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Estimation of Engel Elasticities from Concentration Curves*

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In 1978 Kakwani computed Engel elasticity from an implicit Engel function based on a special type of concentration curve. Extending his idea, a new formula in computing Engel elasticity based on general type of concentration curves has been developed. A non-linear functional form of the concentration curves for total expenditure and expenditure on each commodity is used to derive the expenditure elasticities for various Australian household consumption items. The empirical results show that the proposed Engel function fits better than the other commonly used Engel functions in terms of *goodness of fit*. The *adding-up* criterion is also approximately satisfied by this proposed Engel function at all levels of per capita total expenditure. Additionally, a method of obtaining weighted elasticity for various family compositions is also presented with empirical illustrations.

I. Introduction

Specification of the functional form is an important issue in computing Engel elasticities from household expenditure data. The importance lies in the fact that the magnitude of elasticities depends largely on the functional form used. A huge body of literature has been devoted to the consideration of the appropriate specification of the functional form. In this regard, Haque (1984) reviewed the relevant literature and discussed the various issues on specification and discrimination of different functional forms.

Engel elasticities can also be estimated indirectly via concentration

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A. Lorenz Curve

The Lorenz curve is defined as the relationship between the cumulative proportion of income and the cumulative proportion of income receiving units when the units are arranged in ascending order of income. This curve is widely used to analyse the size distribution of income and wealth.

Mathematical definition of the Lorenz curve

Let X be the per capita total expenditure (income) of a family. If x is assumed to be a random variable with probability density function f(X), then the proportion of families having per capita total expenditure (income) less than or equal to x is given by

$$(1)\quad P(x) = \int_0^x f(X)dX$$

Further, if it is assumed that the mean μ of the distribution exists then the first moment distribution function is defined by

$$(2)\quad Q(x) = \frac{1}{\mu} \int_0^x X f(X)dX$$

This Q(x) is interpreted as the proportion of per capita total expenditure (income) made (received) by those families whose per capita total expenditure (income) is less than or equal to x.

The relationship between P(x) and Q(x) is known as the Lorenz curve. Inverting (1) and eliminating x from (2), one could obtain the Lorenz curve provided the function (1) is invertible, which is true for most commonly used income density functions f(X). On the other hand, the same curve is obtained by plotting Q(x) as the ordinate and P(x) as the abscissa for different arbitrary values of x. This curve is commonly represented in a unit square.

The first and second derivatives of Q with respect to P are respectively given by

$$(3)\quad \frac{dQ}{dP} = g'(p) = \frac{dQ/dx}{dp/dx} = \frac{x}{\mu} > 0$$

$$(4)\quad \frac{d^2Q}{dP^2} = g'''(P) = \frac{1}{\mu f(x)} > 0.$$

These two positive derivatives show that the Lorenz curve is an increasing convex function of P. This convexity along with the fact that Q(0, 0) = 0

The first and the second derivatives of Q(h(x)) with respect to P(x) are given by

$$(6) \quad \frac{dQ(h(x))}{dP(x)} = \frac{h(x)}{E(h(X))} > 0$$

$$(7) \quad \frac{d^2Q(h(x))}{dP^2(x)} = \frac{h'(x)}{E(h(X))} \cdot \frac{1}{f(x)}.$$

The sign of the second derivative depends on h′(x) as both E(h(X)) and f(x) are positive.

Therefore, if $h'(x)>0$ for all $x \geqslant 0$ then $Q(h(x))<P(x)$ and hence the concentration curve falls below the egalitarian line (i.e., the line passes through the points (0, 0) and (1, 1) in a unit square). On the other hand, if $h'(x)<0$ for all x, the curve is concave and will lie above the egalitarian line. If $h'(x)=0$ then the concentration curve and the egalitarian line coincide. When $h(x)=x$ we obtain the Lorenz curve.

III. Estimation of Engel Elasticities from Concentration Curves

Lew $W_i(X)$ be the Engel function of the *i*th commodity. Now if we substitute $W_i(X)$ for h(X) and Q_i for Q in (5), we have

$$Q_i(W_i(x)) = \frac{1}{E(W_i(X))} \int_0^x W_i(X)f(X)dX$$

which could be explained as the proportion of the per capita expenditure on the *i*th commodity made by those families having per capita total expenditure (income) than or equal to x. Hence, the relationship between P(x) and $Q_i(W_i(x))$ is known as the concentration curve of the *i*th commodity.

