Some Theoretical Issues of Indirect Estimation of Capital Stock Series in Korea

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1. Introduction

Most of econometric models constructed and estimated for underdeveloped economies give emphasis on aggregate supply capacity of the economies and, consequently, resort very extensively to the uses of production functions. The rationale of laying emphasis on the aggregate supply of an economy is contrasted to the fact that an econometric model of a developed economy rarely explains the production functions.[1] With a usual attempt of avoiding overspecification of an econometric model, the econometric models of developed economies frequently employ the approach which explains aggregate demand with number of structural demand functions and equilibrium identities.

In estimating a production function a number of input variables have to be considered as explanatory variables. One may consider input of labour, input of capital, input of technology of various types, input of land, input of climatic factors, etc. It is generally agreed that the two most important input variables are labour and capital stock when disembodied technological progress is hypothesized. In almost every case of constructing an econometric model for an underdeveloped economy, one encounters the problem of estimating capital stock variables to be used as explanatory variable in a production function, when there are no such statistical data available. This paper aims at providing some alternate indirect methods of estimating capital stock variables and clarify resulted theoretical issues to be considered along with the estimation, with examples of the Korean economy.

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1 See Lawrence R. Klein (1965).

2. Indirect Estimations

Suppose we have a simple production function with two inputs, L labour and K capital as

$$Y = f\ (L, K, u) \qquad (1)$$

Where Y denotes the amount produced by L and K inputs, and u stochastic disturbance term. When Equation (1) is estimated based on cross-section data, the problem of estimating the capital variable K is avoidable since an ordinary census data contain industrial or sectoral capital stock statistics. However, when time-series data are used in estimation of the function in a simultaneous equation system, the variable has to be estimated or, at least, the role of the variable in the function has to be captured somehow.

2.1. Linear Transformation Method

Let us define the capital stock variable at the beginning of the period t, K_t, as

$$K_t = K_{t-1} + I_{t-1} - D_{t-1} \qquad (2)$$

where I denote gross investment in the form of fixed capital formation and D capital depreciation. Let us further define depreciation in terms of investment and constant depreciation rate as

$$D_{t-1} = \frac{\lambda}{2} I_{t-1} + \lambda(1 - \frac{\lambda}{2}) \sum_{\tau=0}^{t-2} (1-\lambda)^{\tau} I_{t-\tau-2} \qquad (3)$$

where λ denotes constant depreciation rate. Investment I_i depreciates in the current year i only ½ of the rate since I_i is a flow and the average stock level of the year i is obtained as one half of I_i. This is proved in following manner. Suppose investment is a flow at rate of ρ at every point of continuous time. The investment made at any interval of time t will be ρt. The total capital stock accumulated, say during a period T, will be likewise ρT. The average accumulation over the time period T is obtained as

$$\overline{\triangle K} = \left(\int_{0}^{T} \rho t dt\right)/T = \frac{\rho T}{2}, \qquad (4)$$

which is one half of ρT, the total capital stock accumulated during the period T.

Substituting Equation (3) into (1) we obtain a first-order nonhomogenous difference equation with respect to K as Equation (5)

$$K_t=K_{t-1}+I_{t-1}-\frac{\lambda}{2}I_{t-1}-\lambda\left(1-\frac{\lambda}{2}\right)\sum_{\tau=0}^{t-2}\left(1-\lambda\right)^{\tau}I_{t-\tau-2} \tag{5}$$

Solving Equation (5), we obtain Equation (6) as

$$K_t=K_0+\frac{2-\lambda}{2}\sum_{\tau=1}^{t}(1-\lambda)^{\tau-1}I_{t-\tau} \tag{6}$$

Equation (6) tells us that the current capital stock K_t is the result of the past accumulation of investment after proper discount of depreciation, depreciation rate being $\frac{\lambda}{2}$ in the first year, $\lambda\left(1-\frac{\lambda}{2}\right)$ in the second year, $\lambda\left(1-\frac{\lambda}{2}\right)(1-\lambda)$ the third year, and, in general, $\lambda\left(1-\frac{\lambda}{2}\right)(1-\lambda)^{n-2}$ in the n^{th} year.

