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The Demographic Transition and Aggregate Savings in Less Developed Countries

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Savings functions estimated with cross section, country level data often find savings rates increasing with income. This paper argues that this result is partly due to the omission of demographic changes which occur as countries develop, rather than to an underlying convexity of the savings function. Using some concepts from demography, a theoretical connection is established between declines in child mortality and subsequential declines in birth rates on one hand and changes in aggregate saving on the other. The magnitude of this effect is then appraised by a variety of methods.

I. Introduction

There has been a great deal of empirical research on aggregate savings functions in less developed countries. A common characteristic of many of these studies is that savings rates are found to be increasing in total income, or, that the savings functions themselves are convex in income (Landau; Chenery and Syrquin), and the survey by Mikesell and Zinser. For aggregate data from say, a cross-section of countries, there may be a number of reasons for this finding. An increasing role of government saving or the shift from agriculture with nonfinancial savings instruments which has difficult to measure to industry where data is easier to obtain, may influence the recorded aggregate sav-

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U-shapes; zero income (or negative savings) for young people, high for people in their productive years and low or negative for the elderly.

The applicability of this life cycle theory of savings to LDC's has come under some criticism (Landau). However, it has been used with some success in certain cases such as Italy (Modigliani and Tarantelli) where it was shown to perform very well empirically and retains a strong intuitive appeal.

To find aggregate savings it is necessary to integrate over all age groups or:

$$S = \int_0^A y(a)s(a)f(a;\bar{y})da,$$

where

$$\bar{y} = \int_0^A y(a)f(a;\bar{y})da,$$

where A is the maximum age attainable — assumed to be fixed for the remainder of the analysis, and $f(a;\bar{y})$ is the proportion of the population of people of age a for any given level of national income $\bar{y}$. That A is assumed to be a constant is inessential. It can be considered an arbitrarily large number.

The term $f(a;\bar{y})$ will be assumed to be generated by what is known to mathematical demographers as a stable population. it is known that a population with fixed, age-specific, birth and death rates will converge to a population with a unique growth rate and age distribution (Lotka). Since the primary source of data on LDC savings functions has been cross-sectional where observations are taken to represent long run equilibria, reliance on the stable population age distribution will be the approach most consistent with the savings side of the story. In a time-series context use of the stable distribution implies that the population convergence occurs faster than changes in the effects of income on birth rates, i.e., that the target moves slower than the approach to it. The long run, stable population distribution can be shown to be of the following form (Coale):

$$f(a;\bar{y}) = \frac{e^{-r(\bar{y})a}p(a;\bar{y})}{\int_0^A e^{-r(\bar{y})a}p(a;\bar{y})da}.$$

cal regularity that these effects occur at income levels higher than those levels which reduce mortality rates. Alternatively, the reduction in death rates (of children particularly) may directly lead to fertility declines. If families calculate a target family size and wish to end the woman's child-bearing years with a certain number of children, it will be necessary to have more children than is ultimately desired since a relatively high proportion are likely to die. As mortality rates are reduced parents may operate on the assumption that they are still up high as in their own generation until new information is available. Thus, the learning of the true probabilities may take as long as a generation or two and results in the lagged decline.

In either case, the problem is to determine the effect on aggregate savings rates of an increase in income which also accompanies the demographic transition. The first part is the decline in child mortality, the second, a decline in fertility.

