(C) = \ln C$

Finally, we assume that the country is small in the world markets for foreign high tech products, hence the relative price p(t) of foreign investment goods is exogenously given and constant:[3] yet the economy is a resource-rich one, therefore, the relative price of the resource p′(t) can be influenced by the extraction rate R_2, i.e. $p'(R_2)$. Hence, we can write,

(8) $p(t) = \bar{p}$

The current-value Hamiltonian of the above system (6) can be written as;

[2] Indeed we adopt these specific functional forms for the sake of tractability. Yet, it should be emphasized that these functions are not *totally* ad hoc. For instance, the justification of the choice of a Cobb-Douglas production function in the resource literature is given by Kemp and Long (1980). They show that as the resource input goes to zero, some more general production functions approach the Cobb-Douglas form. For the use of a logarithmic utility (or constant elasticity utility) function see, Dasgupta, Eastwood and Heal (1978), Takayama (1980), Chiarella (1980), among many others.

[3] Again a constant relative price $\bar{p}$ is not crucial for the solution of the optimal program. An exogenously given price p(t), which is consistent with our small country assumption, will suffice to get tractable results — except that it will introduce an additional parameter for the growth rate of p(t).

(9) $$H = \ln C + \lambda_1[Y - C - \bar{p}F + p'(R_2)R_2] + \lambda_2(-R_1 - R_2) + \lambda_3\, g(F)$$

from which we can get the following conditions for the maximization of the welfare functional.[4]

Optimality Conditions:

(10a) $$\frac{\partial H}{\partial C} = \frac{1}{C} - \lambda_1 = 0$$

(10b) $$\frac{\partial H}{\partial R_1} = \lambda_1 \frac{\partial Y}{\partial R_1} - \lambda_2 = 0$$

(10c) $$\frac{\partial H}{\partial R_2} = \lambda_1\left[\frac{dp'(R_2)}{dR_2}R_2 + p'(R_2)\right] - \lambda_2 = 0$$

(10d) $$\frac{\partial H}{\partial F} = -\lambda_1 \bar{p} + \lambda_3\, g'(F) = 0$$

Costate Equations:

(11a) $$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial K} = \rho\lambda_1 - \left(\frac{\partial Y}{\partial K}\lambda_1\right)$$

(11b) $$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial S} = \rho\lambda_2$$

(11c) $$\dot{\lambda}_3 = \rho\lambda_3 - \frac{\partial H}{\partial T} = \rho\lambda_3 - \left(\lambda_1 \frac{\partial Y}{\partial T}\right)$$

Transversality Conditions:

(12a) $$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t)K(t) = 0;\ \lim_{t\to\infty} e^{-\rho t}\lambda_2(t)S(t) = 0;\ \lim_{t\to\infty} e^{-\rho t}\lambda_3(t)T(t) = 0$$

(12b) $$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t) \geqq 0;\ \lim_{t\to\infty} e^{-\rho t}\lambda_2(t) \geqq 0;\ \lim_{t\to\infty} e^{-\rho t}\lambda_3(t) \geqq 0$$

In order to be able to get tractable results we have to specify the form of the function $g(F)$ as we have done in the case of the welfare and production functions. Therefore we assume that $g(F)$ is of the form,

(13) $$\dot{T} = F^{\alpha}, \quad 0 < \alpha < 1$$

[4] These necessary conditions are also sufficient by virtue of the concavity of the maximand.

Concentrating on the interior solution of the system (9) and denoting by g_x the growth rate of x, i.e., $g_x = \frac{\dot{x}}{x}$ where $\dot{x} = \frac{dx}{dt}$, we can get the following results.

Differentiating (10a) logarithmically with respect to time, we get,

(14) $$\frac{\dot{\lambda}_1}{\lambda_1} + \frac{\dot{C}}{C} = g_{\lambda_1} + g_c = 0$$

Using (11a) with (14) we can write

(15) $$g_c = \frac{\partial Y}{\partial K} - \rho = Y_K - \rho$$

Equation (15) simply states that the discounted marginal utility of per capita consumption grows at a rate equal to the marginal product of capital; this is the classical Ramsey savings rule.