Now substituting $W_i(x)$ for h(x) in (6) and (7) we have

$$(8) \quad \frac{dQ_i}{dP} = g_i'(P) = \frac{W_i(x)}{E(W_i(x))}$$

$$(9) \quad \frac{d^2Q_i}{dP^2} = g_i''(P) = \frac{W_i'(x)}{E(W_i(x))} \cdot \frac{1}{f(x)}$$

where $E(W_i(x))$ is the mean expenditure of the *i*th commodity.

In section II we have already indicated the drawbacks in constructing the concentration curve from the available income density function. Kakwani and Podder (1973) constructed the concentration curve from its properties. In this study we have constructed the concentration curve directly from its properties so that it fits the data reasonably well.

The purpose of this section is to choose a functional form for the concentration curve. Choice of the concentration curve is very important, since this will be used to compute the Engel elasticities using the formula (10).

We have fitted a number of concentration curves and the following functional form of the concentration curve is chosen for our present analysis, since it fitted better than other concentration curves.

$$(12)\quad Q = (1-(1-P)^{\alpha})^{1/\beta};\ 0<\alpha,\ \beta<1.$$

This form of the concentration curve is suggested by Rasche, Gaffney, Koo and Obst (1980). This is popularly known as the generalized Pareto concentration curve. Because, if we put $\beta = 1$, the above curve becomes the equation of the concentration curve when the distribution of income follows Pareto's Law.[5]

Now differentiating (12) with respect to P twice we have

$$(13)\quad \frac{dQ}{dP} = (\alpha/\beta)\ (1-P)^{\alpha-1}\ (1-(1-P)^{\alpha})^{\left(\frac{1-\beta}{\beta}\right)}$$

$$(14)\quad \frac{d^2Q}{dP^2} = (\alpha/\beta)\ (1-P)^{\alpha-2}\ (1-(1-P)^{\alpha})^{\left(\frac{1-\beta}{\beta}\right)}$$

$$[\alpha(\frac{1-\beta}{\beta})\ (1-P)^{\alpha}\ (1-(1-P)^{\alpha})^{-1}-(\alpha-1)].$$

For all $\alpha<1$, the value of the first derivative would be zero and infinite if it is evaluated at zero and one respectively. On the whole, the function (12) satisfies all the general properties of a concentration curve. Hence, this curve is taken as the general concentration curve to compute Engel elasticities for various consumption items.

V. Empirical Illustrations

Australian HES 1975-76 data are used to illustrate the above method.

[5] $\alpha = 1-1/\delta$ where $\delta>0$, the scalar parameter in the Pareto distribution. The Lorenz curve for the Pareto distribution is defined iff $\delta>1$ which implies that $0<\alpha<1$.

penditure on each item. The estimated parameters are presented in Table 1, together with the residual sums of squares. It is noted that the residual sums of squares are generally low for all commodity items.

Engel elasticities at various per capita income levels are presented in Table 2. It is noted that the elasticities for almost all the commodities including total foods are reasonable. It should be mentioned that the elasticities for each food items were estimated and it was found that elasticities for almost all the food items remain the same for those families whose per capita weekly incomes are $250 or more.[8] This is a quite encouraging result since the level of saturation must be attended by all the food items.

Further, it is also noted that the expenditure elasticity for meals in restaurants and hotels decreases as income rises, although from a common sense point of view, expenditure elasticity for this item should increase with the rise of income. This might happen due to the fact that the good may become stable after income and the consumption of the commodity reach a certain level. The rate of increase in its consumption should diminish progressively as income rises and hence the elasticity of demand falls. It is widely known that the expenditure elasticities of demand for some staple foods will eventually become negative. Therefore, the expenditure elasticity of demand for a specific kind of food like meals in restaurants and hotels generally should fall rather than rise.

The weighted average of the elasticities of all the commodities is given in the last row, the weights being the budget shares to the expenditures on the items. The adding up criterion asserts that the total expenditure elasticities weighted by the budget shares for all the commodities should be equal to 1 at all income levels (see Cramer (1973, p. 147), Nicholson (1949)). From the last row of Table 2, it is seen that the adding up criterion is approximately satisfied at all per capita income levels. The maximum error is about 8% at the extreme minimum and maximum per capita income levels $30 and $750. The systematic decrease of the weighted average total expenditure elasticity is due to grouping according to per capita income rather than per capita total expenditure. Low income earners sometimes spend more than their income (by borrowing or some other means) and higher income people spend less than their income. Hence, the weighted average of the elasticities is over and underestimated for the low and high income groups respectively. Weighted average of the elasticities at the mean value is 1.0105 which is very close to 1.

