Sequential Auctions in Informal Credit and Savings Societies:
Asian Auctions

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I. Introduction

It is quite common to find financial auctions in the informal financial market of South Asian countries, called "pia huey" (PH). The bidding procedure is similar to the English auction, that is, the highest sealed bid wins the item being auctioned off which, in our case, is the mutual funds of the PH members. Recently, the phenomenon of this type of auction has increased at a rapid rate. Among reasons given are the common regulation in interest-rate ceilings in the formal financial market and the high inflation rate; consequently, households save less in the formal financial institutions, preferring instead to participate in PH auctions which yield the market rate of interest; the individual household then can obtain the level of maximum satisfaction, while the aggregate welfare is better off with the expansion of sequential auctions in PH (Chotigeat, 1982, 1983).

The purpose of this paper is to develop a model of competitive bidding in a rotating credit and savings society in which each individual bidder's objective relies on the sequential auctions over a PH cycle. That is, the winning bidder's payoff may depend upon his personal preferences, the preferences of others, and the information of the past auctions in the PH cycle in which he is participating, the expected rate of return of the winning auction

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mutual funds is described as follows. Assume that there is a total of twelve persons (including the organizer) in this PH, each having a contribution share of $100 to lend out at each of twelve auctions to be held at one-month intervals (this interval is the most popular, although it could be one day, one week, two weeks, or longer).

The first auction results in each of the twelve members contributing the full price tag of $100 to a fund totalling $1,200, which is in turn given to the organizer as a reward for his organizing the PH; no bidding takes place during this round. The transaction, then, is equivalent to the organizer receiving an $1,100 interest-free loan from the remaining eleven PH members.

At the second auction, the eleven members (total members minus the organizer) each submit a sealed interest bid they are willing to pay, usually ranging between 7% and 40%. At each auction, all members who have already taken a loan are excluded from the bidding process; therefore, the respective positions of debtors and creditors are affected after each auction. The high interest bids come from members who have a greater need for the use of the money, while the low bids indicate the bidders' preferences to lend (save) to a bidder of a high rate. If there is a reluctance on the part of the bidders to bid high, the trend will be for a member to bid as quickly as possible at a rate close to (but not in excess of) the going bank rate of interest in order to take the loan and deposit it in the bank for the duration of the PH.

In this example at the second auction, if Member A wins the bidding with 40%, he will receive $100 from the organizer (each former bidding winner pays the full price tag of $100 at each auction for the rest of the cycle) and $60 = $(100-40) from each of the other eleven members, himself included, for a total of $760. In other words, the winner pays the interest rate of 40% to other members immediately at the point of borrowing (winning is like taking a loan).

If Member B is the winner of the bids at the third auction with 30%, he receives $100 from both the organizer and Member A, while he get $70 = $(100-30) from each of the remaining 10 members, himself included for a total of $900.

Subsequent auctions result in continually smaller numbers of bidders involved and ever-increasing loans, since the number of it, and pay it back gradually over the next 11 months. Without participating in PH he has to save $100 per month for twelve months before having $1,200 plus interest.

The second column in Appendix 1 indicates that after the second auction, a bidder wins with an interest bid of 40%. He would receive a loan of $760, i.e., $100 from the former winning bidder (the organizer), and $60 (100–40) from the other eleven members, himself included. Again if this bidder deposits his loan in a bank at an interest rate of 8%, he would earn $50.67 at the end of the PH cycle. However, after the second auction, the second winner has to pay a price tag of $100 at each auction until the end of the PH. If all of his paid installments had been put in the bank earning 8% interest, he would have earned $41.33 (see Column 2-S). Since he has opted to give up such a chance, the PH represents an opportunity loss for him. Therefore, he actually receives a net return of $50.669-400 = -$349.331, approximately -67.713% rate of return on installment savings of $1,160 by having participated in PH and taken a loan of $1,123 at the bidding rate of 40%. Although the $1,123 borrowed is used for benefits at 8% and paid back in installments, the bidding of 40% is too high, which is why he receives the negative rate of return. In reality this is not the case, which will be shown in the next section. The explanation of Columns 3 to 12 would be the same as that of Column 2. For example, as Column 11 indicates, the bidder borrows $1,120 for one month and pays the 40% bid interest rate; at the end of the PH cycle, he would earn net interest of 89.311% on the $1,120 loan, saving gradually at each auction for a total of $800. In sum, the last row of the table indicates all the net earned interest rates of each bidder at the end of the PH cycle. The bidder who wins the bid last (the saver) receives a high net rate of interest and vice versa for the borrowers. Therefore, there is a positive relationship between net rates of interest and the length of time of borrowing.