For the early stage, expression (1) is differentiable with respect to income, noting the effect on the population growth rate and the function p(a). Simplification leads to:

$$\frac{\partial Q}{\partial y} = \int_0^A q(a)f(a;\bar{y})\left\{-a\frac{dr}{dy} + \frac{1}{p(a)}\,\frac{dp(a)}{dy}\right\}da$$

$$-Q\int_0^A f(a;\bar{y})\left\{-a\frac{dr}{dy} + \frac{1}{p(a)}\,\frac{dp(a)}{dy}\right\}da.$$

To simplify this expression further, a commonly used (Coale) approximation to the equilibrium growth rate can be applied.

$$r \approx \frac{\log(G) + \log p(\bar{m})}{\bar{m} - \sigma^2\,\dfrac{\log(G)}{2\bar{m}}}$$

where

G : gross rate of reproduction

$\bar{m}$: mean child-bearing age

$\sigma^2 = \int_{m_i}^{m_2} (m-\bar{m})^2 c(a)da$: variance of child bearing age

$$\frac{dp(a)}{p(a)} = \frac{dp(b)}{p(b)} \quad \text{for all } a > b.$$

Hence

$$\frac{1}{p(a)} \cdot \frac{dp(a)}{d\bar{y}} = \frac{1}{p(m)} \cdot \frac{dp(m)}{dy} = \varepsilon.$$

This indicates that if mortality rates decline up to a maximum age b, that is, only infants and children are affected by the decline, the proportionate increase in the probability, evaluated at birth, of surviving to all ages older than b is the same. If we make the further assumption that children up to age b do not themselves save (as is reasonable since b should be about five years of age), then

$$\frac{\partial Q}{\partial y} = \int_0^A a(q)f(a)\ 1 - \frac{a}{T}\varepsilon\ da - Q\int_0^A f(a)\ 1 - a\frac{A}{T}\varepsilon\ da$$

$$= -\frac{\varepsilon}{T}\left\{\int_0^A a\,q(a)\,f(a)da - \int_0^A q(a)f(a)da \int_0^A a\ f(a)\ da\right\}$$

Since the function f(a;ȳ) has all the properties of a probability density function, this last term may be interpreted as

$$E(a\cdot s(a)) - E(a)\bar{E}(s(a)) = \text{cov}(a, s(a))$$

or

$$\frac{\partial Q}{\partial y} = -\frac{\varepsilon}{T}\,\text{cov}(a, s(a)).$$

Thus, the effect on the savings rate of the decrease in infant and child mortality due to the demographic transition depends on the covariance of age and the age-specific savings function. If the covariance is positive, the decrease in mortality will tend to decrease savings by increasing the proportion of the population in the younger, low saving age groups. If the covariance is negative the effect of the first part of the transition is to increase savings as rates. In cross-section, country level data, this will show up as convexity in aggregate savings.

III. Quantitative Impact of Age Distribution

Having established the theoretical possibility of the effect of the demographic transition on aggregate savings, the probable size of the effect can be appraised. For this it is necessary to have information on the stable age structure of countries passing through to demographic transition. Age specific savings functions are also required. Since this information is difficult to obtain, three methods will be used to approximate these effects. These three approximations are: (1) reconstruction of age structures by empirical birth and death rates, (2) use of published tables of stable populations and (3) estimation of aggregate savings functions with population structure taken into account.

A. Method 1

The first technique employs the construct of the Leslie matrix from the demography literature (Leslie). If the population is divided into a finite number of discrete age groups and the number of members of each group arrayed in a vector, the Leslie matrix summarizes the birth and death rates which generate the population vector in the next period. Formally:

$$\begin{bmatrix} k_1 \\ \vdots \\ k_i \\ \vdots \\ k_n \end{bmatrix}_t = \begin{bmatrix} b_1 & \cdots & b_i & \cdots & b_n \\ f_1 & & & & \\ & \ddots & f_i & & 0 \\ & & & \ddots & \\ 0 & & & f_{n-1} & f_n \end{bmatrix} \begin{bmatrix} k_1 \\ \vdots \\ k_i \\ \vdots \\ k_n \end{bmatrix}_{t-1} \quad \text{or } k_t = Mk_{t-1},$$

where b_i is the birth rate for age group i and f_i is the proportion of people in age group i who survive for the next period, i.e., it is one minus the death rate. The k_i's are the proportion of the population in age group i. The population growth rate and the stable age distribution can be derived by solving the determinant equation:

where m is the infant mortality rate. Since there are still too many free parameters, it is necessary to specify the value of f_3 beforehand. These values were varied within a wide empirically relevant range and the results were found to be quite insensitive to changes in these parameters. The age distribution and remaining variables result from the solution of the above equations using empirical birth, death and population growth rates. The typical age distribution to result from this procedure is:

