# Infrastructure, External Diseconomies and Turbulent Economic Growth

Richard H. Day\*

The impact of technological innovation upon the economic process consists of both an industrial rearrangement and ... often of a structural change in society.

Nicholas GEORGESCU-ROEGEN1

We must face the fact that the form of the equations of a realistic model may ... be ... a function of the variables involved.

Trygve HAAVELMO

The implications for economic growth of (1) different socio-technical infrastructures and (2) diseconomies of population numbers, includes the emergence of differing development regimes. The transition from one such regime to another with "larger" infrastructures can occur smoothly, but can also be preceded by "chaotic" fluctuations or, prior to a resumption of growth, by a collapse and reversion to a less "advanced" regime. These propositions are illustrated by a stylized growth model which can in an abstract way mimic many of the varied patterns of development in the archaeological and historical record.

I.

The Tinbergen-Solow-Swan growth model set forth nearly four

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<sup>&</sup>lt;sup>1</sup> I have not been able to find from which of his works I extracted this quotation, probably somewhere in the essays collected in Georgescu-Roegen (1966).

decades ago was motivated by a striking stylized fact: if you disregard relatively short-run fluctuations, the time series of measured aggregate output for the various industrial countries displays a remarkably stable rate of growth. The theory produces exponential trends and, when augmented by a constant rate of technological improvement, exhibits paths of output that fit the data extremely well.

Given a sufficiently rapid increase in productivity, such trends can continue — within the framework of the model — forever. The mind-boggling numbers of people that appear with the passage of time can enjoy an undiminished standard of living — even in the presence of an essential, exhaustible resource, as Solow (1974) subsequently demonstrated.<sup>2</sup>

Economic historians of the 19th Century had focussed on a quite different stylized fact about the "long run," namely, that economies of record had passed through epochs with characteristics of production, exchange or of social organization so different as to set off the dynamics of one epoch from that of another. For them economic growth in the long run had to be described by these stages of development.<sup>3</sup>

In the meantime, archaeologists have viewed human progress from a still longer perspective, that of the lifetime of our species as a whole, a lifetime that has grown from the biblical 5000 years or so to the modern conception (for homo sapiens sapiens) of some 50-100,000 years. (For us economists this is a very long time in the past to contemplate, although we routinely think about an infinitely long future.) What has economic growth been like in this very long run?

It has been anything but steady. Individual economies emerged; some grew to prominence, flourished for a time and then gradually, or more often, from an archaeological perspective, declined precipitously. Many disappeared altogether, leaving only traces in the buried remnants of their most durable products. Others transformed themselves into new, quite different forms and continued their expansion. Some appear to have fluctuated erratically, perhaps for many centuries before making the transition to a different form, while still others, after a period of growth and

<sup>2</sup> Tinbergen (1959), Solow (1956), Swan (1963).

<sup>3</sup> See the American Economic Association volume on economic history for a sample of classic statements. The most recent example is Rostow (1971). Gerschenkron (1968) found the weakness in this general approach: some economies experience different stages, pass through them in an atypical order; exhibit several at the same time or skip some altogether. The present theory overcomes these weaknesses while incorporating the germ of truth the stage-making hypotheses contain.

oscillation, seem to have collapsed, reverting to an earlier economic form; then, after perhaps experiencing alternating periods of fortune in that stage, have reentered a more advanced form and resumed an upward trajectory.

This picture is a much more complex one than the "stage-makers" had described. There indeed appear several stages of growth, but growth need not occur uniformly within a given stage. It may ebb and flow. The stages may be traversed in varying orders and with switching (or skipping) among them. Still there has been a discernable progression in all this, from relatively simple forms involving relatively small numbers of people in any one unit to successively more intricately structured organizations for production and exchange.

The literatures of archaeology and of economic development contain descriptions of various aspects of this grand process, and explanations involving innovation, diffusion, resource exhaustion, population growth and so on. <sup>4</sup> The possibility of a formal theory of such complex interactions would seem remote. Certainly, no simple mathematical system can be devised that both captures all the phases of economic evolution and explains individual human actions and how they add up to form the panoply of human development.