Let us now go back to Equation (1), and consider an explicit linear production function[2] as

$$Y_t=\beta_0+\beta_1L_t+\beta_2K_t+u_t \tag{7}$$

Substituting Equation (6) into Equation (7), we obtain a typical distributed lag equation as

$$Y_t=\beta_0+\beta_1L_t+\beta_2\left[K_0+\frac{2-\lambda}{2}\sum_{\tau=1}^{t}(1-\lambda)^{\tau-1}\ I_{t-2}\right]+u_t \tag{8}$$

Applying a Koyck-type transforamtion to Equation (8), we derive a simplified linear production function as

$$Y_t=\gamma_0+\gamma_1L_t+\gamma_2L_{t-1}+\gamma_3I_{t-1}+\gamma_4Y_{t-1}+v_t \tag{9}$$

where

$$\gamma_0=\lambda(\beta_0+\beta_2K_0) \tag{10-1}$$

$$\gamma_1=\beta_1 \tag{10-2}$$

$$\gamma_2=-(1-\lambda)\beta_1 \tag{10-3}$$

$$\gamma_3=\left(\frac{2-\lambda}{2}\right)\beta_2 \tag{10-4}$$

$$\gamma_4=-(1-\lambda) \tag{10-5}$$

$$v_t=u_t-(1-\lambda)u_{t-1} \tag{10-6}$$

By using Equation (9), capital stock variable may be avoided but

2 A linear production function does not seem to be suitable for purposes of theoretical reasoning. However, it is widely estimated in a nation-wide econometric model.

its role in a linear production function can be measured by indirectly estimating β_2. When an estimator of γ_3 is a maximum likelihood estimator, the estimator of β_2 obtained from the relation designated by (10-4) is also a maximum likelihood estimator.

The use of Equation (9), however, poses a number of theoretical issues: First, the production function has to be a priori linear function. Assumption of linearity in a production relation seems to be sometimes unrealistic when we consider the fact that a linear production function assumes infinite elasticity of substitution between labor and capital. If linearity assumption is not made, we know that Koyck transformation cannot be applied and, therefore, the distributed lag model cannot be arranged in a neat function as Equation (9).

Second, in the process of Koyck transformation, the stochastic error term is specified as shown by Equation (10-6). Unless the original error term u_t is autocorrelated based on a Markov first-order scheme, i.e.,

$$u_t = u_{t-1} + \epsilon_t \tag{11}$$

application of ordinary least square method in estimating γ's does not guarantee efficiency of estimators. However, when the original error term is autocorrelated as Equation (11), ordinary least square estimators themselves are generalized least square estimators, and the efficiency problem is automatically solved.

Third, since the system shown by Equation (10-1) through (10-5) does not generate a unique set of solutions for λ, estimators may be obtained either by Equation (10-5) and Equation (10-3). When the two estimators of λ are equal to each other, there is no problem related to λ estimation. However, there is hardly any theoretical guarantee that the two λ estimators are necessarily the same. In addition, the system does not give solution for β_0 and K_0. When technological progress is treated in the production function, the linear version of the function of the type is almost useless unless some exquisite alteration is made.

2.2. Initial Stock Estimation Method

With obvious restrictions of the method elucidated in the above 2.1., one has to design an alternative method to estimate capital stock variables. Since the official statistics for depreciation and investment are published in Korea, again one has to utilize them in order to estimate the capital stock series. Let us construct a simultaneous equation as

$$D_0 = \lambda\left(K_0 + \frac{I_0}{2}\right) \tag{12-1}$$

$$D_1 = \lambda\left[(1-\lambda)\ \left(K_0 + \frac{1}{2} I_0\right) + \frac{1}{2} I_1\right] \tag{12-2}$$

where the subscript 0 denotes the first year in which official statistics are available in Korea. The first equation (12-1) simply expresses that the capital depreciation in the year 0, denoted by D_0 is a fraction of the capital stock of the year 0, plus one half of the gross fixed capital investment for the year since, as shown by Equation(4), the average increment of capital stock during the year is one half of the gross investment for the year. The second equation (12-2) explains the same relations as does the first equation, the only difference being that all figures are for the year 1. Since we have statistics for D and I, we have two unknowns, λ and K_0 which can be solved via these two equations. A simple derivation results in Equation(13) as

$$K_0=\frac{D_0I_1-2D_0^2}{2(D_1-D_0)}-\frac{1}{2}I_0 \tag{13}$$

Once the intial capital stock is estimated, the time-series of the capital stock or the consecutive years can be estimated by Equation (2)[3]. Estimated series are shown in the appendix. The capital stock time-series estimator of Equation (13) has to be investigated for their stochastic natures.