B. The Simple Bidding Decision

An individual bidder in PH is essentially a profit maximizer, he would bid at the interest rate, b, where his wealth over the entire period of PH is optimum. This is because over the PH cycle, the bidder's wealth is involved from the beginning to the end. In Table 1, in the first row of the formula on the left, it is shown that

$$[(T(j) + T(1-b_{ij})(n-1)](1+r_{ij}) > [(T(n-j) + T(1-b_1) + T(1-b_2) + \ldots + T(1-b_{ij})]$$

At a break-even point, then,

$$(1) \quad b_{ij} = \frac{r_{ij}(n+j-1)+(j-1)+(b_1+b_2+\ldots+b_{j-1})}{n(1+r_{ij})-r_{ij}+2}$$

$$b_{ij} = f(r_{ij}, n, j, b_1, b_2, \ldots, b_{j-1})$$

Consequently, the bidding rate b_{ij} depends on the actual rate of return on the loan, the number of bidders, the auction number j, and the bidding rates before the jth auction. Observing closely, the nominator of the fraction in the equation (1) can be large if r_{ij} and n are large and many auctions have been held, yielding many b's. Meanwhile, the denominator remains constant as j increases (coming closer to the end of the PH cycle). As a result, the bidding rate is larger due to the increase of the nominator when the jth auction approximates the end of the PH cycle. Near this point, the winner of the auction has little time to invest his winning mutual fund, but he has already earned interest by paying less than a full price tag (price tag minus the interest rate bid). In other words, the late bidders are more likely to be savers (than borrowers) who receive interest in adavance; therefore, the net return rate is rather high for this group.

However, since a bidder generally would like to receive at least a real positive return, $\bar{r}_{ij}$ must be greater than or equal to the inflation rate (p). This means, at least $\bar{r}_{ij} = r_{ij}M/(n-j-1) = p$, thus $r_{ij} = \frac{p(n-j-1)}{M}$ Substituting $p(n-j-1)/M$ in equation (1), the bidding rate b now turns out to be the minimum rate that guarantees a non-negative real rate of return to the bidder, one b for each auction. This means that the bidder can adjust for inflation in his bidding.

$$(2) \quad b_{ij} = f(\bar{r}_{ij}, n, j, b_1, b_2, \ldots, b_{j-1})$$

Therefore, the model developed has shown that an individual kind or all of the second kind as a special case.

The subscript i, where i = 1, 2, 3, 4, ..., n, is used as an index for individual bidders. The value of bidder i of a unit of the jth auctioned funds is denoted by $V_{ij} = (T(1) + T(1-b_i)(n-1))(1+r_{ij})$ (from Table 1). Assume that V_{ij} is drawn from the uniform probability distribution on the interval (0,v). Each bidder is assumed to know his own auction value before he submits his bid and to know only the probability distribution from which his rival's values were drawn.

The amount bid by bidder i at the jth auction is denoted by $P_{ij} = T(n-j) + T(1-b_1) + T(1-b_2) + ... + T(1-b_{ij})$ (from Table 1) where b_{ij} is the bidding interest which bidder i promises to pay until the end of PH cycle. If P_{ij} derived from b_{ij} is a winning bid then the ith bidder receives the money income profit or loss, $V_{ij} - P_{ij}$. Likewise, P_{ij} is not a winning bid then he has to pay his price tag T minus rate of the winner's bid, say b_{ij}, and wait to bid the next time in which the number of bidders is then less by the number(s) of winners. Let G(P) be the probability that a bid in the amount P = f(b) will be a winning bid. Then, using (1), the expected utility to bidder i of a bid in the amount b is

$$(4) \quad U_i(P_{ij}) = G(P_{ij})(V_{ij} - P_{ij})^{k_i},$$

The first-order condition for an interior maximization (2) is

$$0 = U_i'(\dot{P}_{ij}) = G'(\dot{P}_{ij})(V_{ij} - \dot{P}_{ij})^{k_i} - K_i G(\dot{P}_{ij})(V_{ij} - \dot{P}_{ij})^{k_{i-1}}$$