We should not be too distressed by this, however, for as Georgescu-Roegen (1966, p. 116) put it, "an economic model is not an accurate blueprint but an analytical simile." Indeed, it is possible, by augmenting classical economic theory with a few structural ingredients, to obtain a multiple-phase dynamic model that is not too complex to comprehend, yet mimics these stylized features of economic growth in the very long run. Proper historians would not put up with it for a moment. They would think of a thousand relevant factors that are overlooked and that are involved, perhaps uniquely, during any one time and place. But from the point of view of theory, the model helps explain why economic processes don't settle down to a steady state in the long run, but change themselves qualitatively in structure and in behavior as time passes. Moreover, the theory may shed some light on contemporary transformations that seem to be taking place on a vast scale.

<sup>4</sup> For a magnificant survey for the nonspecialist, see Barroughclough (1984). See also the references in Day and Walter (1989) for a start into this vast literature.

II.

If simplification is a sin, then (to paraphrase Luther) let us sin boldly. Consider time in the classical unit of a human generation, a quarter century, say. Each period is represented by a population of adults and their children who inherit the adult world in the next generation. Assume that each generation must provide its own capital goods which only last the period. The output possible is then a function of the number of adults. The number of children who survive (that is, who become adults the next period) depends on the per capita production of goods. Given the classical assumption of eventually diminishing returns and given bounded net birth equations that rise from zero after some minimal income threshold, we obtain the standard classical results: population, beginning at a small enough level, rises at an exponential rate during a phase of relative abundance, then slows and converges to a stationary state at which the level of well being is sufficient to motivate and sustain the formation of families just big enough to replace themselves generation after generation. It has been shown how this result changes to one involving periodic or nonperiodic (chaotic) population fluctuations merely by changes in the birth threshold. or by the imposition of absolute production diseconomies to population size 5

Now augment this picture in the following way. Suppose there are several regimes. The technology of each regime is represented by a production function which relates output to the workforce of a given generation, but which can only be effective if a part of the population forms a social infrastructure upon which the use of the given technology depends. Such an infrastructure mediates the human energy devoted to coordinating production and exchange, to providing social cohesion for effective cooperation, and for training and inculturating the workforce. Call the part of the population that makes up this infrastructure the *infrastructural force*.

Esther Boserup called the knowledge upon which the social infrastructure is based, the *administrative technology*. Given this as a prerequisite for any given regime, and given that the combination of production and administrative technology, the *techno-infrastructure* (say), requires a significant block of human resources, it follows that for any given regime to be feasible, the population within a given society must exceed a given bound.

When population within a given reigme passes some range, the in-

<sup>&</sup>lt;sup>5</sup> See Day, Kim and Macunovich (1989) who, it should be noted, derive the classical birth equation from household preferences in the standard neoclassical way.

frastructural force becomes progressively less capable of fulfilling its function. Productivity within the workplace declines, at first marginally, then absolutely and at some still higher level, precipitously. When population becomes too large for a given techno-infrastructure, it may find a way to divide or split to form additional more-or-less independent economic units each with a similar techno-infrastructure, or it can switch to a new regime, the possibility for which must reside in the knowledge accumulated during the past.

The splitting of societies was a common phenomenon in the hunting and food collecting stage. It can be augmented by shedding and fusion, a process in which members from various societies emigrate and fuse to form a new society or are absorbed in another one. In any case, the multiplication of societies with a given techno-infrastructural regime must come to an end when the global bounds are reached. Then an external diseconomy caused by excessive total population (over all societies) is reached.

The results are alternatively (i) the switch to a new regime that allows for renewed growth and permits a further expansion of population; (ii) a sudden decline in well being and population and a resumption of growth within the regime; (iii) a disintegration to a larger number of smaller societies whose infrastructure requirements are smaller.

Within any given regime growth can occur followed by fluctuations and/or the switch to a "higher" or "lower" regime, depending on all the parameters of the techno-infrastructure and demoeconomic behavior.