For the convenience of expanding and deriving formulas to be used in the investigation, let us redefine the variables used in Equation (13). Let

$$\begin{pmatrix} I_1 \\ D_1 \\ D_2 \end{pmatrix}=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} \chi_1+u_1 \\ \chi_2+u_2 \\ \chi_3+u_3 \end{pmatrix} \tag{14}$$

Where χ_i is the true value of x_i and u_i observation errors with the following properties:

$$E\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{15}$$

$$\operatorname{cov}(u)=\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \tag{15}$$

Equation (14) and (15) indicate that the mean values of u_i are zero and u_i is correlated among themselves. It is quite plausible that observation errors of investment and depreciation variables are correlated since the variables are functions of each others. If the

3 Irma Adelman and M. J. Kim assumed in their model that K_0 in Korea (capital stock at the beginning of 1953) is equal to zero. See I. Adelman and M. J. Kim (1969).

variables did not contain any observation errors the estimated capital K_0 would be also a true value without an error as

$$K_0 = K_0 \frac{\chi_1\chi_2 - 2\chi_2^2}{2(\chi_2 - \chi_3)} - \frac{\chi_1}{2} \tag{16}$$

The estimated K_0 is unbiased when the covariance σ_{ij} is equal to zero ($i \neq j$) And the mean value of the K_0 is obtained as follows:

$$\begin{aligned}
E(K_0) &= E[g(x_1, x_2, x_3)] \\
&\approx E[g(\chi_1, \chi_2, \chi_3)] + \frac{\partial g}{\partial x_1}\Big|_\chi (x_1 - \chi_1) \\
&+ \frac{\partial g}{\partial x_2}\Big|_\chi (x_2 - \chi_2) + \frac{\partial g}{\partial x_3}\Big|_\chi (x_3 - \chi_3) + \frac{\partial^2 g}{\partial x_1 \partial x_2}\Big|_\chi \frac{(x_1 - \chi_1)(x_2 - \chi_2)}{2!} \\
&+ \frac{\partial^2 g}{\partial x_1 \partial x_3}\Big|_\chi \frac{(x_1 - \chi_1)(x_3 - \chi_3)}{2!} + \frac{\partial^2 g}{\partial x_2 \partial x_3}\Big|_\chi \frac{(x_1 - \chi_1)(x_3 - \chi_4)}{2!} \\
&= g(\chi_1, \chi_2, \chi_3) + \frac{1}{2}\mathrm{cov}(x_1, x_2)\frac{\partial^2 g}{\partial x_1 \partial x_2}\Big|_\chi \\
&+ \frac{1}{2}\mathrm{cov}(x_1, x_3)\frac{\partial^2 g}{\partial x_1 \partial x_3}\Big|_\chi + \frac{1}{2}\mathrm{cov}(x_2, x_3)\frac{\partial^2 g}{\partial x_2 \partial x_3}\Big|_\chi \\
&= \frac{\chi_1\chi_2 - 2\chi_2^2}{2(\chi_2 - \chi_3)} - \frac{\chi_1}{2} - \frac{1}{2}\sigma_{12}\frac{\chi_3}{2(\chi_2 - \chi_3)^2} \\
&- \frac{1}{2}\sigma_{13}\frac{\chi_2}{2(\chi_2 - \chi_3)^2} - \frac{1}{2}\sigma_{23}\frac{\chi_2(\chi_2 + \chi_3) + 4\chi_2\chi_3}{2(\chi_2 - \chi_3)^2}
\end{aligned} \tag{18}$$

In the same manner, the variance of the estimator is obtained as:

$$\begin{aligned}
&\mathrm{var}\ [g(x_1, x_2, x_3)] \\
&\approx \mathrm{var}[x_1]\left\{\frac{\partial g}{\partial x_1}\Big|_\chi\right\}^2 + \mathrm{var}[x_2]\left\{\frac{\partial g}{\partial x_2}\Big|_\chi\right\}^2 \\
&+ \mathrm{var}\ [x_3]\ \left\{\frac{\partial g}{\partial x_3}\Big|_\chi\right\}^2 + 2\mathrm{cov}\ [x_1, x_2]\ \left\{\frac{\partial g}{\partial x_1}\Big|_\chi \cdot \frac{\partial g}{\partial x_2}\Big|_\chi\right\} \\
&+ 2\mathrm{cov}\ [x_1, x_3]\ \left\{\frac{\partial g}{\partial x_1}\Big|_\chi \cdot \frac{\partial g}{\partial x_3}\Big|_\chi\right\} + 2\mathrm{cov}\ [x_1, x_3]\ \left\{\frac{\partial g}{\partial x_1}\Big|_\chi \cdot \frac{\partial g}{\partial x_3}\Big|_\chi\right\} \\
&= \sigma_{11}\left[\frac{\chi_2}{2(\chi_2 - \chi_3)} - \frac{1}{2}\right]^2 + \sigma_{22}\left[\frac{-\chi_1\chi_3}{2(\chi_2 - \chi_3)^2}\right]^2 \\
&+ \sigma_{33}\left[\frac{\chi_1\chi_2}{2(\chi_2 - \chi_3)^2} - \frac{\chi_2^2}{(\chi_2 - \chi_3)^2}\right]^2 \\
&+ 2\sigma_{12}\left[\frac{\chi_2}{2(\chi_2 - \chi_3)^2} - \frac{1}{2}\right]\left[\frac{-\chi_1\chi_3}{2(\chi_2 - \chi_3)^2}\right] \\
&+ 2\sigma_{13}\left[\frac{\chi_2}{2(\chi_2 - \chi_3)} - \frac{1}{2}\right]\left[\frac{\chi_1\chi_2}{2(\chi_2 - \chi_3)^2} - \frac{\chi_2^2}{(\chi_2 - \chi_3)^2}\right] \\
&+ 2\sigma_{23}\ \left[\frac{-\chi_1\chi_3}{2(\chi_2 - \chi_3)}\left(\frac{\chi_1\chi_2}{2(\chi_2 - \chi_3)^2} - \frac{\chi_2^2}{(\chi_2 - \chi_3)^2}\right)\right]
\end{aligned}$$