Now dividing through by $(\dot{V}_{ij} - \dot{P}_{ij})$, then

$$0 = G'c(\dot{P}_{ij}) - K_i G(\dot{P}_{ij} \;/\; (V_{ij} - \dot{P}_{ij})$$

Therefore, the V inverse of the bid function is

$$(5) \quad V_{ij} = h(P_{ij}, K_i) = P_{ij} + k_i G(P_{ij})/G'(P),$$
$$\text{where } P_{ij} = f(T, j, n, b_1, b_2, \ldots, b_{ij}).$$

To arrive at equation (5), one then must assume that each i maximizes expected utility and that all i have the same probability ex-

Substituting (9) in (7) (respectively (9) in (8)), then, in conjunction with (5), this implies that (5) is the inverse of an equilibrium bid function for $P \leqslant \overline{P}$ (respectively $P > \overline{P}$). For instance, if all bidders $i \neq j$ and bid according to (5), then the probability that a bid of P^*_{ij} by i will win is $G(P^*_{ij})$, given by (7) and (9) if $P \leqslant \overline{P}$ (respectively $P > \overline{P}$), If i now choose P^*_{ij} to maximize $G(P_{ij})(V_{ij}-P_{ij}),^{k_i}$, he will bid according to (5). Therefore, if all $i \neq 1$ bid according to (5) then i can make no bid better than following (5).

Through the mathematical manipulation of equations (7) and (9) (see Appendix 2) and substituting both of them into (5) yields

$$(5') \quad V_{ij} = P_{ij} + k_i E(K)^{-1} \overline{V}(I(F(P_{ij})))^{-E(k)^{-1}} \int_0^{F(P_{ij})} I(Y)^{E(K)^{-1}} dY,$$

where $I(\cdot)$ is the Kernel of the incomplete Beta integral. Furthermore, substituting the expression for $G(P_{ij})/G'(P_{ij})$ in (5) into the manipulated (7), one obtains

$$(7') \qquad F(P_{ij}) = \overline{V}^{-1}(P_{ij} + ((V_{ij} - P_{ij})/k_i\, E(k)),$$

Therefore, the equilibrium bid function is possibly derivable from equation (5′) and (7′).

In the seqential auctions in a PH cycle, since each auction has only one mutual fund to be bid and if one assume that all bidders are risk neutral (each $= k_i = 1$) and further assume that $E(K) = 1$, then the equilibrium bid function for the single unit discriminative auction (the highest sealed-bid winning the auction) and for the first auction in a PH cycle is

$$(8') \quad P_{i0} = ((n-1)/n)V_{i0}$$

Let $L = (n-1)/n$, since in PH there are n bidders, but the organizer is not bidding even though receives an award for the first auction. Hence, the auction immediately next to it is called the 1st auction and then has $L_1 = (n-2)/(n-1)$. The second auction will have $L_2 = (n-3)/(n-2)$. Hence, L of the jth auction is $L_j = (n-j-1)/(n-j)$. Note that the value of lj is decreasing as j in-

$\ldots b_{j-1}$), in the same PH cycle, before the jth auction in which he is going to bid. The information of the last two points, (c) and (d), is available to every bidder. Since the PH auction is a type of first-price auction, a bidder whose information is also available to some others must have zero expected profit at equilibrium. The bidder who has access only to some information of others (the more poorly informed), but makes an independent estimate, may have a positive expected profit.

Therefore, the bidding rate of individual i at the jth auction under 3 different models (or circumstances) which we have just described above can be summarized in a general form as in Table II.

