# The Tradeoff Between Economies of Scale and Reliability in The Electric Power Industry

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# I. Introduction

There is a trade-off between economies of scale and reliablity standards in the electric power industry. Because of economies of scale in investment cost it is desirable to build a single large plant instead of several small ones. Because of reliability<sup>1</sup> it is desirable to build several small plants instead of one large plant. Garza, Manne and Valencia, Gately and Rowse have included economies of scale in capacity expansion planning in the electric power industry by using integer variables, but reserve capacities were estimated in a deterministic way in their studies. Consideration of plant size along with uncertainties in system reliability has been articulated and explored by some researchers.<sup>2</sup> However, this has usually been done by using simulation models. Also, some of these studies are operations rather than investment models. Only a few efforts have been made to incorporate economies of scale and uncertainty into an optimization framework. Scherer and Joe have

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<sup>2</sup> Arnoff and Chambers, Baldwin, Desalvo and Limmer, Billinton, Booth, Kirchmayer, Mellor, O'Mara and Stevenson, and Vassell and Tibberts.

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<sup>&</sup>lt;sup>1</sup> An excellent survey of this basic concept was published in 1960 as an AIEE Committee Report.

mathematical programming packages. Chebyshev's Inequality shows that for any h>0,

$$(2.2) \quad \text{Prob. } (\mid X - E(X) \mid > h\sigma_X) \leq 1/h^2$$

where

X = a random variable, E(X) =the expected value of X, and  $\sigma_X =$ the standard deviation of X.

We can get the following reliability constraint function from equations (2.1) and (2.2).<sup>7</sup>

$$(2.3) \ \overline{\mathbf{Y}}_t - \overline{\mathbf{L}}_t \geq \frac{1}{\sqrt{\mathbb{R}}} \ (\sigma_{\mathbf{Y}_t}^2 \ + \ \sigma_{\mathbf{L}_t}^2)^{1/2}$$

where  $\overline{Y}$  and  $\overline{L}$  denote mean values of the random variables Y and L and  $\sigma^2$  denotes the variance of these random variables.

# B. Formulation of the Model

The symbols used in the model can be categorized into sets and indicies, variables, and parameters, as follows:

Sets and Indicies

ieI: plant type j, keJ: region

leL: block on the load duration curve t, τεT: time interval and time period

meM: transmission line peP: mode of operation

<sup>&</sup>lt;sup>7</sup> See Kang 98-101 for this derivation.

# 1. Objective Function

The total cost to be minimized is the discounted sum of capital costs (both plant and transmission line), fixed operating costs and variable operating costs (fuel cost).

Objective Function

$$(2.4) \quad \xi = \sum_{t \in \mathbf{T}} (\delta_{kt} \phi_{kt} + \delta_{\lambda t} \phi_{\lambda t} + \delta_{\psi t} \phi_{\psi t} + \delta_{\pi t} \phi_{\pi t}) \quad t \in \mathbf{T}$$

This can be translated to

total cost = capital cost for plant + capital cost for transmission + fixed operating cost + variable operating cost

where  $\delta_{nt}$  = the discount factor which is appropriate to the *n*th cost term, (see Kang 135-138.); and

(2.5) 
$$\phi_{kt} = \sum_{i \in J} \sum_{i \in I} \sum_{\tau=J}^{t} \eta_i \beta_{ki} c_i y_{ji(\tau - w_i)}$$

(2.6) 
$$\phi_{\lambda t} = \sum_{m \in M} \sum_{\tau=1}^{t} \eta_{i} e_{m} \beta_{\lambda} b \alpha_{m} h_{m(\tau-w_{m})}$$

$$(2.7) \quad \phi_{\psi t} = \sum_{j \in J} \sum_{i \in I} v_i k_{ji} + \sum_{j \in J} \sum_{i \in I} \sum_{\tau=1}^{t} \left[ \beta_{\psi i} v_i \right]$$

$$(g_i y_{ji(\tau - w_i)} + k_{ji\tau}^p) - v_i s_{ji\tau}$$

(2.8) 
$$\phi_{\pi t} = \sum_{j \in J} \sum_{i \in I} \sum_{p \in P} \beta_{\pi i} f_i \theta_p u_{jipt}$$

(2.9) 
$$\eta_i = \frac{\sigma}{1 + (1 + \widetilde{p})^{-2}i}$$

### where

k: total initial capacity,

kmean: mean initial capacity,

 $\overline{d}_t$ : mean value of peak demand at time t,  $\sigma_{dt}^2$ : variance of peak demand at time t

 $\sigma_h^2$ : variance of initial capacity,

kp: plant capacity already under construction at time zero

R: maximum allowable risk

## b. Demand Constraint

The demand for electricity in a region at time t must be satisfied, either by the current regional production or the power transmitted (less transmission loss) from the other regions.