### m.

To formalize all this, measure the size of an economic unit (band, tribe, nation, civilization) by the number of families, x. Each household supplies one adult equivalent of effort to society, either as a part of the work force or as part of the infrastructural force; one adult equivalent of effort is utilized in household production, childrearing and leisure. If the size of a "production unit" is G, then = M + L where M is the number of adult equivalents in the infrastructural force and L the number in the work force.

Planning, coordination and control of economic activity becomes increasingly difficult as population grows within a unit. Let the maximum number compatible with an effective socioeconomic order be denoted by N. The term N-G=S represents the social "space" or "slack." If S is large the unit may increase in size without depressing productivity very much. When S is small, increase in size begins to lower productivity — at first

marginally, then absolutely. When  $S \le 0$  the group cannot function. This is the internal diseconomy of group size.

Suppose now that the productive activity within a group can be represented by a group production function continuous in the arguments L and S. Thus, Y = h(L, S). Substituting S = N - G and L = G - M we get

(1) 
$$Y = h(G-M, N-G) \equiv g(G)$$
.

In Figure 1a the standard power production function is illustrated. In Figure 1b the infrastructural force M has been added and in Figure 1c the slack term N-G to get a single-peaked production function in terms of labor.

Allowing for the splitting of units, the total population is organized into  $2^k$  groups of average size  $G = x/2^k$  in such a way as to achieve a maximum output (to avoid extreme complications). Thus,

(2) 
$$Y = f(x) := 2^k g(x/2^k) = \max_{n \in \mathbb{N}^+} \{2^n g(x/2^n)\}$$

gives the output Y of a population x that possesses a given technoinfrastructure.

The external diseconomy, that becomes increasingly important when the absorbing capacity of the environment is increasingly stressed, is expressed by a monotonically decreasing function

(3) 
$$p(x,\bar{x}) \begin{cases} = 1 & , x = 0 \\ \in (0, 1), \ 0 < x < \bar{x} \\ = 0 & , x \ge \bar{x}. \end{cases}$$

The social production function is then defined to be

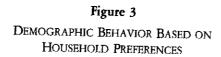
(4) 
$$F(x) \equiv f(x)p(x,\bar{x}).$$

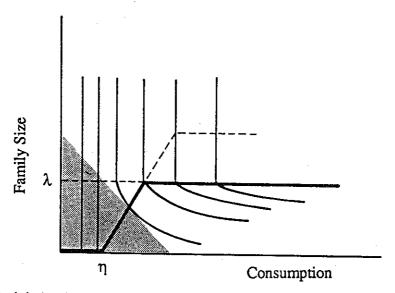
It is illustrated in Figure 2. Whether or not it is smooth as in Figure 2a, or kinked as in Figure 2b, or nonoverlapping as in Figure 2c depends on the size of M.

The family function b(y) that determines the average number of surviving children per family is assumed to depend on the average level of well being

$$(5) y = Y/x$$

Figure 1 Figure 2 THE PRODUCTION FUNCTION THE PRODUCTION FFUNCTION (a) (a) Overlapping, almost smooth **(b)** (b) Overlapping, kinked (c) (c) Non overlapping





The dark piecewise linear line is the demo economic line. The shaded triangula area is the "budget set" for a given income. The vertical lines that turn into curved lines are indifference curves.

and satisfies

(6) 
$$b(y) = 0, \ 0 \le y \le \eta$$
$$0 < b(y) \le \lambda, \ \eta < y$$

The parameter  $\eta$  is called the birth threshold. Note that the family function can be derived from the expansion path of a household preference function as shown in Figure 3.