Table II

Bidding Rate at Each Auction:
b_{ij} = Individual i would bid at jth auction

	Risk Neutral	Semi-Risk Neutral	Risk Averse
Auction #	Under no risk of return for investment and not include the probability to win the bid	Under no risk of return for investment but include the probability to win the bid	Under risk of return for investment and include the probability to win the bid
1	$b_{i1} = \frac{r_{i1}(n)}{n(1+r_{i1}) - r_{i1} + 2}$	$b_{i1} = \frac{L_1(r_{i1}+1)(n) - n}{L_1(r_{i1}+1)(n-1) - 1}$	$b_{i1} = \frac{A_{i1}(r_{i1}+1)(n) - n}{A_{i1}(r_{i1}+1)(n-1) - 1}$
		where $L_1 = \frac{n-3}{n-2}$	where $A_{i1} = \frac{n-2}{n-2+k_i}$
2	$b_{i2} = \frac{r_{i2}(n+1) + b_1}{n(1+r_{i2}) - r_{i2} + 2}$	$b_{i2} = \frac{L_2(r_{i2}+1)(n+1) - n + b_1}{L_2(r_{i2}+1)(n-1) - 1}$ where $L_2 = \frac{n-3}{n-2}$	$b_{i2} = \frac{A_{i2}(r_{i2}+1)(n+1) - n + b_1}{A_{i2}(r_{i2}+1)(n-1) - 1}$ where $A_{i2} = \frac{n-3}{n-3+k_i}$
⋮	⋮	⋮	⋮
j	$b_{ij} = \frac{r_{ij}(n+j-1) + (j-1) + \sum_{j=1}^{j-1} b_j}{n(1+r_{ij}) - r_{ij} + 2}$	$b_{ij} = \frac{L_j(r_{ij}+1)(n+j-1) - n + \sum_{j=1}^{j-1} b_j}{L_j(r_{ij}+1)(n-1) - 1}$ where $L_j = \frac{n-j-1}{n-j}$	$b_{ij} = \frac{A_{ij}(r_{ij}+1)(n+j-1) - n + \sum_{j=1}^{j-1} b_j}{A_{ij}(r_{ij}+1)(n-1) - 1}$ where $A_{ij} = \frac{n-j-1}{n-j-1+k_i}$; k_i = risk aversion of bidder i

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APPENDIX 1 (Continued)

PH: Deduction of Interest at the Moment of Bidding
All win at 40%

Winner # / Auction #	7		8		9		10		11		12		# months earn int.
	L	S	L	S	L	S	L	S	L	S	L	S	
1		100		100		100		100		100		100	11
2		60		60		60		60		60		60	10
3		60		60		60		60		60		60	9
4		60		60		60		60		60		60	8
5		60		60		60		60		60		60	7
6		60		60		60		60		60		60	6
7	960	60		60		60		60		60		60	5
8		100	1000	60		60		60		60		60	4
9		100		100	1040	60		60		60		60	3
10		100		100		100	1080	60		60		60	2
11		100		100		100		100	1120	60		60	1
12		100		100		100		100		100	1160	60	0
Total	960	960	1000	920	1040	880	1080	840	1120	800	1160	760	
Capital gain (L–S)	0		80		160		240		320		400		
If invested at 8%	32	32	26.67	30.933	20.80	30.133	14.396	29.6	7.47	29.333	0	29.333	
% of benefit at the end of the PH	8		27.587		48		68.756		89.311		109.092		

By substituting (f) into the equilibrium inverse bid function (equation(5) in the text). We obtain

$$\text{(g)}\quad V_{ij} = P_{ij} + k_i \frac{I(F(P_{ij}))}{I'(F(P_{ij}))F'(P_{ij})}$$

Differentiate(h) and substituting the resulting expression for $I'(F)F'$ into (g) we get

$$V_{ij} = P_{ij} + k_i\, \theta \overline{V}(I(F(P_{ij}))^{-\theta} \int_0^{F(P_{ij})} I(Y)^{\theta}\, dY,$$

where $\theta = 1/E(k)$.