Elasticity estimates for different commodities at mean total expendi-

[8] These results are presented in Table A1 in Appendix A1.

ture based on the proposed curve are compared with many commonly used Engel functions and are presented in Table 3. In their pioneering work, Prais and Houthakker (1955) investigated the linear, hyperbolic, semi-log, double-log and log-inverse functional forms. These functions were also used by Podder (1971) to describe household consumption expenditure pattern in Australia. The share semi-log Engel function was used by Working (1943) for the United States budget data. Later, this function was generalized by Leser (1963). In recent times, the Leser, Working functions became more popular. In fact, Bewley (1982) used the Working function to analyse the same survey data, using 180 observations cross classified by 12 average weekly household income and 15 family compositions. Giles and Hampton (1985) also used the same Working function for New Zealand Household Expenditure Survey data. Goreux (1960) selected the log log-inverse functional form for this larger FAO study, while Sinha (1966) used this form to estimate income elasticities for food in India. The double semi-log Engel function is used by Haque (1984) to analyse the Australian Household Expenditure Survey data.

The last row of Table 3 gives the weighted average of elasticities for all the functions. This shows that the adding up criterion is not satisfied for the hyperbolic, semi-log and log-inverse functional forms. However, this property is approximately satisfied by all other functional forms. In general, the proposed curve performs better than the other Engel functions except the linear, double semi-log, Working and Leser functional forms, where adding up criterion is supposed to be exactly satisfied.

The weighted residual sum of squares is used to compare the goodness of fit among the alternative functional forms, taking the proportion of the estimated population in each per capita income class as weights. It is clear from Table 4 that the proposed curve is superior to all other curves as judged by the minimum weighted residual sum of squares criterion. This curve is selected for seven out of ten items.

However, for clothing and footwear the log-inverse functional form gives a lower value of the weighted residual sum of squares than the proposed cuve. It is interesting to note that the double semi-log functional form performs well and occupies the position next to the proposed curve. Earlier Haque (1984) showed that this function performed better than the other usual functional forms on the basis of the minimum weighted residual sum of squares.

The choice of an appropriate functional form for an Engel curve on theoretical grounds depends on the relative weights attached to the various properties by the investigator. In this analysis, we base our choice of an appropriate functional form on the minimum weighted residual

Table 4

WEIGHTED RESIDUAL SUM OF SQUARES FOR DIFFERENT FUNCTIONAL FORMS

Commodity Groups	Engel Curves									
	Working Curve	Leser Curve	Linear	Semi-log	Hyper-bolic	Double-log	Log-Inverse	Log-log Inverse	Double Semilog	Proposed Curve
Total of all food	.36	.14	.15936	.36128	1.21909	.13838	.79372	.12618	.11968	.13356
All alcohol and tobacco	.16	.14	.13058	.18795	.56335	.16224	.23651	.13902	.10747	.09656
Current housing costs	2.90	2.06	2.69392	6.17366	10.7208	3.16981	8.31379	2.27769	1.90825	2.14052
Fuel and power	.02	.01	.00890	.01334	.02209	.01373	.01936	.00988	.00890	.00890
Clothing and footwear	.99	.47	.82649	.46048	.67704	.92386	.42736	.44811	.44814	.45514
Household equipment and operation	6.17	4.85	6.15969	9.51387	12.53506	6.62704	11.03941	5.74109	4.47597	4.30417
Medical care and health expenses	.03	.03	.03042	.02865	.05416	.02738	.03659	.02721	.02400	.02400
Transport and communication	2.03	.43	1.14965	.687282	3.39518	2.17402	.75765	.41066	.41294	.35238
All recreation and education	.70	.63	.60556	.90148	1.98184	.70918	1.02257	.60458	.58035	.55846
Miscellaneous goods and services	.82	.75	.71898	1.06392	2.20721	.87529	1.15117	.73228	.70384	.69056

case is 1.02 which is not significantly different from unity. Hence, any bias in the estimated total expenditure elasticity based on the per capita income classes may be considered as negligible.

The proposed method of estimation of Engel elasticity has not yet been developed to take into account the effects of other variables. However, it is important to note that if the data are classified according to total expenditure (income) for different family composition then the effect of family composition and economies of scale could be considered by computing the total expenditure (income) elasticities for different family compositions and then these elasticities could be combined to find the average total expenditure elasticity of demand for the whole population.[9] Some data of this nature are in fact available for 1974-75 HES and the relevant results have been presented in Table A2 in Appendix A2.