On the other hand differentiating logarithmically (10b) with respect to time and using (11a) and (11b) we obtain

(16) $$g_{Y_R} = Y_K \text{ where } Y_K = \frac{\partial Y}{\partial K}, \ Y_R = \frac{\partial Y}{\partial R}$$

which says that the proportional rate of change in the marginal product of the exhaustible resource should be equal to the marginal product of capital along the optimal path,: this is the well-known Solow-Stiglitz condition for the efficient use of natural resources.[5]

To study the dynamic behavior of the system (6) we have to define some new variables as it is often done in the exhaustible resource literature. Hence we introduce the following new variables defined as

(17) $$M = \frac{C}{K};\ N = \frac{Y}{K};\ P = \frac{R_1}{S};\ Q = \frac{F}{K};\ Z = \frac{F^{\alpha}}{T};\ N' = \frac{R_2^{\gamma}}{K}; P' = \frac{R_2}{S}$$

Finally, following Dasgupta et al. (1978), we assume an inverse demand function for the resource in the following form,

$$p'(R_2) = R_2^{\gamma-1},\ 0<\gamma<1$$

[5] These results are the same for a closed resource economy as shown by Stiglitz (1974), Solow (1974), Suzuki (1976), and Chiarella (1980).

where $1/\gamma-1$ is the usual price elasticity of demand for the resource. Using the appropriate growth rates and the newly defined variables in (17), it is possible to obtain the following simultaneous differential equations system (see the Appendix for details),

$$(18) \qquad g_M = M + (\alpha_2-1)\, N + \bar{p}Q - \gamma N' - \rho$$

$$(19) \qquad g_N = \frac{\alpha_1 Z}{1-\alpha_3} + \left\{\frac{1-\alpha_2-\alpha_3}{1-\alpha_3}\right\} (M + \bar{p}Q - \gamma N') + (\alpha_2-1)N$$

$$(20) \qquad g_P = \frac{\alpha_1 Z}{1-\alpha_3} - \frac{\alpha_2}{1-\alpha_3} (M + \bar{p}Q - \gamma N') + P + P'$$

$$(21) \qquad g_Q = \left\{\frac{\alpha\alpha_1 Z}{\bar{p}Q(\alpha-1)} - \frac{\alpha_2}{\alpha-1}\right\} N + M + \bar{p}Q + \gamma N'$$

$$(22) \qquad g_Z = \left\{\frac{\alpha^2\alpha_1 Z}{\bar{p}(\alpha-1)Q} - \frac{\alpha_2}{\alpha-1}\right\} N - Z$$

$$(23) \qquad g_{N'} = \left\{\frac{\gamma\alpha_2}{\gamma-1}\right\} N + M + \bar{p}Q - \gamma N'$$

$$(24) \qquad g_{P'} = \frac{\alpha_2 N}{\gamma-1} + P + P'$$

Equations (18)-(24) form a system of simultaneous differential equations whose steady-state solution can be found by equating the growth rates to zero, i.e., $g_M = g_N = g_P = g_Q = g_Z = g_{N'} - g_{P'} = 0$. Taking account of the nonlinearity of Equations (21) and (22) and noting that P and P′ are not independent in light of Equations (20) and (24), we can solve the system for the steady-state values $\bar{M}$, $\bar{N}$, $\bar{Q}$, $\bar{Z}$, $\bar{N}'$.[6]

$$(25) \qquad \bar{M} = \frac{\alpha_1\alpha_2 AB - 2\bar{p}\gamma}{\alpha_3 A}$$

$$(26) \qquad \bar{N} = \frac{\alpha_3\rho\,(1-\gamma)}{\alpha_2}$$

$$(27) \qquad \bar{Q} = \frac{(2A-\alpha_2 C)\gamma}{\alpha_1\alpha_2\bar{p}}$$

$$(28) \qquad \bar{Z} = \frac{(\alpha\alpha_1 - \bar{p}A)\rho}{\alpha_3(1-A)}$$

6 We have used the computer package program MAPLE to solve this system of equations (18)-(24). This program is capable of solving simultaneous equations symbolically.

(29) $$\bar{N}' = \frac{(1-\alpha_3)A - \alpha_1 \bar{p}}{\alpha_1 \alpha_3 \gamma}$$

and

$\bar{p} + \bar{p}' = \rho\alpha_3$ where
$$A = \alpha_1 + \alpha_2 + \alpha_3$$
$$B = \alpha\alpha_1 + \alpha_2 - \alpha_3$$
$$C = \alpha_1 + 2\alpha_2 - 2\alpha_3$$
$$D = 2\alpha_1 + 2\alpha_2 - \alpha_3$$

Furthermore, by using the definitions of new variables in Equation (17) and equating to zero their growth rates for the steady-state solution, we obtain

(30) $$g_K = g_C = g_Y = g_F = \frac{g_T}{\alpha}$$

and

(31) $$g_{R_1} = g_{R_2} = g_S$$

Finally, substituting the steady-state solutions of Equations (25)-(29) into the growth rates of C, Y, K, F, T (see the Appendix for these growth rates), we can get

(32) $$g_C = g_K = g_Y = g_F = \frac{g_T}{\alpha} = \rho\alpha_3(1-\gamma) - \rho$$

Equation (32) simply implies that consumption, output, the flow of foreign products and the technological level of the country all increase exponentially at a constant rate.