Now begin with an initial population  $x_0$ . Putting this into (1) we get the number of groups and into (4) the total output. This takes account of both the internal and external diseconomies of population size. This gives average welfare y from (5) which using (6) yields the next generation of families  $x_1$  and so on. This process, when carried out generation after generation, yields the difference equation

(7) 
$$x_{t+1} = \theta(x_t) := \frac{1}{2} x_t b (F(x_t) / x_t).$$

### · IV.

Suppose there are several quite different techno-infrastructures available which we may denote by a set of indexes  $\tau$ : = (1,2,3, ...,j,...). The various components and parameters are then indexed accordingly so that a given system can be indicated by

$$S^{j}:=\{g_{j}(\cdot),\mathbf{M}^{j},\mathbf{N}^{j},x_{j}(\cdot),x^{j},\mathbf{b}_{j}(\cdot),\eta^{j},\lambda^{j}\},j\in\tau.$$

Suppose as before that society is organized so as to maximize output for any given population (again, to simplify the analysis). Then

(8) 
$$f_{i,k}(x) := \max_{j \in \tau} \max_{n} \{2^{n} g_{j}(x/2^{n})\}$$

and

(9) 
$$F_{i,k}(x) := f_{i,k}(x) \cdot p_i(x, x^i)$$

The index pairs I(x) = (i, k) gives the efficient techno-infrastructure i and the efficient number of economic units  $2^k$  for each population x.

Using the birth function (6) indexed to indicate the system to which it applies, we get a difference equation for each regime

(10) 
$$x_{t+1} = \theta_{i,k}(x_t) := \frac{1}{2} x_t b_i (F_{i,k}(x_t) / x_t)$$

Let  $X_{i,k}$  be the set of populations for which the efficient infrastructure and number of units is the pair (i, k). Then

$$I(x) = (i, k)$$
 for all  $x \in X_{i,k}$ .

In this way we arrive a difference equation

(11) 
$$x_{t+1} = \theta(x_t) := \theta_{I(x_t)}(x_t)$$

which is a mathematical expression for Haavelmo's observation quoted at the beginning of this paper.

Each phase structure  $\theta_{I(x)}(\cdot)$  characterizes growth in a given regime, which consists of a given number of economic units and a given technoinfrastructure, and determines the population in the succeeding generation when the current population x belongs to a phase zone  $X_{i,k}$ .

If  $I(x_{t+1}) = I(x_t)$  then there is no structural change, just growth, fluc-

tuation or decline as the case may be. If  $I(x_{t+1}) = I(x_t)$  then there is a phase change, that is a switch in regime.

If  $\theta^{t}(x)$  is the population of generation t when x is the initial condition, then the sequence

$$r_t = I(x_t) = I(\theta^t(x)), t = 0, 1, 2, \dots$$

gives the history in terms of the sequence of regimes.

V.

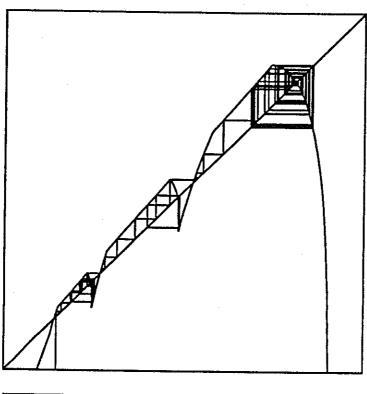
The study of possible trajectories of growth that pass through the various possible regimes is a tedious and somewhat intricate business. But the upshot is, I think, more-or-less obvious and intuitively reasonable. The underlying production function over all techno-infrastructures is a piecewise, single-peaked function, essentially like that already shown in Figure 1c. It leads to a dynamical system that can have a scalloped form like those shown in the top diagrams of Figures 4-7 that follow below.

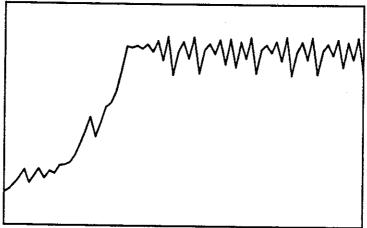
These figures show examples, computed by my student, Gang Zou, for a model with just 3 regimes. In each case we see several periods of more-orless exponential growth interspersed with irregular fluctuations of varying duration with switching and reswitching among neighboring regimes. Note that in Figures 4 and 5 the model differs only by a very small difference — indiscernible on the graph — in the initial condition. In Figure 6 such a large collapse in population occurs that a reverse in regime is followed by fluctuation at a much lower level. In Figure 7 the initial condition is slightly different than in Figure 6. In this case a collapse also occurs, but it is followed by a resumption on growth in the higher stage.

