Leisure-Income Choice and the Development of a Dual Economy

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I. Introduction

Development of the closed dual economy has been an important and controversial focus of theoretical development economics since the publication of Lewis' seminal contribution.[1] Dual economy growth involves interaction between a traditional agricultural sector, employing backward production methods, and a modernizing industrial sector utilizing capital and more advanced production techniques. Reallocation of rural labor to the higher productivity industrial sector has been the essential mechanism of dual economy growth since Lewis.

Industrialization, led by the growth of manufacturing, absorbs rural labor into industrial employment. An agricultural surplus, in the form of subsistence goods, must be provided to maintain the industrial sector labor force. The pace of industrialization is

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1 The literature includes many contributions. Fei and Ranis elaborated Lewis' classicial model retaining the surplus agricultural labor assumption. Jorgenson's (1964) neoclassical model abandoned the surplus labor assumption and raised the issue of viability of dual economy growth. Zarembka (1970, 1972) considers viability from the viewpoint of non-zero price and non-unitary income elasticities of demand for food. Marino evaluates growth prospects by introducing a production function where only the usual neoclassical restrictions apply. Models by Sato and Niho, and McIntosh incorporate endogenous population growth. Stern addresses the question of optimal development of a dual economy.

balanced growth strategy stressing productivity improvements in agriculture would be more favorable to both overall growth and reallocation of labor.

Dual economy growth models have failed to realistically portray the farm sector by assuming labor hours are fixed. A more realistic approach, adopted here, permits variable work hours, with each farmer maximizing utility by choice between leisure and income. A number of authors, including Sen, Stiglitz, Zarembka (1972), Huang and Bhatia, have incorporated variable work hours in models of a developing country's agricultural sector and rural-urban migration. The next step is taken in this paper by incorporating variable labor hours in a neoclassical growth model of a dual economy. It is shown that if the price elasticity of demand for food is less than zero, manufacturing sector growth *may* have adverse effects on the size of the food surplus. In the empirically valid case of zero price elasticity, the effect of manufacturing sector growth is unambiguously negative.

II. A Model of Dual Economy Growth

In this section we formulate a dual economy growth model,[3] incorporating leisure-income choice in agriculture. The following sections are devoted to solution of the model and analysis of its results. A closed dual economy is assumed; food is assumed to be the only agricultural output.

Production:

We assume Cobb-Douglas production functions for both food and manufacturing output. At any point in time food output, Y, is determined by the rate of neutral technological progress in agriculture, α_1, the fixed amount of land, $\bar{L}$, and labor hours, ℓA, (ℓ is labor hours per farmer and A is the agricultural labor force):

$$(1) \quad Y = e^{\alpha_1 t} \bar{L}^{\beta_1} (\ell A)^{1-\beta_1}$$

[3] For convenience of the reader, all symbols used in the model are listed and defined in Appendix I.

hours. The first order condition for utility maximization is:

$$(4) \quad \frac{\partial Z_A}{\partial \ell} = 0 = \frac{\partial U(\frac{Y}{A}, q)}{\partial (\frac{Y}{A})} \quad \frac{\partial (\frac{Y}{A})}{\partial \ell} - V'(\ell)$$

or:

$$(4') \quad \frac{\partial Y}{\partial (\ell A)} = \frac{V'(\ell)}{U_1 (\frac{Y}{A}, q)}$$

Equation (4') implies that a farmer supplies labor hours up to the point where marginal porduct of labor hours is equal to the individual rate of indifferent substitution between income and labor hours, the latter being the real cost of labor.

We assume that industrial workers have utility function, Z_M, identical to that of farmers, so that:

$$(5) \quad Z_M = U(\frac{w}{q}, q) - V(\bar{\ell})$$

where w is the wage rate in the manufacturing sector in terms of manufacturing output.

The wage rate in the manufacturing sector, w, is determined competitively and is equal to the marginal product of labor. Using equation (2) the wage rate is represented as:

$$(6) \quad w = \frac{\partial X}{\partial M} = (1-\beta_2)\frac{X}{M}$$

Allocation of labor between agriculture and industry is given by the relative net utility that a worker derives by working in either sector. A farmer's opportunity cost of leaving for the city is the net utility he was deriving in agriculture (Z_A). A farmer will migrate for industrial employment as long as the net utility derived by working in the manufacturing sector (Z_M) is greater than the opportunity cost of leaving agriculture and *vice versa*. Intersectoral migration will be zero only if Z_M is equal to Z_A. Therefore, the labor allocation rule between the two sectors is:

$$(7) \quad Z_M = Z_A$$

the market for manufacturing goods.[4] Therefore, only the market equilibrium condition is specified. Since identical utility functions have been assumed for farmers and industrial workers, their demand functions will also be identical. Let us define demand for food by any worker as a function of his real income in terms of food and the relative price of food. Furthermore, the demand functions are assumed to be of the Cobb-Douglas form. Then, equality of demand and supply of food can be written as:

$$(12) \quad Y = A\left(\frac{Y}{A}\right)^{\theta} q^{-\sigma} + M\left(\frac{w}{q}\right)^{\theta} q^{-\sigma}$$

where θ is the income elasticity and σ is the price elasticity of demand for food. Question may be raised whether such a demand function follows from a utility function. We show in Appendix II that this is true under certain conditions, and these conditions apply rather well to the case of a typical developing country.

Population is assumed to grow at an exogenously given rate, ε. Assuming initial population to be equal to one, total population at any point in time is represented as:

$$(13) \quad P = e^{\varepsilon t}$$

The model is closed by stating the identity:

$$(14) \quad P = A + M$$

The model has ten endogenous variables, Y, X, ℓ, A, M, w, I, $\dot{K}$, K, and ten equations, (1′), (2′), (4′), (6), (8), (9), (1 (11), (12), and (14). Equation (13) determines total population dependently of the system. The model is summarized in Tab

III. Solution of The Model

The solution of the system involves the derivation of t

4 By Walras' law, if n-1 markets are in equilibrium, then the n^{th} market m equilibrium, which permits us to eliminate one market. In our case, we market for manufacturing goods.

which is constant since $\dot{M}/M$ is constant in the long-run. Using this expression at the cost of some distortions involved, but simplifying the solution greatly, we obtain:

$$(22)\qquad \frac{\dot{X}}{X} - \frac{\dot{M}}{M} = \frac{\alpha_2}{1-\beta_2}$$

From equation (14) we derive:

$$(23)\qquad \frac{\dot{A}}{A} = \varepsilon\left(1+\frac{A}{M}\right) - \frac{\dot{M}}{M}\,\frac{M}{A}$$

which shows the feasible values of the growth rates of agricultural and industrial labor forces for any given rate of population growth and distribution of population.

Using equations (22) and (23) in (19), we obtain equation (24). This and the remaining equations (25) through (28) comprising the model's solution are grouped together at the end of this section. Equation (24) is the model's fundamental differential equation showing the rate of growth of industrial employment, $\frac{\dot{M}}{M}$.

Equation (25) showing the rate of growth of the agricultural labor force, $\frac{\dot{A}}{A}$, is obtained by substituting equation (24) in (23). The rate of growth of per capita food output, $\frac{\dot{Y}}{Y} - \frac{\dot{A}}{A}$, is given by equation (26) which is derived by using equations (1′), (18), (22) and (25). Substituting equations (22) and (25) in (18) yields an expression for the growth rate of agricultural labor hours, $\frac{\dot{\ell}}{\ell}$, in equation (27). Finally, the rate of growth of the terms of trade, $\frac{\dot{q}}{q}$, is derived in equation (28) by using equations (17), (22), (26) and (27).

$$(24)\quad \frac{\dot{M}}{M} = \varepsilon + \frac{A}{M}\,\frac{(\alpha_1 - \varepsilon\beta_1)\left\{(1+n+hu)(1-\theta-\sigma) - \sigma u(1-m-h)\right\}}{\Delta_2}$$

$$+\ \frac{A}{M}\,\frac{(\sigma+k)\frac{\alpha_2}{1-\beta_2}\Delta_1}{\Delta_2}$$

agriculture, a growth strategy biased toward the manufacturing sector may be self-defeating.

We begin by noting that the signs of Δ_1 and Δ_2 are positive. Observe that a rise in the relative price of food, ceteris paribus, has a negative impact on labor hours because of the assumptions of diminishing marginal productivity of labor hours and diminishing marginal utility of income with respect to real income defined in terms of food as well as the relative price of food.[6] Also observe that, ceteris paribus, growth of manufacturing output increases the relative price of food (see equation (17)) which subsequently reduces labor hours in agriculture. This is shown by the term $h(\dot{X}/X-\dot{M}/M)/\Delta_1$ in equation (18) or by $\frac{h}{\Delta_1} \cdot \frac{\alpha_2}{1-\beta_2}$ in equation (27). Clearly these terms will be negative since $h<0$, provided that Δ_1 is positive.