Moreover Equation (31) together with the growth rates of S, R_1 and R_2 reveals that

(33) $$g_{R_1} = g_{R_2} = g_S = -\rho\alpha_3$$

In other words, the growth rate of resource extraction for both domestic production and export is the same and declines exponentially at a constant rate. Therefore, the resource is depleted asymptotically.

III. Conclusion

The paper analyzed the optimal growth of a resource rich open economy that extracts its resource for both domestic production and export. The export of high tech foreign product is assumed to enhance the technological level of the country. It has been shown that when certain parametric conditions are satisfied, it is possible for the economy to enjoy growing per capita consumption, output and capital, despite the finite stock of its exhaustible resource.

Appendix

To get the growth rates of the newly defined variables in Equations (18)-(24) we have to obtain first the growth rates of the variables in the maximization problem and then find the growth rates of the newly defined variables in Equation (17).

Dividing (6a) by K, we obtain,

(1) $$g_K = N - M - \bar{p}Q + \gamma N'$$

Similary, by dividing (6b) by S, (6c) by T, we get

(2) $$g_S = -P - P'$$

(3) $$g_T = Z$$

Nothing that $Y_K = \alpha_2 \frac{Y}{K}$ Equation (15) can be written as

(4) $$g_C = \alpha_2 N - \rho$$

In a similar vein, we can express Equation (16) as

(5) $$g_Y - g_{R_1} = \alpha_2 N$$

On the other hand, by differentiating logarithmically the production function in Equation (1) with respect to time,

(6) $$g_Y = \alpha_1 g_T + \alpha_2 g_K + \alpha_3 g_{R_1}$$

Using Equations (1), (3) and (22), we can find g_Y and g_{R_1} as,

(7) $$g_Y = \frac{\alpha_1 Z}{1-\alpha_3} - \frac{\alpha_2}{1-\alpha_3}(M + \bar{p}Q - \gamma N') + \alpha_2 N$$

(8) $$g_{R_1} = \frac{\alpha_1 Z}{1-\alpha_3} - \frac{\alpha_2}{1-\alpha_3}(M + \bar{p}Q - \gamma N')$$

As long as $F>0$, we know from Equation (10d) that

(9) $$\lambda_1 \bar{P} = \lambda_3 \alpha F^{\alpha-1}$$

Differenting logarithmically Equation (9) with respect to time and using Equations (10d), (11a) and (11c), we obtain

(10) $$g_f = \frac{\alpha F^{\alpha-1}}{\bar{p}(\alpha-1)} Y_T - \frac{Y_K}{(\alpha-1)}$$

Furthermore, remembering that $Y_T = \frac{\alpha_1 Y}{T}$ and $Y_K = \frac{\alpha_2 Y}{K} = \alpha_2 N$ it is possible to write

(11) $$g_F = \frac{\alpha\alpha_1 F^{\alpha-1} Y}{\bar{p}(\alpha-1)T} - \frac{\alpha_2}{(\alpha-1)} N$$

Also, noting that

(12) $$\frac{NZ}{Q} = \frac{Y}{T} F^{\alpha-1}$$

We can write Equation (11) using Equation (12) as

(13) $$g_F = \frac{\alpha\alpha_1}{\bar{p}(\alpha-1)} \frac{NZ}{Q} - \frac{\alpha_2}{\alpha-1} N$$

Now using the definitions of M, N, N′, P, P′, Q and Z from Equation (17), we can get a system of simultaneous differential equations. Using Equations (1) and (4) and noting that $g_M = g_C - g_K$ we can get

(14) $$g_M = M + (\alpha_2 - 1) N + \bar{p}Q + \gamma N' - \rho$$

Similarly from $g_N = g_Y - g_K$, in view of Equations (1) and (7), g_N can be expressed as

(15) $$g_N = \frac{\alpha_1 Z}{1-\alpha_3} + \frac{1-\alpha_2-\alpha_3}{1-\alpha_3}(M + \bar{p}Q - \gamma N') + (\alpha_2 - 1)N$$