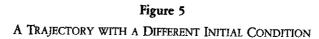
The general picture is one of growth interspersed with periods of fluctuation of a more-or-less irregular nature. The transition from one regime to another can be smooth rather than turbulent, but *evolution requires* each regime to be unstable. If any one regime is globally stable, then once it is entered, escape is impossible.

The present model has some serious limitations. For example, capital is treated as if each generation must provide its own stock which wears out at the end of the period. Exhaustible resources are not treated explicitly but implicitly in the externality term that limits the total size of any given regime. The infrastructure is represented only by a fixed infrastructural force for each regime. It should probably be broken into social overhead

Figure 4
A TRAJECTORY WITH MULTIPLE PHASE DYNAMICS







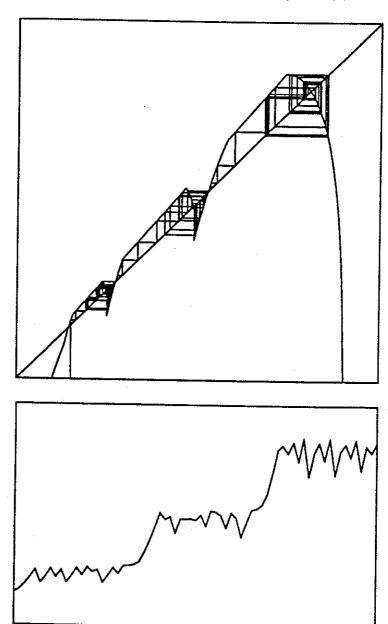


Figure 6
GROWTH, FLUCTUATION AND COLLAPSE

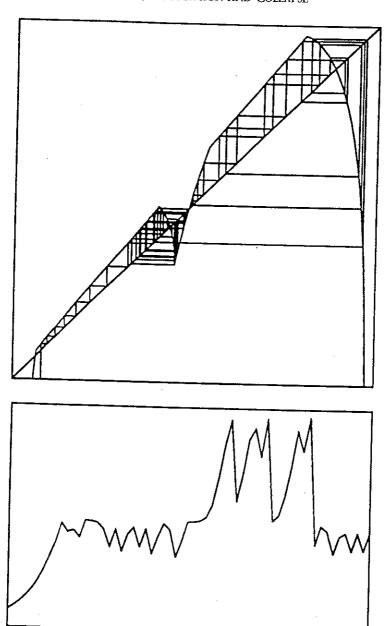
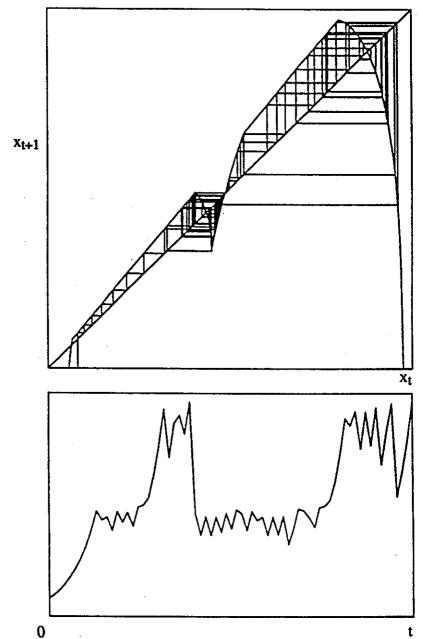


Figure 7

AN EPOCHAL PROGRESSION
(Growth interspersed with chaos and phase switching)



(fixed) and variable components. One would like to incorporate geographical heterogeneity and trade relations between economies. Technological change is treated only by the switch among regimes. It would be desirable also to allow for continuous technical advance within given regimes. Overcoming these deficiencies would complicate the analysis without, however, changing the basic results. Still, it would be of considerable interest to expand the analysis in all of these directions.