To show that Δ_2 is positive, we note from the demand function that a rise in the relative price of food reduces food demand. Ignoring any effect on labor hours, reduced food demand would induce migration of agricultural labor to the industrial sector to maintain food market equilibrium. The term $\sigma \frac{\alpha_2}{1-\beta_2} \cdot \frac{\Delta_1}{\Delta_2}$ in equation (24) captures this effect. Since this term is positive, Δ_2 must be greater than zero.

The term $\frac{h}{\Delta_1} \frac{\alpha_2}{1-\beta_2}$ in equation (27) represents the negative effect on the farmer's labor hours from a rise in the relative price of food caused by manufacturing sector growth. Any change in labor hours induces change in both demand for and sup-

6 From equation (4′) $\frac{\partial Y}{\partial(\ell A)} = \frac{V'(\ell)}{U_1(\frac{Y}{A}, q)}$

Assuming no change in A, total differentiation of (4) gives:

$$\frac{\partial \ell}{\partial q} = \frac{V'' - U_1 A \frac{\partial^2 Y}{\partial(\ell A)^2} - (\frac{\partial Y}{\partial(\ell A)})^2 U_{11}}{U_{12}} < 0$$

since $V'' > 0, \partial^2 Y/\partial(\ell A)^2 < 0, U_{11} < 0, U_{12} < 0.$

dustrial employment, through the agricultural surplus, depends on the sign of $\sigma + k$.

Empirically, the price elasticity of demand for food is close to zero ($\sigma = 0$). Where this is true, term $\sigma + k$ turns out to be $h(1-\beta_1)(1-\theta)$ which is clearly negative since h is less than zero. Realistically, therefore, a development strategy biased toward industrial sector growth will have negative effects on industrial employment through the relative price-reduced labor hours mechanism. This means that if the agricultural sector cannot generate an adequate food surplus to accommodate labor transfer called for by industrial growth, the capacity will not be enhanced by promoting industrial expansion. In fact, prospects for increasing the agricultural surplus will worsen, causing reallocation of labor to slacken and perhaps even cease.

This analysis clearly points to the need for a balanced development strategy aimed at promoting growth of the agricultural surplus. If σ is close to zero, as we believe to be true, such a strategy will enhance growth of industrial employment. This is shown by the term $(\alpha_1 - \varepsilon\beta_1)\left\{(1 + n + hu)(1 - \theta - \sigma) - \sigma u(1 - m - h)\right\}$ in equation (24). With $\sigma \mp 0$, this term will be positive if $\alpha_1 - \varepsilon\beta_1$ is positive.[8] A balanced development strategy should be designed to affect technological conditions in agriculture; i.e., by raising α_1 or lowering β_1, or perhaps even attacking the population growth rate, ε. Successful policies of these types will raise the agricultural surplus, thus accelerating growth of the manufacturing sector.

Appendix I

Symbols Used in the Model

Y = agricultural output (food)
$\bar{L}$ = fixed amount of land

[8] Note that a positive value of $\alpha_1 - \varepsilon\beta_1$ is Jorgenson's (1967) condition for generation of an agricultural surplus and viability of the dual economy.

σ = price elasticity of demand for food

$P(=|A+M)$ = total labor force (equal to total population)

u = elasticity of Υ with respect to labor hours in agriculture

m = elasticity of marginal utility of income with respect to real income in agriculture

n = elasticity of marginal disutility of labor hours in agriculture

h = elasticity of marginal utility of income with respect to relative price of food.

Appendix II

Justification for Cobb-Douglas Demand Function

Kazuo Sato uses a utility function of the form $U = \Sigma a_i q_i^{c_i}$ from which he derives a demand function of the form: $q_i = (I/P_a)^{\theta_i} \, (P_i/P_m)^{\sigma_i'}$

where q_i represents demand for the i^{th} commodity, θ_i and $\sigma_i'$are the corresponding income and price elasticities of demand. P_a and P_m are the price indices defined below.

Let us assume two goods: food (Y) and manufacturing output (X). Then, the price indices are: $P_a = P_Y^{\psi_1} P_X^{\psi_2}$ and $P_m = P_Y^{\mu_1} P_X^{\mu_2}$, where P_Y and P_X are the prices of food and manufacturing sector's output, $\psi_1(\psi_2)$ and $\mu_1(\mu_2)$ are proportion of income spend on food (manufacturing output) and the marginal propensity to consume food (manufacturing output).

In the present paper, we have used demand functions which are identical to Sato's under some approximations. For example, a farmer's food demand has been expressed as: $D_Y = (Y/A)^{\theta_1} q^{-\sigma}$

where Y/A is the real income per farmer in terms of food, q =

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