19)

As it is shown by Equations (18) and (19), in obtaining the mean and variance of K_0, approximation by Taylor expansion method is used.[4] K_0 is necessarily a baised estimator if the error terms are correlated. Efficiency of the estimator is not considered here, but the approximated variance seems to be very large. Again, when $\sigma_{ij}(i \neq j)$ is assumed to be equal to zero, the approximated variance reduces to the first three terms of the right-hand side of Equation (19).

Even though it is complicated to derive the distribution of K_0, [5] in estimating a confidence interval or hypothesis test with regard to K_0, Chebyschev inequality may be used.

3. Some Further Issues

Even though the deriving procedures of the above two methods seem to be somewhat tedius and sophisticated, the obtained results may be conveniently used in calculating estimates of capital stock or other proxy variables. The second method of estimating initial capital stock, if not used directly as operational capital stock, may be utilized to check consistency of directly measured capital series from a census survey.

In addition to the theoretical issues directly concerned with the indirect estimation methods mentioned above, there are some further practical issues to be considered. First, the capital stock series obtained by the second method are an aggregate concept. Most of econometric models are, however, composed by a number of production functions instead of one aggregate function. In order to explain sectoral production behavior, one has to establish sectoral production functions with variables such as sectoral capital input, sectoral labor input, etc. Since the official national income statistics do not have sectoral depreciation rate, the sectoral capital stock variables are not estimated. Various assumptions may be made to solve the issue. However, the more number of assumptions are made, the greater will be estimation error.

Second, since 1962 when the First Five-year Economic Plan was initiated, foreign capital, either in forms of loan or joint venture, has been very actively introduced in Korea. Fixed capital formation made so far in Korea composes a large portion of imported foreign capital. Homogeneity of capital stocks from the two different sources must be tested.[6] If homogeneity hypothesis were rejected statistically, the capital stock originated from foreign capital investment should be se-

4 See A. Mood, F. Graybill and D. Boes (1974), pp. 180-181.
5 Existence of a distribution has not been investigated.
6 See S. B. Yoon (1971).

gregated from the domestic capital stock. The estimation of foreign capital stock raises various unexpected problems, and a production function has to be respecified with explicit distinction of foreign capital from domestic capital.

Third, making of a time-series of capital stock variables becomes a more complicated problem when technological change is hypothesized as varying with vintage of the capital investment. Not only physical depreciation has to be considered, but also obsolescense should be explicitly treated along the different vintage of capital stock, Hence, the indirect estimation of capital with all these consideration is practically hardly possible.

Appendix

Investment, Depreciation and Estimated Capital Stock
(in billion 1965 constant Won)

Year	I_t	D_t	K_t^*
1953	69.56	20.05	16.26
1954	57.87	23.54	30.98
1955	61.34	22.49	49.10
1956	57.29	22.91	75.59
1957	87.91	24.26	105.09
1958	77.72	26.60	142.14
1959	57.83	30.07	173.33
1960	62.84	29.44	202.55
1961	72.95	29.44	234.82
1962	77.99	34.60	270.58
1963	137.27	37.39	320.03
1964	114.41	39.77	388.59
1965	118.48	45.73	442.15
1966	207.38	51.25	514.06
1967	241.72	58.64	653.44
1968	344.12	66.56	826.89
1969	451.47	82.33	1065.80
1970	455.58	84.24	1434.94

Source: Bank of Korea, *Economic Statistics of Korea* 1969-1972, Seoul.
* Estimations.

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