$$(2.11) \sum_{p=1}^{Q} \sum_{i \in I} u_{jipt} + (1-r) \sum_{k \in J, k=j} x_{kj} Q_{t} - \sum_{k \in J, k \neq j} x_{jk} Q_{t} = \sum_{k \in J, k \neq j} x_{jk} Q_{t}$$

$$(2.11) \sum_{p=1}^{Q} \sum_{i \in I} u_{jipt} + (1-r) \sum_{k \in J, k=j} x_{kj} Q_{t} - \sum_{k \in J, k \neq j} x_{kj} Q_{t}$$

$$(2.11) \sum_{p=1}^{Q} \sum_{i \in I} u_{jipt} + (1-r) \sum_{k \in J, k=j} x_{kj} Q_{t} - \sum_{k \in J, k \neq j} x_{kj} Q_{t}$$

# which translates to

power generated within region+power received from others - power sent to others ≥ projected demand level.

# c. Capacity Constraint

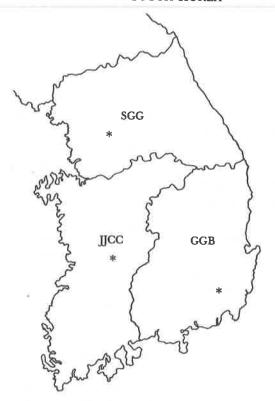
The power capacity constraint requires that production levels not exceed the power capacity available at each plant in each time period. There is no explicit energy capacity constraint. Since all plants but hydroelectric plants could be operated virtually continuously if enough fuel were provided, the power capacity constraint provides an effective energy capacity constraint for coal or nuclear power plants. The capacity constraint is

<sup>11</sup> Garza, Manne and Valencia use energy constraints in their study for Mexico.

through the use of a three year load duration curve which is composed of five rectangles. The shape of the load duration curve is unchanged over the planning period and the same load duration curve is used in the different regions. Hydroelectric production is exogenous to the model.

 Four plant types are chosen as candidates for addition to the initial system. These are a 600MW nuclear plant, a 900MW nuclear plant, a 500MW coal plant, and a 900MW coal plant.

Figure 1
REGIONS OF SOUTH KOREA



SGG = Seoul-Gyeonggi-Gangwon.

GGB = Gyeongbuk-Gyeongnam-Busan.

JJCC = Jeonbuk-Jeonnam-Chungbuk-Chungnam.

\* = weighted center point.

Table 2

COMPARISON OF TWO CASES

(Million Dollars)

Risk Index	Objective Value	Plant Type
0.04	13,749.9	All large plants
0.025	14,257.8	Large and small plants

# B. Results

In order to analyze the trade-off between reliability and economies of scale, it is useful to vary the reliability criteria. Thus the model was solved twice with a maximum allowable risk (R) of 0.04 and 0.025. Though it would have been desirable to solve the model many times and trace out the trade-off curve, it was not possible to do so because of the expense of solving the nonlinear mixed integer programming problems.

Table 2 lists the objective value for the two cases. As expected, the cost is higher when the maximum allowable risk is decreased. Also, since the smaller maximum allowable risk indicies require larger reserve requirements, it is reasonable to expect that small plants will be favored<sup>13</sup> in moving from 0.04 to 0.025. The optimal investment decision with a risk index of 0.025 indicates that small plants will be installed to reduce the value of the variance for capacity, thereby minimizing the capital cost by reducing the reserve requirement. The objective value in the 0.025 case is higher than in the 0.04 case, due to the diseconomies of scale in building small plants, in addition to the increased capacity itself. The increased cost of 508 million dollars due to employing a higher standard of reliability, can be traded off against the increased reliability.

<sup>12</sup> For the input data which are used in this study, see Kang, Appendix C, 181-189.

<sup>18</sup> Given a required capacity, the more plants we have, the less reserve capacity we need because of the smaller variance for capacity.

only case is less than in the unconstrained case due to the construction of small plants, which give more flexibility in satisfying the reliability and capacity constraints. The objective value in the large-plants-only case is also higher than in the unconstrained case because of the necessity to increase capacity due to the large variance in capacity. The capital cost in the large-plants-only case is lower in the unconstrained case due to economies of scale. However, fixed operating cost in the large-plants-only case is much higher than in the unconstrained case. The objective value in the small-plants-only case is higher than the objectie value in the large-plants-only case. From these comparisons, one can say that there exists a trade-off between the reliability standard and economies of scale. If one wants to increase the standard of reliability, it is necessary to sacrifice some benefits from economies of scale in capital costs.

## IV. Conclusion

The trade-off between reliability and economies of scale in the electric power industry has been examined. An empirically important trade-off between the standard of reliability and the economies of scale in capital costs was found. Reliability constraints play a significant role in investment decisions in the case examined. In particular, different indicies of the maximum allowable risk lead to different choices of plant size and different combination of plant sizes are adopted as an optimum solution according to the trade-off between reliability and economies of scale. The investment decisions are sensitive to the treatment of the reliability criteria in the model. Therefore, more attention should be given to the appropriate treatment of reliability constraints in electric power investment planning models.

### References

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