	Low income	High income
k_1	0.643	0.547
k_2	0.268	0.311
k_3	0.089	0.142

From this point, the effect of the alternative age distributions on predicted savings rates can be calculated if age specific savings functions were known. Here, again, the data leave something to be desired. To overcome this deficiency, a variety of alternative savings functions can be used. With three assumptions the range of possible effects can be explored. These are: (1) that only people in the oldest two generations save, (2) the middle generation saves at least as much as the oldest taking both income and propensity to save into account (relaxing this assumption strengthens the predicted effect) and (3) the savings rate of the poorer country is normalized to its empirical level of about 13% (again using World Bank analyses). If the total savings rate of the economy is $\sum_i s_i^* k_i$, the results below show the savings rates when: (A) $s_2 = s_3$, (B) $s_2 = 3s_3$, and (C) $S_3 = 0$, i.e., when the middle generation performs all the savings.

Case	Low income	High income
A	0.13	0.165
B	0.13	0.156
C	0.13	0.151

The effect of age structure changes consistent with empirical population growth data can range from 2.1 to 3.5 percent between poor and rich countries. The empirical difference between these countries is about 8 percent (Chenery and Syrquin). So,

Table 1

Income \ Age	0-1	1-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
Low	0.039	0.129	0.137	0.118	0.103	0.088	0.075	0.064	0.054
High	0.024	0.090	0.103	0.095	0.088	0.081	0.074	0.068	0.062

Age	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80 +
Low	0.046	0.039	0.032	0.026	0.020	0.014	0.009	0.005	0.002
High	0.057	0.051	0.046	0.041	0.036	0.030	0.024	0.017	0.015

Entries are the proportion of the population in each age group.

where S/Y: savings rate
Y/N: per capita income (in 1970 dollars)
G : income growth rate for the previous decade
DP : dependency rate (proportion of the population younger than 15 and older than 65).

Standard errors are in parentheses. Data is from The World Bank.

When the dependency rate is ommited, the regression equation becomes

$$\ln\frac{S}{Y} = \underset{(0.4340)}{-3.4943} + \underset{(0.0768)}{0.2172}\ \ln\frac{Y}{N} + \underset{(0.1110)}{0.0195}\ \ln(G)$$

$$S.E. = 0.4808$$
$$R^2 = 0.0967.$$

These results conform quite well to the original study.

To illustrate the direct effect of income on demographic structure, the following regressions were run:

$$\ln(DP) = \underset{(3.6172)}{59.1027} - \underset{(0.5954)}{2.0778}\ \ln\frac{Y}{N}$$

$$S.E. = 3.741$$
$$R^2 = 0.1898$$

$$\ln(DP) = \underset{(19.471)}{-4.7018} + \underset{(6.6182)}{19.8548}\ \ln\frac{Y}{N} - \underset{(0.5550)}{1.8455}\left(\ln\frac{Y}{N}\right)^2$$

$$S.E. = 3.4247$$
$$R^2 = 0.3341$$

The second equation illustrates the entire demographic transition in the sample. Dependency (mostly due to large numbers of children) increases at low levels of income and decreases thereafter as outlined in the theoretical discussion. The turning point occurs at income levels of about $215 per capita in the lower end of the sample. The first equation, without the quadratic term, shows the direct effect of income on dependency as it is specified in the savings equations. The negative coefficient indicates that over most of the sample, the second phase of the demographic transition dominates.

by an upward shift in the age specific savings function itself. The resulting interaction would be in the direction predicted in the above results. A preliminary attempt at this problem appears in Hammer.

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