From $g_P = g_{R_1} - g_S$, using Equations (2) and (8), we can write g_P as

(16) $$g_P = \frac{\alpha_1 Z}{1-\alpha_3} - \frac{\alpha_2}{(1-\alpha_3)}(M + \bar{p}Q - \gamma N') + P + P'$$

Also from $g_Q = g_F - g_K$, using Equations (1) and (13) we can get

(17) $$g_Q = \left\{\frac{\alpha\alpha_1 Z}{\bar{p}(\alpha-1)Q} - \frac{\alpha_2}{\alpha-1} - 1\right\} N + M + \bar{p}Q - \gamma N'$$

From $g_Z = \alpha g_F - g_T$, using Equations (3) and (13), it is easy to see that

(18) $$g_Z = \left\{\frac{\alpha^2\alpha_1 Z}{\bar{p}(\alpha-1)Q} - \frac{\alpha\alpha_2}{(\alpha-1)}\right\} N - Z$$

Finally, using similar substitution, one can get $g_{N'}$ and $g_{P'}$ as

(19) $$g_{N'} = \left\{\frac{\gamma\alpha_2}{\gamma-1} - 1\right\} N + M + \bar{p}Q - \gamma N_1$$

and

(20) $$g_{P'} = \frac{\alpha_2 N}{(\gamma-1)} + P + P'$$

References

Bardhan, P.K., "Optimal Foreign Borrowing," in K. Shell (ed.), *Essays in the Theory of Optimal Economic Growth*, the MIT Press, 1967, 117-128.

Borts, G.H., "A Theory of Long-Run Capital Movements," *Journal of Political Economy*, 1964, 341-359.

Chiarella, C., "Optimal Depletion of a Nonrenewable Resource When Technological Progress is Endogenous," in M.C. Kemp and N.V. Long (eds.), *Exhaustible Resources, Optimality and Trade*, North Holland, 1980, 80-93.

Clifton, E.V., *International Capital Movements and Capital Accumulation in Less Developed Countries: Optimal Foreign Borrowing Policies*, unpublished Ph.D. Dissertation, Indiana University, Bloomington, Indiana, 1980.

Dasgupta, P., R. Eastwood and G. Heal, "Resource Management in a Trading Economy," *Quarterly Journal of Economics,* 1978, 297-306.

Hamada, K., "Economic Growth and Long-Term International Capital Movements," *Yale Economic Essays,* Spring 1, 1966, 49-96.

———, "Optimal Capital Accumulation by an Economy Facing an International Capital Market," *Journal of Political Economy,* 1966, 684-697.

Hanson, J.A. and P.A. Neher, "The Neoclassical Theorem Once Again: Closed and Open Economies," *American Economic Review,* 1967, 869-878.

Hochman, O. and A. Razin, "Debt and Capital Accumulation in a Two-Sector Model with Non-Traded Goods," *European Economic Review,* 1976, 165-180.

Kamien, M.I. and N.L. Schwarz, "Optimal Exhaustible Resource Depletion and Endogenous Technical Change," *Review of Economic Studies,* 1978, 179-196.

Kemp, M.C. and N.V. Long, "A Modest Defense of the Use of Cobb-Douglas Production Functions in the Analysis of Exhaustible Resources," in M.C. Kemp and N.V. Long (eds.), *Exhaustible Resources, Optimality and Trade,* North Holland, 1980, 247-250.

Koizumi T. and K.J. Kopecky, "Economic Growth, Capital Movements and the International Transfer of Technical Knowledge," *Journal of International Economics,* 1977, 45-65.

Moussavian, M., "Growth Rates with an Exhaustible Resource and Home Goods," *Journal of International Economics,* 1985, 281-299.

Onitsuka, Y., "International Capital Movements and the Patterns of Economic Growth," *American Economic Review,* 1974, 24-36.

Solow, R.M., "Intergenerational Equity and Exhaustible Resources," *Review of Economic Studies,* 1974, Symposium, 29-45.

Stiglitz, J.E., "Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths," *Review of Economic Studies,* 1974, Symposium, 123-137.

Suzuki, H., "On the Possibility of Steadily Growing Per Capita Consumption in an Economy with a Wasting and Non-Replenishable Resource," *Review of Economic Studies,* 1976, 527-535.

Takayama, A., "Optimal Technical Progress with Exhaustible Resources," in M.C. Kemp and N.V. Long (eds.), *Exhaustible Resources, Optimality and Trade,* North Holland, 1980, 95-110.