It should be pointed out that the regression estimates for grouped data are biased for the non-linear Engel functions because of the use of arithmetic means as proxies for geometric and harmonic means for logarithmic or reciprocal relationships. As a result, Engel elasticities computed from non-linear Engel functions are biased. The present method could be used to compute Engel elasticity from grouped data without such an added problem.

In this paper, we have extended Kakwani's method of estimating Engel elasticity from implicit Engel function based on non-linear concentration curves, using grouped data. The empirical results show that the proposed Engel curve fits better than the other commonly used Engel functions on the grounds of goodness of fit. The adding-up criterion is also approximately satisfied by this proposed Engel function at all levels of per capita total expenditure. Hence, our method of estimating Engel elasticities based on concentration curves may be considered as superior over the usual method of least squares in computing Engel elasticities.

[9] See Appendix A2 for more details about this technique.

Appendix A2

The Method of Obtaining Weighted Elasticities

In many household expenditure surveys the data are classified according to household income rather than per capita income. In such cases, our method fails to reflect the family size effect on consumer patterns. However, family size or family composition effects can be taken into account in the estimation of Engel elasticities from concentration curves, when the data are classified according to family income and size/composition. The method is given below.

Suppose that the total population is divided into several homogeneous groups. Let us see how total expenditure (income) elasticity for the entire population would be obtained from the total expenditure (income) elasticity for various groups. The problem is relatively simple since it amounts to finding the weighted elasticities. The procedure is discussed below.

Let us assume that we have N Engel functions with respect to all family total expenditure (income) i.e. $Y_1 = f_1(X)$, $Y_2 = f_2(X)$, ... $Y_N = f_N(X)$ with total expenditure (income) elasticities η_1, η_2, ..., η_N respectively, where Y_i's are the expenditures for different family compositions on a particular item. Then the elasticity of the sum of the functions, $f_1(X) + f_2(X) + \ldots.. + f_N(X)$ is given by

$$\eta = \frac{\eta_1 Y_1 + \eta_2 Y_2 + \ldots.. + \eta_N Y_N}{Y_1 + Y_2 \ldots.. + Y_N}$$

This is true by definition since we know that

$$\eta_1 = f_1'(X) \frac{X}{Y_1}$$

$$\eta_2 = f_2'(X) \frac{X}{Y_2}$$

$$\vdots$$

and $\eta_N = f_N'(X) \dfrac{X}{Y_N}$

and hence the elasticity of the sum of the functions $f_1(X)$ ..., $f_N(X)$ is

Table A2

TOTAL EXPENDITURE ELASTICITIES FOR VARIOUS HOUSEHOLD COMPOSITION AT MEAN FOR 1974-75 HES DATA†

	Household Composition								
Commodity Groups	Head only	2 adults no children	3 or more adults no children	Head with children	2 adults 1 child	2 adults 2 children	2 adults & 3 or more children	3 or more adults with children	Total of all house-holds
All foods	.4655	.4462	.6177	.7174	.6182	.6163	.4270	.6944	.5892
	(.4359)	(.4446)	(.5818)	(.7819)	(.6042)	(.5705)	(.3963)	(.6604)	(.5686)
Total alcohol & tobacco	1.6015	1.0224	1.1121	1.0170	.7074	1.4470	.5345	.9673	1.0203
	(1.6645)	(1.0508)	(1.0984)	(1.1603)	(1.0820)	(1.2773)	(.4462)	(1.0825)	(1.0708)
Current housing costs	1.1067	1.2948	1.0567	.5588	.6366	.7921	.8918	.5602	.8405
	(1.1235)	(1.3010)	(1.0236)	(.5157)	(.7485)	(.8924)	(.8042)	(.4919)	(.8433)
Fuel and power	.1237	.1444	.2631	.5375	.5929	.8803	.5354	.6192	.4996
	(.1020)	(.1428)	(.2378)	(.6306)	(.4668)	(.7897)	(4998)	(.6441)	(.4770)
Clothing and footwear	.8148	1.2309	1.0984	1.4133	1.5409	1.3491	1.1928	1.1675	1.2314
	(.8384)	(1.2007)	(1.0499)	(1.9656)	(1.5984)	(1.6771)	(1.3530)	(1.1645)	(1.3399)

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