Equilibrium theorists might prefer to see the time path based on an infinite horizon objective function as is often done in optimal growth theories. Certainly, one could imagine the existence of optimal paths with optimal switching among regimes, and such an analysis might be of some interest. But the application of intertemporal equilibrium seems to me to be too farfetched as an explanation of economic history over long time periods. For explanatory purposes one doesn't need such an assumption so there is no need to invoke it.

## VI.

Suppose we have been presented by scientists with a qualitative history described by a sequence of epochs, each with certain techno-infrastructural characteristics and numbers of societies, and during each of which certain characteristic patterns of output and population are supposed to have occurred. These patterns would involve

- growth and switching to a higher or lower stage
- growth with fluctuations, then switching to a higher or lower stage
- convergence to an endless fluctuation or steady state within a "final" regime
- collapse and demise of the system.

Within a given reasonable class of functions for technology and demographic behavior, can trajectories for such qualitative histories exist? Are such qualitative histories sensitive to small changes in any of the parameters of the system or to the initial condition?

What is needd here is a new kind of *qualitative* econometrics: do there exist reasonable parameters, ones consistent with whatever shreds of evidence are available from which histories of the prescribed kind can be generated numerically? Such investigations are in their infancy, but the work done so far along the lies illustrated in the diagrams above is suggestive. In particular it shows that many of the vicissitudes of economic development, formerly attributed to random shocks and other more-or-less ad hoc exogenous variables, can also be explained by intrinsic, socio-

economic forces based on plausible hypotheses and represented by reasonably straight forward functional forms.

VII.

In the long run we are all dead.

John Maynard KEYNES

Economists are fond of citing Keynes' famous dictum about the long run. What must he have thought about the very long run? Surely, his sense of history was keen enough for him to have recognized that what once seemed a very long time in the future does come to pass, and to contemporaries of that eventual time, it is very much the present.

So it is that remnants of paleolithic peoples whose ways of life have persisted for thousands of years, are coming to a close now in our century. The various great empires of the ancient world that continued to flourish and grow for centuries, have long ago broken up and vanished; the British Empire on which in Keynes' time the sun did not set, has all but vanished; one of its last vestiges (Hong Kong) about to end in just a few years.

And now we experience, too, the disintegration of the Soviet Empire and the incipient integration of the European nations, the emergence of a world economy of multi-national firms, of internationally coordinated financial policies and of global information and cultural flow. A time, perhaps, of chaos and of collapse — and, perhaps, a time of transition to a new regime.

The implication of all this seems to me to be that effective policies of the here and now should be based not just on forces of the short run, but on those, too, of the very long run which have after all brought the world to its present state. It is a challenge for economic sicence to understand these forces so it may inform those who make policy. It must also be a challenge, for those who hope to shape history, to raise policy-making to this new level.

For development policy in particular, several implications standout. First and foremost is that *infrastructure matters;* both for making development possible and for allowing it to continue. Those who would see these countries grow will focus adequate attention on transportation, health, education, science, adequate water and clean air supplies and waste disposal. Moreover, they will attempt to design political systems and conduct public life so the willing participation of their fellows is engaged.

Second is that infrastructures must not merely be renewed, they must be developed and, from time to time, in new ways that can ameliorate new problems that emerge out of the process of successful growth. Humans do not eventually come to live happily ever after on blanaced growth paths.

Third is that excessive population relative to its techno-infrastructural base is a potential cause for instability and economic decline. Development could conceivably continue in a smoother fashion than in the past if population growth could be reduced while, at the same time, attention to science, education and innovation was reinforced.

Finally, expansion within a given infrastructure is almost surely limited. As the many regions of the world become ever more intimately linked through economic transactions, the ratio of infrastructural resources to population may become inadequate. New public agencies organized along international lines with reduced sovereignty for individual countries may be required to facilitate further growth and moderate economic instability.

One can scarcely claim prescience for these inferences. All seem quite plausible in the light of contemporary experience.

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